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HYDRODYNAMICAL STUDIES ON THE ST. LAWRENCE RIVER

Gabriel Godin

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ABSTRACT

Use has been made of the fundamental concepts of hydrodynamics to study the steady and tidal flows in the St. Lawrence River.

The Equations of Hydrodynamics take on a very simple form for steady flow as it is the case between Montréal and Trois Rivières. The definition and the meaning of the quantities they relate such as the mean depth D, the inclination of the river bed I, of the surface S_x, have been carefully reviewed. The mean inclination of the surface of the river S_x, which is always positive, is found to be remarkably constant from year to year in spite of the variations occurring in the discharge. Using this fact it is possible to deduce a value of the Chézy coefficient between Montréal and Sorel.

A plot of the mean level for all the stations located between King Edward Pier in Montréal and Pointe au Père shows the mean downwards slope of the surface of the river when undisturbed by the tide. The monthly fluctuations of this mean level decrease downstream since the capacity of the river bed increases in this direction. The mean flow is found to be subcritical and therefore cannot impede the propagation of the tide upstream.

The propagation of the tide may be understood using the time dependent form of the Equations of Hydrodynamics. An elementary solution of these equations shows that under the influence of linearized friction, the tide is damped exponentially and is retarded in its progress upstream. The actual occurrence of quadratic friction in the equations implies the creation of fast and slow shallow water constituents along the river; the speed of progress of the tide upstream is ascertained from observations. The use of the current observations in conjunction with the time dependent equations of motion allows the deduction of the value of the Chézy coefficient which prevails in the part of the river which extends between Trois-Rivières and Québec.

The spectral analysis of the water levels recorded by the water level gauges reveals that the low frequency band contains oscillation of 15, 9, 7 and 5 days which may be traced to the interaction of the tidal constituents. The change in phase and amplitude of the major tidal constituents deduced from the same analysis has been plotted and supplies the material which has to be reproduced in a mathematical model of the river. The current observations carried out by G.C. Dohler and the Ship Channel Division have been analyzed as well, whenever possible. The suggestion by W.D. Forrester that the average one dimensional currents in a river could be indirectly deduced from the readings on the tide gauges and the equation of continuity rather than by direct observations has been confirmed by our analyses.

A one dimensional numerical model of the St. Lawrence River has been elaborated. In it, the contribution of the tributaries, especially the Saguenay and the bifurcation of the river bed at Isle d'Orléans have been neglected. Still the model reproduces adequately the main features of the major tidal constituents; it reproduces qualitatively but not quantitatively, the shallow water constituents. It indicates that the fluctuations in the discharge are felt only upstream of the Québec constriction. The amplitude of the tide is little affected but its progress may be retarded by up to 3 hours. The slow shallow water constituents are increased when the discharge is increased but in the range of the value of the friction considered ($C=58m^{1/2}/sec$ to $C=52m^{1/2}/sec$), the amplitude of the fast and slow shallow water constituents is decreased when friction is increased. Finally the model indicates that blockage anywhere in the river automatically involves an enhancement of the tide downstream.

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THE FUNDAMENTAL PARAMETERS OF A RIVER

The regime of a river may be characterized by the following quantities:

1) Q: the discharge. The amount of water which flows through a vertical cross section of the channel per unit time. This quantity is not easy to measure directly but it may be estimated by integrating the values of the velocity of the current measured at various points of the cross section. A change in discharge will cause a change in the depth of water, larger discharges normally implying greater depths at a given point (as seen in paragraph 4).

Units: meters^3/second.

2) D: the depth of water. D varies at every point of the river depending on the bottom configuration and it normally varies with the discharge. If a portion of the river channel is schematized in such a way that a vertical cross section is a rectangle of width B and depth D, the mean depth D acquires a precise meaning.

Units: meters.

3) I: the inclination of the bed of the river. The water can flow in a river only if the bed of the river slopes downward and if it is remote from a large body of water with which it would be in hydrostatic equilibrium. The inclination of a river bed is not at all easy to visualize from ordinary chart soundings and it is necessary to schematize the river into channels of rectangular cross section in order to get some idea of it.

Units: dimensionless.

4) B: the width of the river. B happens to be an easy quantity to visualize but if marshes, reeds and islands are scattered near the shores, the river may "feel" a width which is quite different from the width which the eye can see.

Units: meters.

5) The friction. Friction is an invisible quantity but it is the major factor controlling the flow of a river. Without it a river would always be dry as the water would run very rapidly towards the ocean. With friction present, there develops an equilibrium between the urge of the water to tumble downstream and the force of friction holding it back. Under steady conditions, a constant velocity of outflow develops and the river surface will be inclined at an angle with respect to the horizontal.

Units of the Chézy friction coefficient: meters$^{\frac{1}{2}}$/second.

STEADY STATE

When nothing changes in time, although the conditions may change from point to point, we wish to study the relations between the various quantities which we have just described and which are descriptive of a river. To measure them we have water level gauges and current meters. Gauges are not difficult to install and they can operate unattended; they give us the elevation of the water surface above a reference level. This quantity seems to satisfy the hydrographers but it does not suffice for our purpose as we are really in need of the quantity D, the actual depth of water. Current meters are more difficult to handle than gauges and cannot be left at one place

indefinitely. Used judiciously they can give us an idea of the value of the discharge Q at a section of the river, a quantity which is fundamental in our investigations.

Gauges strung along the river, if they are all calibrated to the same reference level, will show the inclination of the surface of the river as it flows downstream. In the portion of the river where the tide makes itself felt they will show the distortion of the water surface as the tidal wave moves in and out. In conjunction with hydrographic charts which give mean soundings undisturbed by the tide or by the irregularities in the discharge, we may compute mean values of the depth D over the river.

In the upstream portion of a river, steady state conditions do not quite prevail. The discharge varies from day to day, ice jams form in the spring which distort the profile drastically for short intervals of time. As a consequence the depth D and the inclination D vary from day to day. There arises the need to average out the fluctuations. The mean values of Q, D, I and B may be considered as characteristic of the particular river under study.

1. The Equations of Hydrodynamics for steady state

For time independent motion, the quantities which we have just discussed are related through the equations (Stoker: Water Waves, p. 456; Dronkers: Tidal Computations, p. 158):

$$Q=Q_o \quad \text{a constant} \qquad (1)$$

$$I - D_x - Q_o^2/C^2D^3B^2 = 0 \qquad (2)$$

where

Q = the discharge in m^3/sec

g = the acceleration due to gravity, 9.80 m/sec^2

I = the inclination of the schematized bed of the river

D = the mean depth of the schematized portion of the channel, in meters

D_x = the rate of change looking downstream of the mean depth of water

C = the Chézy coefficient of friction in $m^{\frac{1}{2}}$/sec

B = the effective width of the river in meters

x = the distance along the river measured positive downstream, in meters

(2) contains an extra convective term $(Q_o^2/gBD)\ (1/BD)_x$ which is usually small compared to the other quantities. x used as an index indicates differentiation with respect to x.

We assume the absence of tributaries in (1) and (2). Equation (1) simply states that the discharge throughout any vertical section of the river is the same everywhere if steady state prevails. This implies in turn that D cannot change locally although it does change along the river as (2) indicates. Essentially, all the information which (2) supplies is that:

$$D_x = I - Q_o^2/C^2B^2D^3 \qquad (3)$$

which tells us that the local rate of change of the depth of water depends on Q_o, on the friction coefficient C and on the width B. Strictly speaking, B and D vary for each value of Q_o and in the tidal portions of the river, vary at each instant of time. However, these quantities oscillate around mean values which are shown in Figs. 24 and 25 and which may be considered as representative of the St. Lawrence River. We had to plot the mean width B on a vertical logarithmic scale because it varies so immensely: from 800 meters near Deschambault to 52,000 meters at Matane. The depth D decreases more or less steadily from 250 meters at Pointe des Monts to 3 meters in Lake St. Peter.

The quantity which we know the least about in (3) is really not D_x which we may calculate indirectly from the readings on tide gauges and hydrographic charts but C the Chézy coefficient of friction. We may therefore actually use (3) to obtain estimates of the Chézy coefficient for the upstream portion of the river where steady state conditions prevail on the average. However, before we succeed in this task, we have to look closer on how we may go about measuring D and D_x from schematized sections of the river, gauge readings and hydrographic soundings and how we can succeed in obtaining some estimates of the inclination of the river bed I.

2. The profile of the river

Gauges strung along a river actually measure h_x, the inclination of the water surface with reference to a horizontal level, the International Great Lakes Datum (IGLD) in the case of

the St. Lawrence River, rather than the rate of change of the mean depth with distance D_x.

Fig. 1 may help to visualize the various quantities involved. This figure refers to a hypothetical small portion of a river made up of many elementary sections of length Δx. The depth has been averaged across the river and over the elements of length Δx. This is what we call the depth D. The dark solid line indicates the actual surface of the river, the hatched line, the bottom of the river, assumedly sloping at a constant angle I downward. D is the depth of water at a given point. The gauge itself measures h, the depth of water above IGLD or its complement S. The dotted line indicates a constant depth D_o which is a possible regime of flow but which does not actually occur in the St. Lawrence River.

Using the diagram we see that we can write

$$D = z + h = z+h_o+\Delta$$

h_o would be the height read on the gauge if the depth were everywhere constant, Δ is the deviation from this constant depth. Differentiation with respect to x gives

$$D_x = z_x+h_{ox}+\Delta_x = I - I + \Delta_x = \Delta_x$$

Δ_x simply measures the deviation of D from a surface of constant depth. Therefore,

$$D_x = \Delta_x \qquad (4)$$

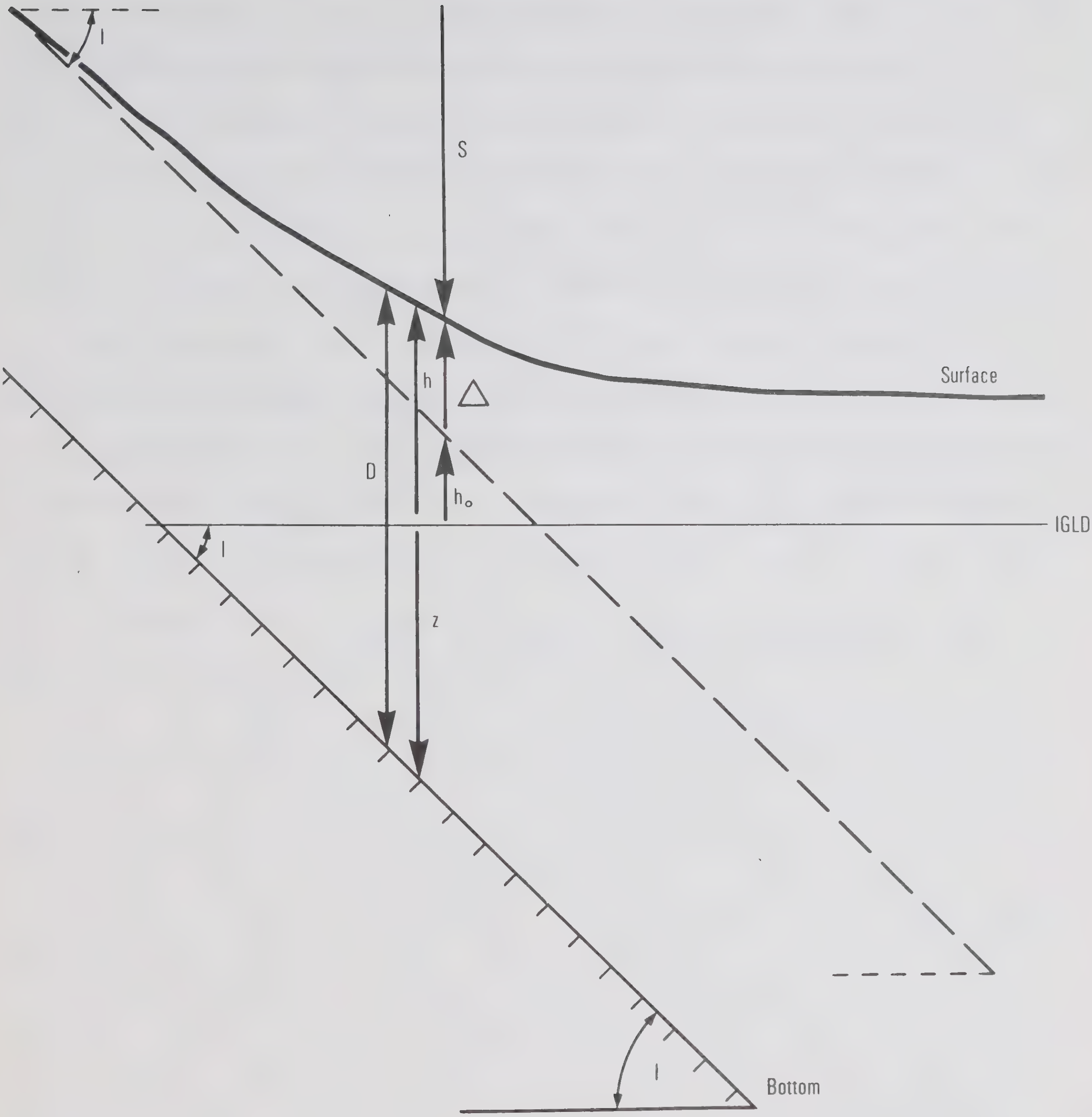


FIG.1 RIVER PROFILE

From a physical point of view we are more interested in h as read by the tide gauge or its complement S. Fig. 1 indicates

$$h_x = h_{ox} + \Delta_x = -I + \Delta_x = -I + D_x = -I+I-Q_o^2/C^2B_2D_3$$

or

$$h_x = -S_x = -Q_o^2/C^2B^2D^3 \qquad (5)$$

$Q_o^2/C^2B^2D^3$ does not change sign and is always a positive quantity; therefore, h_x is always negative downstream, i.e. the water surface must always slope downward in steady state conditions or equivalently, S_x is always positive.

3. The significance of the quantities D and I

Although we have used D and I in the previous paragraph, we have not yet given them a precise meaning: this is now our task before we proceed to use formula (5).

D is meant to be the depth of water in the river at a given section corresponding to a value of x; in fact it varies at every point across and along the river. Hydrographic charts show a fair collection of depth samples which have been reduced to an arbitrary datum. The depths shown are "safe" depths in the sense that the actual depth of water at any given time will usually exceed the depth shown on the map. Where there is a tide, the hydrographic depth will be the depth of water at one of the lowest low water; where there is no tide, the depth shown is related to the average water level. The first step in order

to obtain a manageable value of the mean depth D which can be utilized in our calculations is to divide the river bed into short sections of fairly uniform depth and width and average the hydrographic soundings over it. The gauge readings at a particular station like King Edward Pier, Frontenac and so on, give the height of the water surface above a given reference level at this particular location. It varies all the time and we have to take its average to compare it with the average depth obtained from the chart soundings for the channel around the station. For instance, the 10 year average at the gauge site of King Edward Pier gives 6.62 meters while the average depth of water in the corresponding channel turns out to be 6.19 meters. We establish the correspondence:

6.62 meters at King Edward implies a depth of water of 6.19 meters in the corresponding channel and so on.

In 1969, the mean annual level read on the gauge at King Edward was 6.86 meters. I surmise that the depth of water in the channel was 6.44 meters. In the regions where there is a tide, the average depth deduced from the soundings should be increased by the mean half range of the tide strictly speaking. In practice though, the actual mean soundings may be kept as a useful representative quantity since taking account of the change of depth during the tidal cycle introduces second order effects which it is not the purpose of this paper to investigate. We are more interested in having first a more general view of the basic mechanism present in the river.

The quantity I is even more difficult to extricate from the gauge readings and the soundings. At King Edward, the

10 year average gauge reading indicates that the surface of the water at this station stands at 6.62 meters above mean sea level (which we assume to coincide with IGLD). From the soundings, we had calculated a mean depth of 6.19 meters for the portion of the river bed enclosing King Edward; we deduce that the river bed stands at .43 meters above msl at this location. The Sorel gauge indicates a 10 year average of 4.69 meters while the average of the soundings in the vicinity gives 7.40 meters; we conclude that the river bed lies at 2.71 meters below msl. In this fashion we may obtain the mean inclination of the river bed between King Edward and Sorel. The distance between the two stations is 68,600 meters and therefore: (see Fig. 2):

$$I=3.14/68600 = 4.6\times10^{-5}$$

With the same information we deduce

$$-h_x=S_x=1.8/68600=2.6\times10^{-5}$$

We notice that the river bottom slopes more rapidly than the river surface in this portion of the river. In practice we are not too concerned with I since it cancels out of (5).

4. The inclination of the water surface

The gauge readings give us a direct indication of S_x, the inclination of the water surface. Such gauge readings are published by the Canadian Hydrographic Service under the Tides and Water Levels Section. We consider that under average

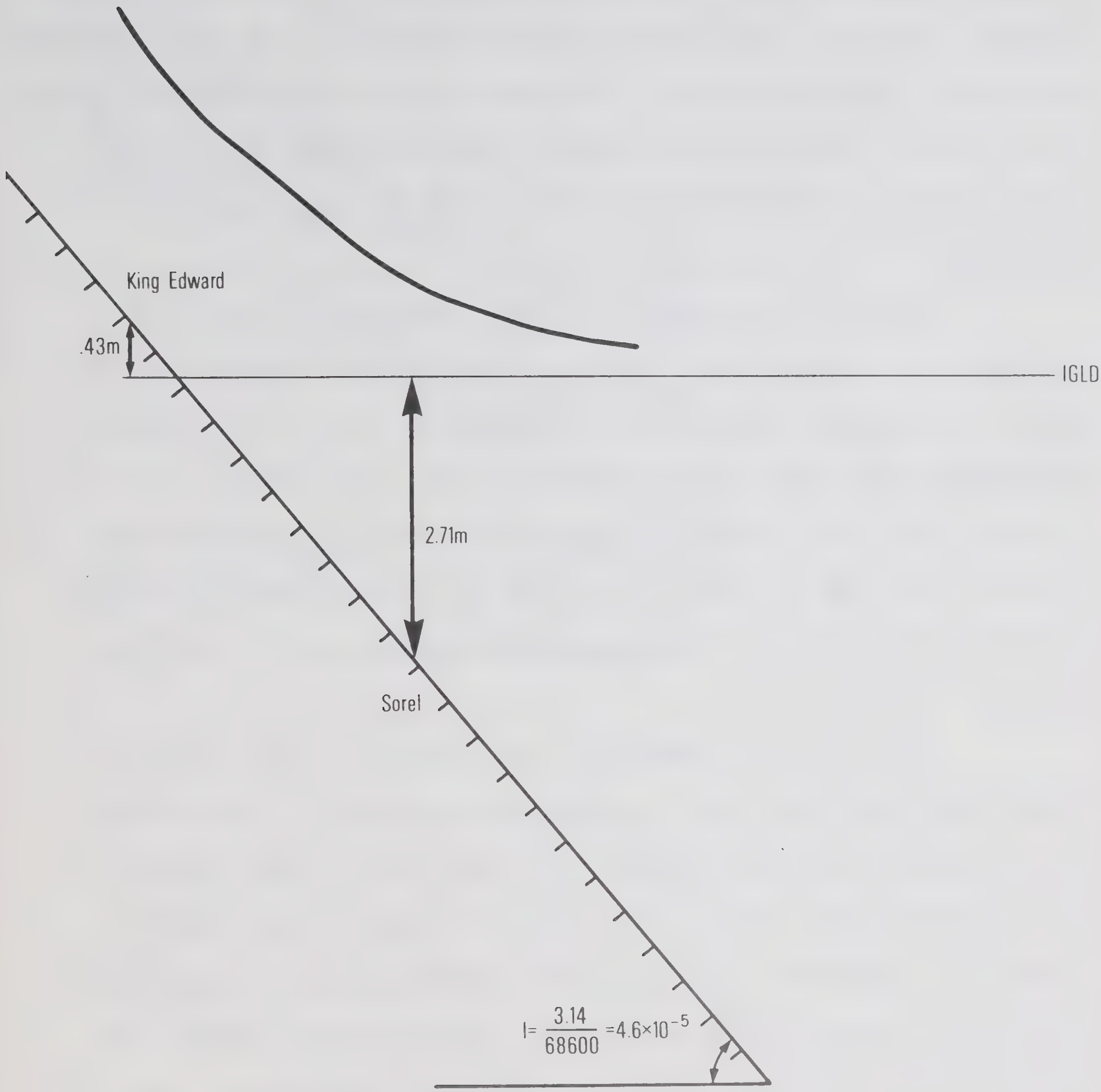


FIG.2 THE INCLINATION OF THE RIVER BED

conditions steady state prevails between Montréal and Sorel; therefore we calculate S_x over this portion of the river. Fig. 3 shows the mean profile S between the years 1962 to 1969; the rate of change of this profile with x gives S_x. We have entered with each curve the value of the mean yearly discharge Q_o; there is an adequate correlation between the discharge Q_o and the overall depth of water in the river, but it certainly is not absolute.

What is remarkable in Fig. 3 is that in spite of D varying quite appreciably with Q_o, the profile itself seems little affected. It differs from point to point like between King Edward and Sorel and between Lanoraie and Sorel, but it changes only very slightly from year to year. S_x oscillates between 2.48 and 2.74×10^{-5} for 1964 to 1969 between Frontenac and Lanoraie while D itself undergoes variation of over one meter.

If S_x is so remarkably uniform from year to year, (5) would indicate that Q_o^2/D^3 is nearly a constant. This may be ascertained by plotting Q_o vs $D^{3/2}$ for some of the stations. Fig. 4 shows this type of plot for Frontenac, King Edward, Lavaltrie, Lanoraie and Sorel from observations on the annual mean taken between 1962 and 1969. The horizontal scale for Lavaltrie is different and this has been indicated on the diagram. This fact in turn indicates that the depth varies directly with the discharge according to the relation:

$$D \propto Q_o^{2/3}$$

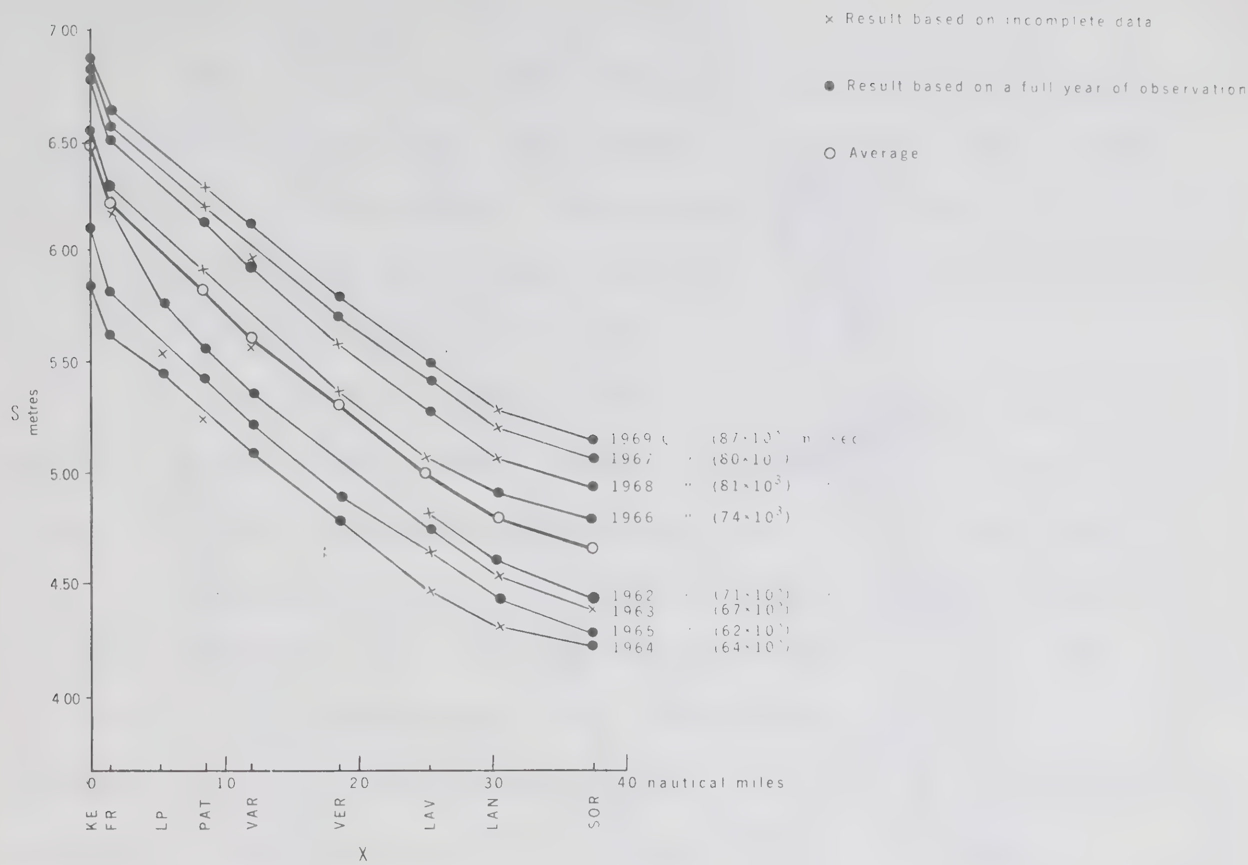


FIG 3 SLOPE OF THE WATER SURFACE S_X

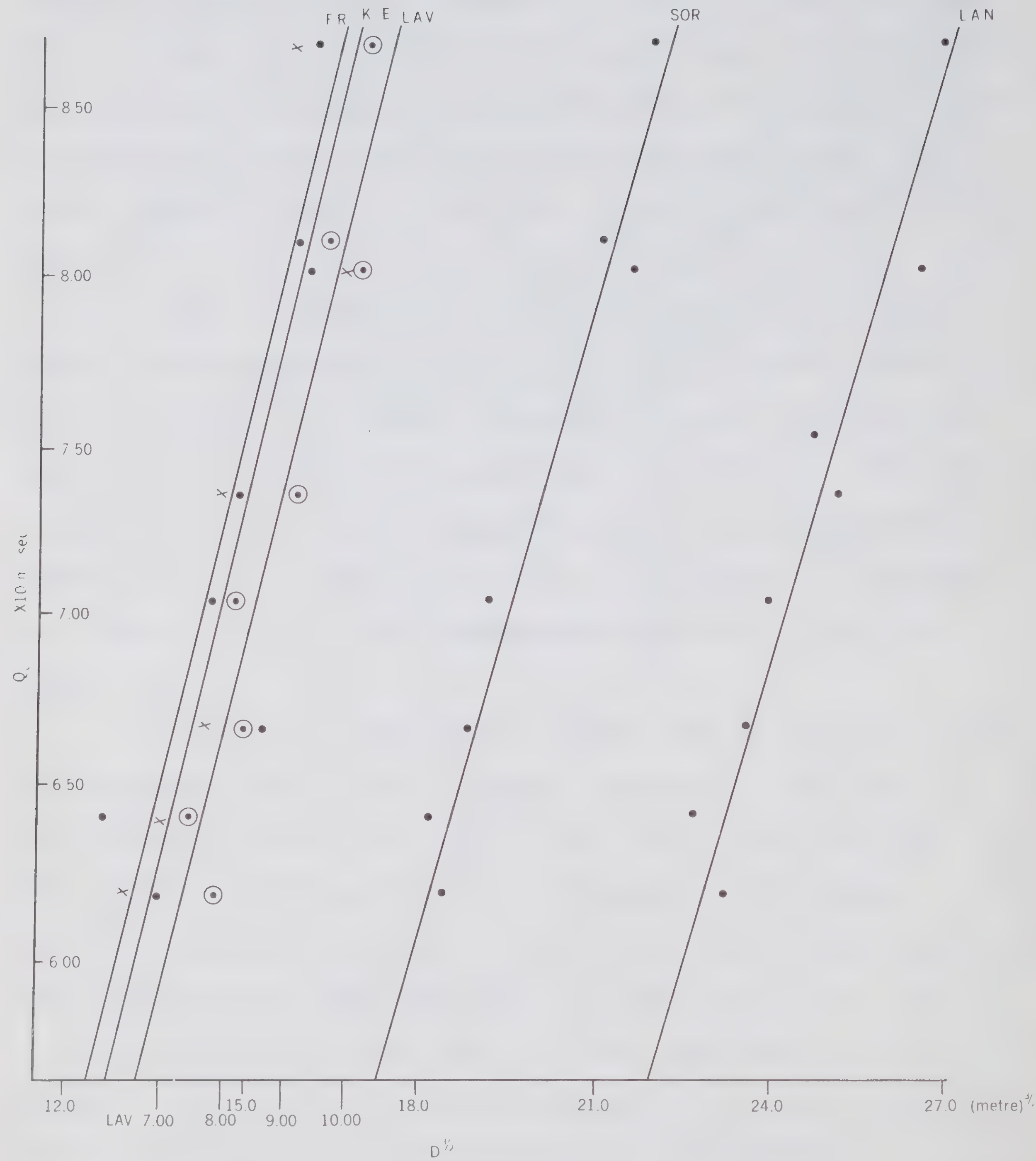


FIG. 4 PLOT OF Q_0 VERSUS $D^{3/2}$

5. The mean level at various stations

The gauges also indicate directly the water levels at the points where they are located. The mean value of the levels has a climatological value and Fig. 5 shows this mean level on a monthly and yearly basis for a 10 year average at the principal stations strung along the river.

The fluctuations in mean levels decrease gradually downstream till they become nearly imperceptible at Pointe au Père. At King Edward and Frontenac, there is a large drop in level in January and February following the freeing of the channel from the ice. The next feature common to all the curves is the increase in level in March and April created by the melting of the snow cover. These variations in level are maximum between the mouth of Lake St. Peter and Québec. Afterwards, the level drops uniformly to reach a minimum. value in September when they start increasing on account of the increased precipitation.

Figs. 6a and 6b show the deviation from the mean yearly level of the monthly levels. As expected these deviations enclose each other like an envelope since the bed of the river expands downstream and is increasingly less affected by the fluctuation in the discharge. The only exception to the rule is Trois Rivières during April which exhibits then larger deviations from mean level than the stations on the other side of Lake St. Peter such as Sorel and Lavaltrie.

The mean level may deviate by as much as 1.3 meters in Montréal while at Québec its deviation seldom exceeds 20

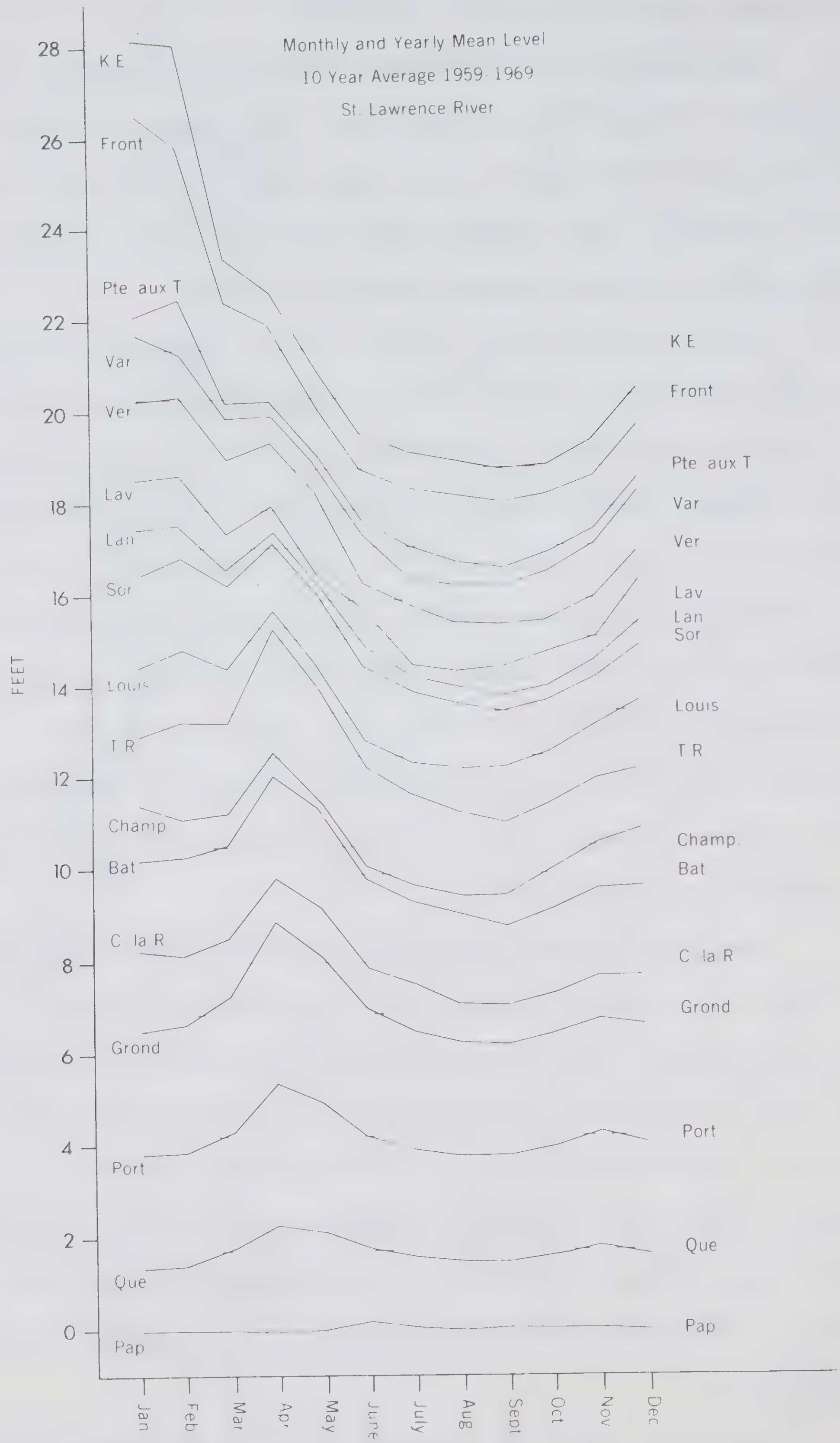

Monthly and Yearly Mean Level
10 Year Average 1959-1969
St. Lawrence River
FEET
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
K E
Front
Pte aux T
Var
Ver
Lav
Lan
Sor
Louis
T R
Champ
Bat
C la R
Grond
Port
Que
Pap
Champ.
Jan
Feb
Mar
Apr
May
June
July
Aug
Sept
Oct
Nov
Dec

Figure 5

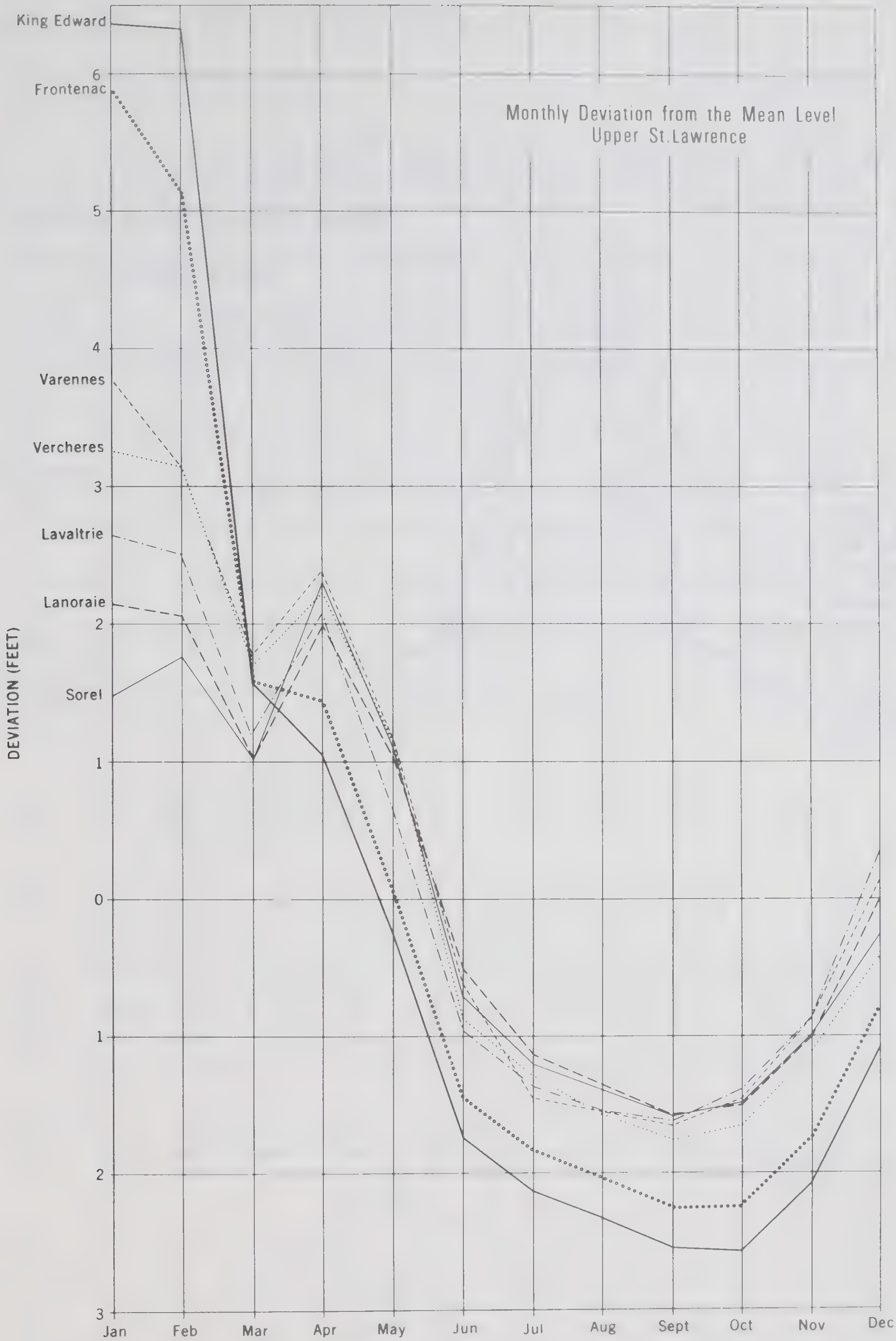

Monthly Deviation from the Mean Level
Upper St.Lawrence
King Edward
Frontenac
Varennes
Vercheres
Lavaltrie
Lanoraie
Sorel
DEVIATION (FEET)
6
5
4
3
2
1
0
1
2
3
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sept
Oct
Nov
Dec

FIG. 6a

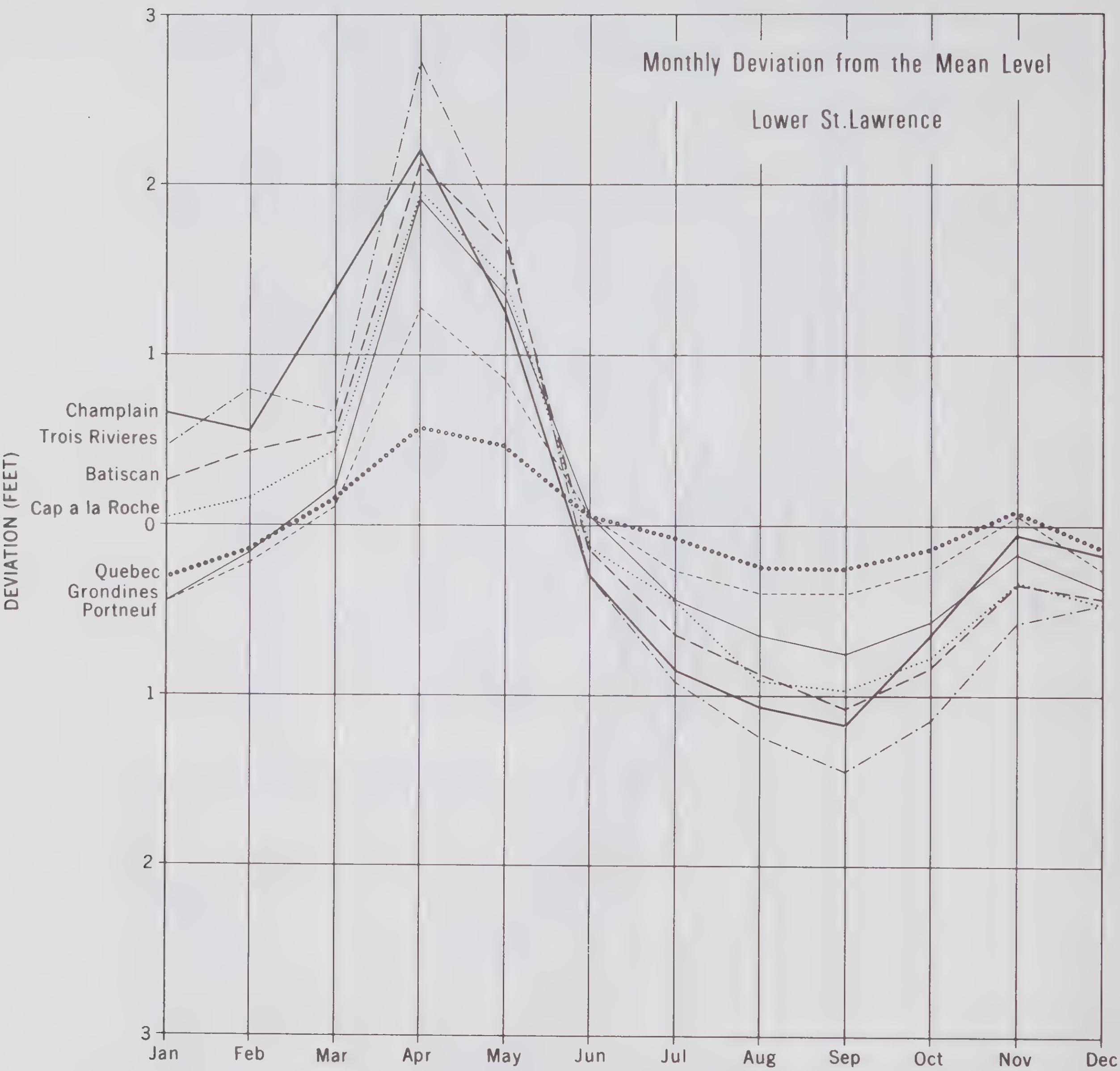

Monthly Deviation from the Mean Level
Lower St.Lawrence
DEVIATION (FEET)
Champlain
Trois Rivieres
Batiscan
Cap a la Roche
Quebec
Grondines
Portneuf
3
2
1
0
1
2
3
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec

FIG. 6b

centimeters. Ice free conditions in Montréal should reduce the fluctuations in level appreciably.

6. The Chézy coefficient

The term $Q_o^2/C^2B^2D^3$ is meant to represent the effect of friction; it does so but very imperfectly. It is adequate for the purpose of our investigations. It depends fundamentally on the value of C, the Chézy coefficient. There exist theoretical estimates for this parameter (Dronkers: Tidal Computations p. 157) but in practice it has to be determined from observations. (5) may be utilized for this purpose since S_x and B are known and since we have established that $Q_o/D^{3/2}$ is approximately a constant of value 483 $m^{3/2}$/sec according to Fig. 4. Between Montréal and Sorel, $S_x \sim 2.6 \times 10^{-5}$ and the average width of the river is 1850 meters so that

$$C \sim 53 \ m^{1/2}/sec$$

7. The rate of change of depth with distance, D_x

According to (3), D_x, the change of depth of water as one moves downstream may be positive, negative or zero depending on the relative magnitude of the two quantities on the right hand side.

$D_x<0$ over a longish stretch of the river would imply that the depth of water would eventually reach zero, a situation which definitely does not occur in the St. Lawrence River.

$D_x>0$ implies that D increases steadily downstream so that in turn the friction term on the right hand side diminishes

steadily and the $D_x \rightarrow I$ for large x which means that the surface of the water eventually becomes horizontal. An inspection of Fig. 1 shows that this is what actually occurs in most rivers and in the St. Lawrence in particular.

The special case $D_x=0$ corresponds to the situation considered by Chézy in which the depth of water is constant throughout the channel. Experiments seem to indicate that such a state of flow is unstable. (Dodge and Thompson: Fluid Mechanics. p. 247). This special case is illustrated by the dotted line in Fig. 1. In this particular instance $S_x=I$ and therefore

$$S_x=Q_o^2/C^2B^2D^3$$

which in turn implies

$$S_x^{\frac{1}{2}} \alpha\, Q_o$$

I did plot $S_x^{\frac{1}{2}}$ vs Q_o in the upper reaches of the St. Lawrence; I could find no correlation at all between $S_x^{\frac{1}{2}}$ and Q_o. It is therefore unlikely that $D_x=0$ holds anywhere in that area.

8. The convective term and the critical velocity

We have dismissed the convective term in (2) as being negligible in the St. Lawrence. We may now verify this.

Throughout the paper we schematize the river bed as having a rectangular section of width B and depth D. We may then write

$$Q_o = BDu_o$$

u_o being the mean velocity of the current along the channel under consideration. The convective term becomes

$$u_o^2 D_x / gD$$

assuming that only the depth varies downstream. We had noted that between Montréal and Sorel

$$I \sim 4.6\text{x}10^{-5} \qquad S_x \sim 2.6\text{x}10^{-5} \qquad D_x = I - S_x \sim 2\text{x}10^{-5}$$

u_o nowhere exceeds 2 m/sec and $D \sim 6$ m. so that

$$u_o^2 / gD \sim 1/90$$

so that the convective term

$$u_o^2 D_x / gD \sim D_x / 90 << D_x, I$$

Similarly

$$Q_o^2 / C^2 B^2 D^3 = u_o^2 / C^2 D = (u_o^2 D_x / gD) . (g / D_x C^2)$$

Now $g/D_x C^2 \sim 181$ so that $Q_o^2/C^2B^2D^3 \sim 181$ times the convective term. The convective term cannot always be neglected. For larger velocities than those quoted for u_o it has to be retained

in equation (2). An analysis of the complete equation (2) including the convective term indicates that D_x steepens to infinity for

$$u_o=(gD)^{\frac{1}{2}}$$

which is known as the critical velocity. At this velocity a hydraulic jump may occur and waves dowstream are effectively prevented from moving upstream. The velocity of the currents in the St. Lawrence is well below the critical velocity; this is the reason why the tide can move so far into the river.

TIDAL MOTION

1. The tide in the St. Lawrence River

From Trois Rivières at the mouth of Lake St. Peter, onwards to the Gulf of St. Lawrence the river flow and the water level become increasingly affected by the tide.

First, the modifications are slight and are appreciable only over intervals of two weeks or more. Fig. 7 shows the changes in level during a month at Trois Rivières. The daily tidal fluctuations ride on the back of a more important semi-monthly oscillation. Downstream of Trois Rivières the diurnal and semidiurnal oscillations become predominant. The change in level is the shape of saw teeth when plotted on a time scale as can be seen for instance in the plot of ΔS in Fig. 12 with a steep and rapid increase in height followed by a more gradual lowering of the level till the cycle is repeated with varying

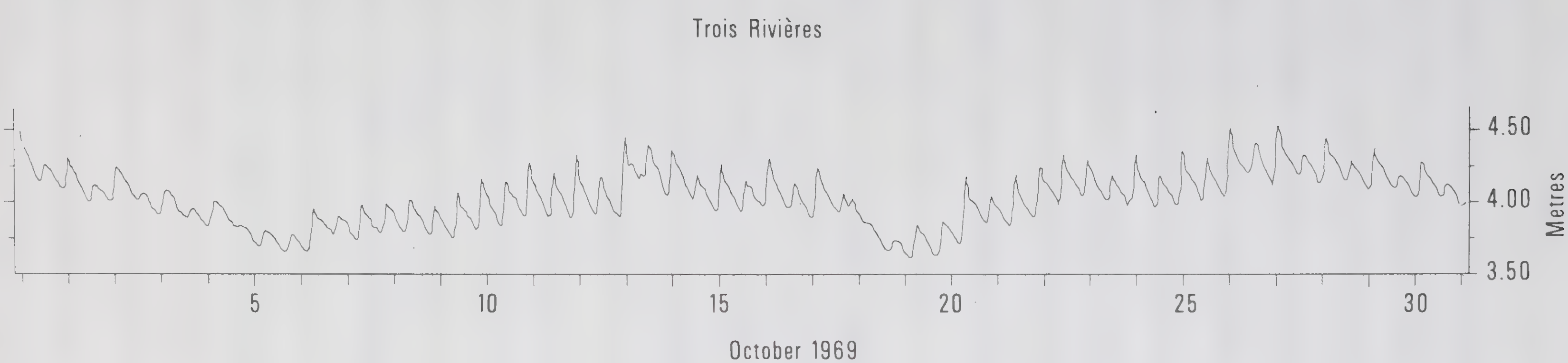

Trois Rivières
4.50
4.00
3.50
Metres
5
10
15
20
25
30
October 1969

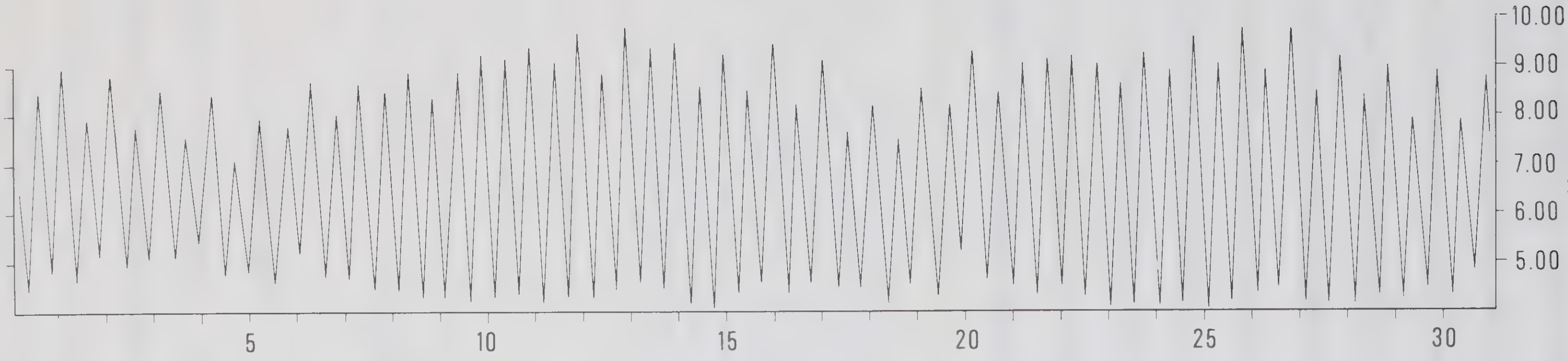
Fig. 7

intensity (upside down). Such a curve necessitates many harmonics to describe it.

The tide reaches its maximum amplitude in the vicinity of the Isle aux Couldres; downstream it decreases and takes a more symmetrical oceanic character. Figs. 8 and 9 show the change in level at Québec and Pointe au Père during one month. Québec is a station where the tide has a shallow water character although the tight time scale of the diagram does not allow to fully realize this fact (look instead at Fig. 12) while at Pointe au Père the tide has a smooth oceanic character.

The surface slope S_x studied in the previous section is no longer constant and it varies from instant to instant. In contrast to (4), it may become negative i.e. the water level is higher downstream than upstream, for instance before high water. The river flow Q as well, becomes an oscillating quantity and below a certain point, located in the vicinity of Grondines and Batiscan, the water may flow upstream during appreciable intervals of time. As the capacity of the river bed increases downstream, the steady flow Q_o contributed by the discharge of the river and its tributaries becomes imperceptible compared to the oscillating flow Q created by the tide. There is an intermediate zone though, which stretches between Trois Rivières and St. François (Isle d'Orléans) where the river discharge Q_o and the oscillating flow are of the same order of magnitude. In this area Q_o and Q will interact with each other through the influence of the friction represented by the Chézy term in the equations of motion. The tide will be damped by the friction of

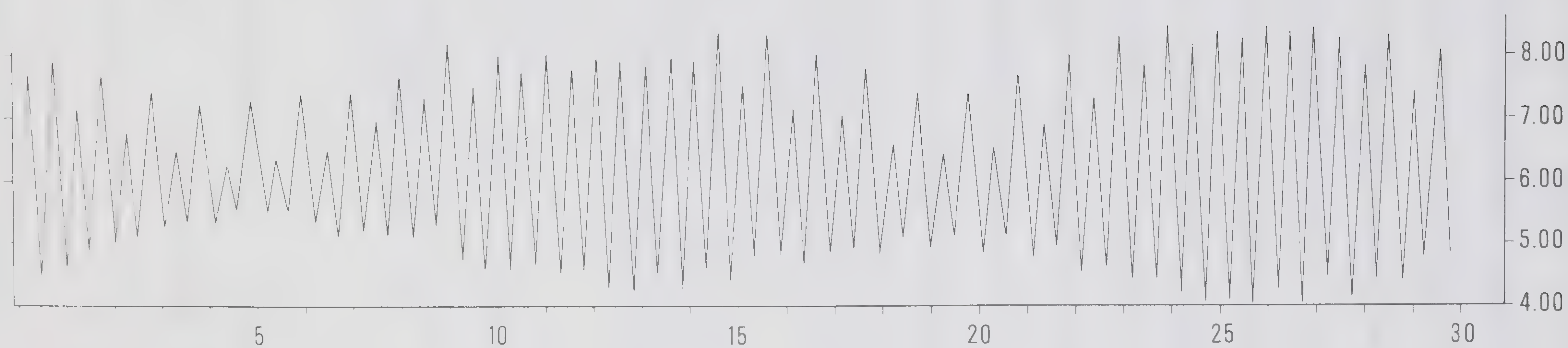
Québec



October 1969

Fig. 8

Pointe au Père



September 1969

Fig. 9

the river bed and its progress will be slowed down by the discharge. High and low water will move more and more slowly upstream and they rapidly decrease past the constriction of Québec City; the interval between low and high water will be shortened while the interval between high and low water will be increased. The interaction between the tidal constituents creates slower oscillations of a period of two weeks and one month which increase in amplitude upstream and eventually dominate the whole tidal motion in the vicinity of Trois Rivières. Eventually these slow oscillations and the more rapid diurnal and semidiurnal oscillations become extremely weak upstream of Lake St. Peter.

2. The Equations of Hydrodynamics for time dependent motion

A fair amount of the motion in the St. Lawrence where the tide is of importance may be described and understood with the help of the simplified Equations of Hydrodynamics:

$$Q_x + BD_t = 0 \tag{6}$$

$$I - D_x - \frac{Q|Q|}{C^2B^2D^3} = \frac{Q_t}{g\ BD} + \frac{2QQ_x}{g\ B^2D^2} \tag{7}$$

(Dronkers: Tidal Computations 196-197). The symbols have already been defined. t stands for time. Equations (6) and (7) reduce (1) and (2) when the time dependent terms are dropped.

Our first task is to investigate which of these terms may be evaluated from observations. The flow Q is the total

flow across a vertical section of the river; it embodies the river discharge Q_o and the oscillations due to the tide. The flow can only be measured directly with the help of current meters; this instrument measures the current at a given point. To obtain the flow Q from such current measurements it is necessary to investigate the horizontal and vertical distribution of the current in the vertical section which unfortunately differs for each stage of the tide and to do a space integration. A sensible technique for getting an idea of Q is to make a set of preliminary measurements at many points across the section at various depths and try to locate the point where the current is approximately equal to the average current going across the section. A self recording current meter can be installed there to study the time variation of Q from observations of the current at a single point of the section.

The width B and depth D can be deduced from the schematization of the river and readings of the water level gauges. The gradient in flow Q_x can only be found if values of the flow are measured at sections which are closely spaced, a costly undertaking. The equation of continuity (6) allows us to re-express Q_x in terms of D_t since

$$Q_x = -B\ D_t$$

Therefore, the useful form of the equation of motion is

$$S_x - \frac{Q|Q|}{C^2B^2D^3} = \frac{Q_t}{gBD} - \frac{2QD_t}{gBD^2} \qquad (8)$$

S_x can be inferred immediately from the differences in readings between the various tide gauges strung along the river.

2.1 The order of magnitude of the terms contained in the Equations of Hydrodynamics

Keeping in mind that all the quantities present in (8) vary at each instant of time, it is still desirable to compare the order of magnitude of the terms present in (8).

If we use the observations of the Ship Channel Division at Québec Bridge in 1968 we notice that the difference in level between Basile Low Light and Québec Bridge oscillates between + and - 40 cm over a distance of two nautical miles (3.7 km), so that

$$S_x \sim 40/3.7 \times 10^5 \sim 10^{-4}$$

To estimate the maximum values of the remaining terms we use the fact $Q = BDu_{mean}$

and

$C = 52\ m^{\frac{1}{2}}/sec$

$B = 1650\ m$

$D = 21\ m$

$u_{max} = 2 m/sec$

$u_{t_{max}} = 2 m/sec/2$ hours

$D_t = 3m/6$ hours

then

$$\frac{Q|Q|}{C^2 B^2 D^3} = \frac{u|u|}{C^2\ D} \sim 7 \times 10^{-5}$$

$Q_t/gBD=u_t/g \sim 10^{-5}$

$2QD_t/gBD^2 = 2uD_t/gD \sim 3x10^{-6}$

The terms are of the same order of magnitude with the exception of the convective term $2uD_t/gD$ which is smaller than the rest. The instantaneous values of the terms considered may be vastly different from the values quoted but here we have tried to estimate their maximum possible values.

2.2 An elementary solution

Equations (6) and (7) cannot be solved analytically; still there is little point in embarking into involved numerical computations before getting some insight into the physical situations they may describe. We may drop the convective term without more ado, now that we have convinced ourselves that it is relatively small. We will replace the troublesome friction term by a linearized term of the form rQ; this is a mathematical artifice which renders (6) and (7) tractable analytically although this excludes from our possible solutions, physical situations which we will have to face eventually such as the creation of shallow water constituents.

(6) and (7) become in their simplified form

$$Q_x + BD_t = 0 \tag{6a}$$

$$I-D_x-rQ = Q_t/gBD \tag{7a}$$

The D at the denominator of (7a) makes things unmanageable as well and we treat it as a constant depth D_o. This latter approximation is adequate in relatively deep water where D_x and D_t are small compared to the mean depth of the water column D_o; finally we drop I compared to D_x since retaining it increases the analytical manipulations without improving our physical understanding. We are now left with

$$Q_x+BD_t = 0 \qquad (6b)$$

$$D_x+rQ+Q_t/gBD_o = 0 \qquad (7b)$$

At this stage, the time and space dependence of D and Q can be treated as separate factors of the form X(x)T(t) and since we work with tides which are periodic oscillations, we write

$$T(t) \sim e^{i\sigma t}$$

We consider a channel or rectangular section of constant depth D_o with width B_o. The solution of (6b) and (7b) for such a channel are

$$D,Q \sim e^{\pm iKx}\, e^{i\sigma t}$$

where

$$K= \sqrt{i\sigma B_o(r+i\sigma/gBD_o)} = \sqrt[4]{\sigma^2 B_o^2 r^2+\sigma^4/g^2 D_o^2}\ \exp\,[i\tfrac{1}{2}\text{arc tan } B_o rgD_o/\sigma]$$

K has the form $K=\alpha+i\beta$ and $e^{\pm Kx}$ contains a factor $e^{\pm\beta x}$. β depends essentially on the friction r and it would vanish if r vanished. Linear friction automatically creates an exponential damping in the elevation and currents associated with the propagation of the tide (the exponential increase does not make physical sense). The α part of K may be interpreted as a wave number; without linear friction it would be $\sigma / (gD_o)^{\frac{1}{2}}$. The wave length would depend on the frequency σ and the depth D_o of the channel. With the addition of linear friction the wave length depends also on r; since $\sigma^2 B_o^2 r^2$ is essentially positive, the wave length is shortened with the occurrence of linear friction.

We could write out explicitly the solution to (6b) and (7b) using the basic relations we have just derived. There is little point in doing this as these solutions are given in innumerable textbooks (Dronker: Tidal Computations p. 225 ff. Defant: Physical Oceanography Vol. 2 p. 142 ff. Stoker: Water Waves pp. 37-54). All that we wish to retain is that linearized friction implies an exponential decay in a tidal wave progressing upstream and a shortening of its wave length.

The solution of (6b) and (7b) is possible for more complicated channels such as a channel whose width and depth vary exponentially, linearly or quadratically. Besides the analytical complications, the physical results are the same. Linear friction damps the amplitude and currents associated with the travelling tidal wave and there occurs a shortening of the wave length.

2.3 The creation of shallow water constituents and their selective damping up the river

We now have to look at the new features brought by the non-linear friction term. A frontal attack cannot do as this term makes (6) and (7) intractable, but one may get some information by using some devious means. We substantiate, with the help of a little bit of algebra, the two following statements:

1) the shallow water constituents are created by the interaction of the tidal constituents with each other and with the river discharge through the friction term $u|u|/C^2D$
2) The low frequency constituents can travel further up the river than the higher frequency constituents before being damped away.

From Trois Rivières to Montréal, the current is one directional and the tidal streams become quite small compared with the steady current due to the river discharge. In this part of the river, we may therefore write:

$$u|u|/C^2D=u^2/C^2D$$

If we consider only two tidal constituents along with the river discharge, we get:

$$u=u_o+u_1\cos(\sigma_1 t-\alpha_1)+u_2\cos(\sigma_2 t-\alpha_2) \qquad (9)$$

and we assume

$$u_1 << u_o \qquad u_2 << u_o$$

The friction term becomes explicitely:

$$u^2/C^2D = u_o^2 + \tfrac{1}{2}(u_1^2+u_2^2) + 2u_o u_1 \cos(\sigma_1 t-\alpha_1) + 2u_o u_2 \cos(\sigma_2 t-\alpha_2)$$

$$+\tfrac{1}{2}u_1^2 \cos 2(\sigma_1 t-\alpha_1) + \tfrac{1}{2}u_2^2 \cos 2(\sigma_2 t-\alpha_2)$$

$$+u_1 u_2 \, \{\cos[(\sigma_1+\sigma_2)t-(\alpha_1+\alpha_2)] + \cos\,[(\sigma_1-\sigma_2)t-(\alpha_1-\alpha_2)]\} \qquad (10)$$

Even taking into account of our very restrictive assumptions about the components of u, (9) still cannot be a solution to equation (8) if the friction term is given by (10). In an iteration process however, assuming that it will converge, we may consider (9) as a first order solution obtained without the friction term while an improved solution may be obtained by retaining the linear terms in (10) and by dropping the non-linear ones:

$$u_2/C^2D \sim u_o^2 + 2u_o u_1 \cos(\sigma_1 t-\alpha_1) + 2u_o u_2 \cos(\sigma_2 t-\alpha_2) \qquad (11)$$

The terms in $u_o u_1$ and $u_o u_2$ represent the interaction of the tidal constituents with the river discharge; the term $u_1 u_2$ represents the interaction of the tidal constituents with each other. If, for the sake of an example, we take as the two

tidal constituents, M_2 and S_2, (10) indicates that their interaction through the friction would create shallow water constituents such as M_4, S_4, MS_4 and MSf. The terms in $u_o u_1$ and $u_o u_2$ will modify M_2 and S_2 themselves. The use of (11) allows us to separate the solutions for u_o, u_1 and u_2. The solution for constituents 1 and 2 has the form:

$$u,h \quad \exp\left[i\sigma t \pm \frac{i\,\sigma}{(gD)^{\frac{1}{2}}} \quad (1 - 2igu_o/C^2\sigma D)^{\frac{1}{2}}\, x\right] \qquad (12)$$

the amplitude of the wave moving up the river is modulated by the function

$$\exp\left[-i\sigma/(gD)^{\frac{1}{2}} \quad (1 - 2igu_o/C^2\sigma D)^{\frac{1}{2}}\, x\right] \qquad (13)$$

which differs from the usual sinusoidal modulation factor:

$$\exp \quad \frac{-i\,\sigma}{(gD)^{\frac{1}{2}}} \quad x$$

for a wave travelling upstream without friction.
The argument of (13) has the form

$$\frac{-i\,\sigma}{(gD)^{\frac{1}{2}}} \quad (\alpha - i\beta) \qquad (14)$$

α is akin to the wave number $K=\sigma/(gD)^{\frac{1}{2}}$ but it is modified by the friction. $\beta\sigma/(gD)^{\frac{1}{2}}$ reduces the amplitude of the wave exponentially.

The explicit expression for α and β is

$$\begin{Bmatrix}\alpha\\ \beta\end{Bmatrix} = [1+(2u_o)^2 g^2/C^4\sigma^2 D^2]^{\frac{1}{4}} \begin{Bmatrix}\cos\\ \sin\end{Bmatrix} (\tfrac{1}{2} \text{ arc tan } 2u_o g/C^2\sigma D)$$

$$\simeq (2u_o g/\sigma D C^2)^{\frac{1}{2}} \begin{Bmatrix}\cos\\ \sin\end{Bmatrix} (\tfrac{1}{2} \text{ arc tan } 2u_o g/C^2\sigma D) \qquad (15)$$

for $\sigma<<1$, which holds for the tidal constituents.

We may insert some numerical values into (13) and check if the calculated phase retardation and the attenuation agree at all with the observations.

Between Lavaltrie and Trois Rivières, the mean depth $D \sim 6.6$ meters, while between Trois Rivières and Batiscan $D \sim 6.2$ meters. We take $u_o \sim 55$ cm/sec, $C \sim 52 m^{\frac{1}{2}}$/sec

$\sigma_{M_2} = 1.41 x 10^{-4}$/sec $\quad \sigma_{O_1} = .73 x 10^{-4}$/sec $\quad \sigma_{MSf} = .049 x 10^{-4}$/sec

The distance between Lavaltrie and Trois Rivières is 82.4 kilometers while the distance between Trois Rìvières and Batiscan is 31.9 kilometers.

Table 1 shows a comparison between the observed and calculated values of the phase retardation and of the attenuation.

The attenuation between Batiscan and Trois Rivières is larger than our estimate but it must be understood that the assumptions underlying (11) do not hold in this area. But even there our qualitative deduction that the damping of MSf should be much less than that of M_2 and O_1 still holds true.

Table 1

Phase Change and Attenuation Between Batiscan and Lavaltrie for M_2, O_1 and MSf

Station		M_2				O_1				MSf			
	Δx km	A cm	ϕ deg	β' km^{-1}	α' deg/km	A cm	ϕ deg	β' km^{-1}	α' deg/km	A cm	ϕ deg	β' km^{-1}	α' deg/km
Batiscan	-31.9	32.9	320.6	4.5×10^{-2}	1.1	7.9	331.5	3.5×10^{-2}	1.1	17.8	62.7	$.8 \times 10^{-2}$	.4
Trois-Rivières	00	7.9	356.4			2.6	7.1			13.8	75.4		
Louiseville	33.8	2.5	89.9	2.4×10^{-2}	2.8	1.4	58.1	1.7×10^{-2}	1.5	11.6	88.8	$.6 \times 10^{-2}$	.4
Sorel	58.1	1.3	128.2	3.1×10^{-2}	2.2	.6	67.2	2.4×10^{-2}	1.1	7.5	92.0	$.7 \times 10^{-2}$	.3
Lanoraie	71.9	1.3	157.2	1.6×10^{-2}	2.2	1.5	74.0	2.2×10^{-2}	.9				
Lavaltrie	82.4	.6	214.7	3.8×10^{-2}	2.7	.2	140.8	2.9×10^{-2}	1.6				
Average between Trois-Rivières and Lavaltrie													
				$\overline{2.7 \times 10^{-3}}$	$\overline{2.5}$			$\overline{2.3 \times 10^{-2}}$	$\overline{1.2}$			$\overline{.6 \times 10^{-2}}$	$\overline{.3}$
Calculated				2.4×10^{-2}	1.7			1.8×10^{-2}	1.1			$.5 \times 10^{-2}$	.3

$\beta' = \log(A_2/A_1)/\Delta x$ A=Amplitude ϕ=phase

$\alpha' = \Delta\phi/\Delta x$

2.4 The speed of propagation of the tide

A wave travelling in a frictionless channel should propagate at a speed:

$$c = (gD)^{\frac{1}{2}} \qquad (16)$$

In the lower St. Lawrence the tidal oscillation is intermediate between a standing wave and a travelling wave. Upstream of Québec the tidal wave takes a definite progressive character.

In a frictionless channel when one is in presence of a mixture of standing and travelling waves, the crest of high water should always travel at a speed in excess of (16). If the crest, anywhere in the channel, travels at a speed less than (16), one is in presence of a damped travelling wave. The use of (12) confirms this fact at least qualitatively, keeping in mind the limitations of its validity. From (12) we deduce that the speed of displacement of a damped wave is

$$c' = (gD)^{\frac{1}{2}}/r \qquad (17)$$

where

r=1.89 for M_2

=2.13 for O_1

=8.17 for MSf

Therefore in the presence of friction, free waves travel at different speeds depending on their frequency and they always travel more slowly than an undamped wave.

Also, an inspection of the speed of travel of high and low water in the upper St. Lawrence indicates that the low water travels more slowly than the high water. The fact that the depth of water D in the channel is less at low water than at high water may explain this fact as the difference in depth between high and low water equals the range of the tide. However, the low water travels much more slowly than would be predicted by simply inserting the appropriate value of the depth in (17) thus indicating that our algebra does explain many facts qualitatively but not quantitatively. A more refined numerical analysis is necessary in order to obtain some quantitative agreement.

The distance between Québec and Grondines is about 93 km and the crest of high water takes an average of $2\frac{1}{2}$ hours to reach Grondines from Québec; therefore, the high water travels at about 10.3 m/sec. The average depth between the two localities is 12 m. and therefore the speed of the free wave in the absence of friction should be c=10.9 m/sec. The actual wave travels more slowly than the free wave and it therefore consists of a damped travelling wave.

To conclude, we give a table of the time of travel for the high and low water at neaps and springs between various stations (Table 2). These averages are based on the occurrences during April and October, 1964. The low water at neap is higher than the low water at spring; this may explain why it travels appreciably faster. The table indicates that the high water at spring reaches Québec before St. François on the Island of

Table 2

Average Time of Travel of the High and Low Water Between Pointe au Père and Various Stations Upstream in Hours.

	A	B	C	D	E	F	G	H
High Water Spring	0	4.3	4.1	5.5	5.9	6.6	6.7	7.3
Neap	0	4.0	4.3	5.4	6.0	6.8	6.8	7.9
Low Water Neap	0	4.1	5.1	6.4	7.4	8.4	8.8	10.3
Spring	0	4.8	5.9	7.4	8.1	9.3	9.5	11.2

A Pointe au Père
B St. François
C Québec
D Neuville
E Portneuf
F Grondines
G Cap à la Roche
H Trois Rivières

Orleans. This is hard to believe and even more difficult to explain, but many observations confirm this fact.

2.5 The critical points $\Delta S=0, u=0, u_t=0$

For a channel of constant width B and depth D, (8) reduces to

$$S_x = u|u|/C^2D + u_t/g - 2uD_t/gD \qquad (19)$$

u being the mean current across the section.

S_x may be written in finite difference as $\Delta S/\Delta x$, where Δx is the distance between two tide gauges while ΔS is the difference in level between them. When $\Delta S=0$, the water level is horizontal all along the portion of the channel considered and this implies $S_x=0$. From (19) we deduce that if $S_x=0$, the friction term $u|u|/C^2D$ should approximately balance the acceleration term u_t/g, neglecting the convective term. In theory therefore one may deduce the Chézy coefficient C by evaluating u and u_t at the instant when $\Delta S=0$. In practice though it is not easy to get an accurate value of u_t.

Finally when $u_t=0$, the currents run at their maximum velocity either at flood or ebb and there should be an approximate balance between the surface gradient S_x and the friction term $u|u|/C^2D$; this case gives us our only real chance to evaluate C when there is tidal motion.

2.6 A review of the current observations carried out by the Ship Channel Division and G.C. Dohler

There is now some material available for the study of (19), namely the work carried out by the Ship Channel Division near the Québec Bridge in 1968 and the current survey of the Upper St. Lawrence carried out by G.C. Dohler in 1960.

In finite difference form (19) becomes

$$\frac{\Delta S}{\Delta x} - \frac{u|u|}{C^2 D} = \frac{\Delta u}{\Delta t}\ \frac{1}{g} - 2\ \frac{\Delta D}{\Delta t}\ \frac{1}{gD}$$

We obtain values for ΔS, $\Delta D/\Delta t$, and u by setting up tide gauges at both ends of the channel and a current meter in the middle where we assume that we effectively measure Q.

In 1968, the Ship Channel Division set up two tide gauges, one at Basile Low Light and the other at Québec Bridge, located exactly two nautical miles apart; they also installed one at Pointe à Basile, located halfway between the two stations. At the same point they installed a self recording current meter at the appropriate depth. Such a set up is the ideal for our present investigation. During the interval of observations the current meter worked quite adequately but unfortunately the results obtained from the tide gauges are questionable.

The tide gauge installed at Pointe à Basile which at first sight looks superfluous could have been used to check the other gauges but unfortunately its indications cannot be used at all: the high and low water which it records occur one or two hours after those at Basile Low Light and Québec Bridge. There

is something basically wrong with the clock mechanism or the abstraction of the data. The gauge at Basile Low Light seems to have worked properly during the interval of interest but I have the definite impression that the gauge at Québec Bridge was damped and had an appreciable time lag behind the true water level. This has unpleasant consequences as the quantity ΔS, the difference in level between Basile Low Light and Québec Bridge is very slight, seldom exceeding 30 cm, and a small error in one or the other water level suffices to make it very nearly impossible to verify (20) in this portion of the river where it would have been extremely interesting to put it to test.

Although the two stations are barely two nautical miles apart there is an appreciable difference in mean level between them: the observations give a mean of 64 cm at Basile Low Light for October, 50 cm at Québec Bridge while the mean level at Québec (Lauzon) was 40 cm. The difference of 14 cm between Basile Low Light and Québec Bridge seems a bit high to me and there might be an error in levelling at one or the other gauge sites considered.

I did plot ΔS and u for this portion of the channel on a time scale for a few days in October when the two gauges seemed to work not too badly, although one of the gauges gave some indication of drifting. The plot of u and ΔS is shown in Fig. 10. The current curve is sensible enough. We notice that the interval of flooding is shorter than the time of ebb and that the current runs at maximum flood only for a very short time whence it turns suddenly to ebb during which it flows at

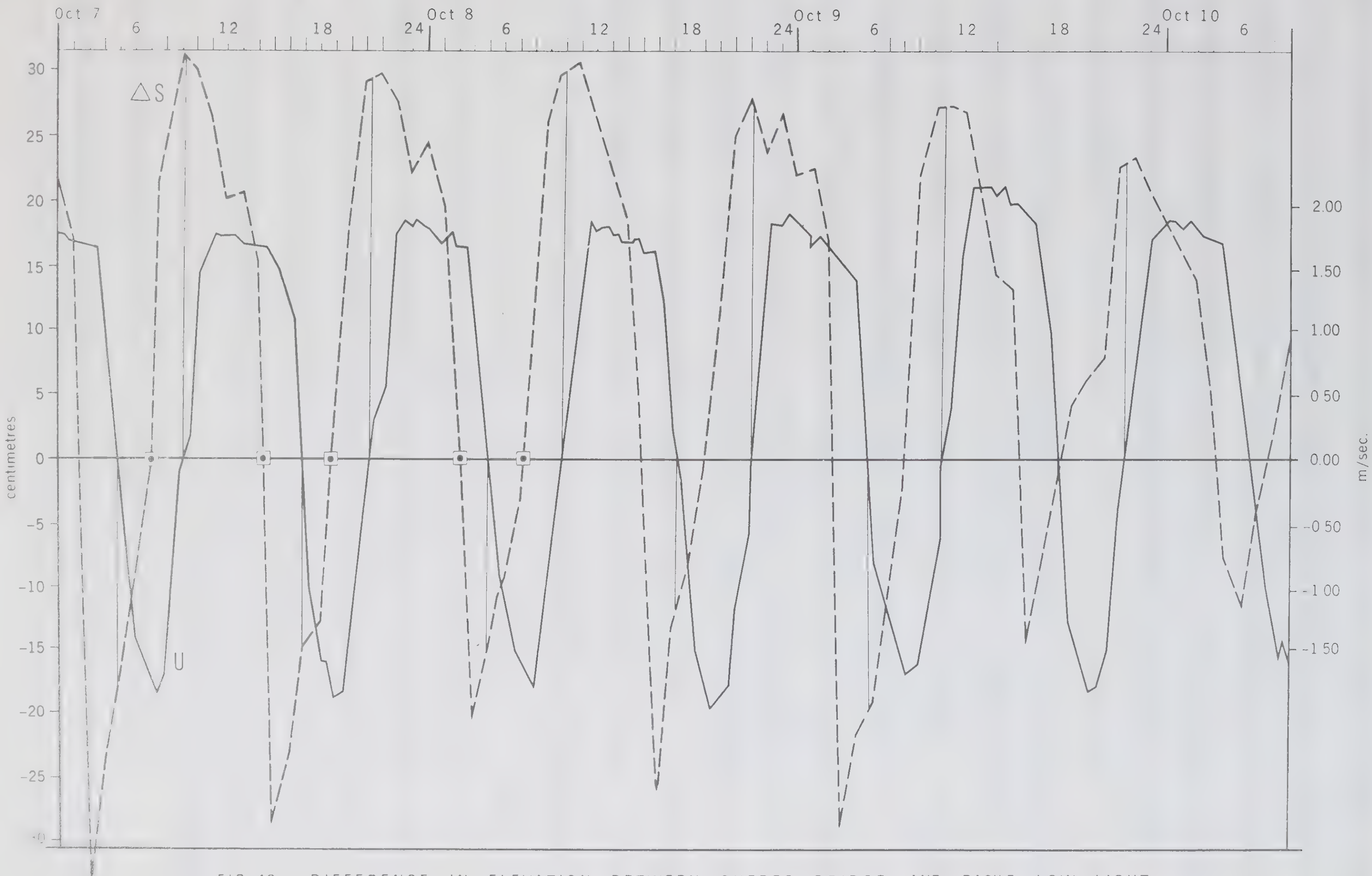


FIG. 10 DIFFERENCE IN ELEVATION BETWEEN QUEBEC BRIDGE AND BASILE LOW LIGHT.

(Current at pte a Basile)

nearly the same velocity for about four hours. The current curve exhibits the saw tooth structure which we mentioned in the paragraph on the tide in the St. Lawrence River. The difference in level ΔS between Québec Bridge and Basile Low Light has the proper shape as well; the level at Québec Bridge is higher than the level at Basile Low Light when flooding is about to take place. However, we will see that the ΔS curve does not seem right on the time scale; it appears to be displaced by a few hours on the left, a fact which I attribute to the lag and the distorted response of the gauge located at Québec Bridge.

Slacks occur when the current curve u intersects the zero abscissa; these points have been marked in Fig. 10. The friction terms $u|u|/C^2D$ vanish then and the surface gradient $\Delta S/\Delta x$ should balance the acceleration term $(1/g)\Delta u/\Delta t$. By inspecting the diagram we notice that $\Delta u/\Delta t$ is approximately the same for two consecutive slacks while the corresponding ΔS is quite irregular, especially for the slack water which precedes the flood.

At maximum current, either at flood or ebb, $\Delta u/\Delta t=0$ or is very small; there should be an approximate balance between the surface gradient $\Delta S/\Delta x$ and the friction $u|u|/C^2D$. In Fig. 10 we notice that at maximum flow at ebb, ΔS has pretty regular values while at flood it is virtually impossible to obtain consistent values of ΔS.

Finally when there is no surface gradient ($\Delta S/\Delta x=0$), the acceleration $(1/g)\Delta u/\Delta t$ should balance the friction term $u|u|/C^2D$. At flood, $\Delta S=0$ coincides with $\Delta u/\Delta t<0$ while the

friction term is positive; therefore there is not even agreement in sign. Naturally $\Delta u/\Delta t$ varies very rapidly near maximum flow at flood but still the general lack of balance between the various terms of the equation of motion suggests to me that one of the two gauges was not working properly. $\Delta S=0$ should fall after the maximum flow at flood and not before as the record seems to indicate.

Now we turn our attention to the set of observations collected by G.C. Dohler and his assistants contained in the publication "Current Survey St. Lawrence River 1960". The currents were measured manually with Ott and Neyrpic type of instruments at various depths and locations; the duration of the observations seldom exceeded 48 hours and the observations showed marked fluctuations. Many of these observations are not contained in Mr. Dohler's publication and they have been kindly communicated to me.

In order to test the equations of motion with such observations I used the hourly values of the water levels at the stations for which tides gauges are operated. These gauges sometimes are more than 50 km apart and the approximation to (19) by finite differences cannot be too good. The current observations show so many irregularities that it is usually impossible to obtain even a fair estimate of u_t and at times even of u when $u_t=0$. Still I found that the observations were of some use at the points $u_t=0$; I have plotted u and ΔS on a time scale and these are shown in Fig. 11 to 15. ΔS is deduced from the records of hourly heights kept in the records of the

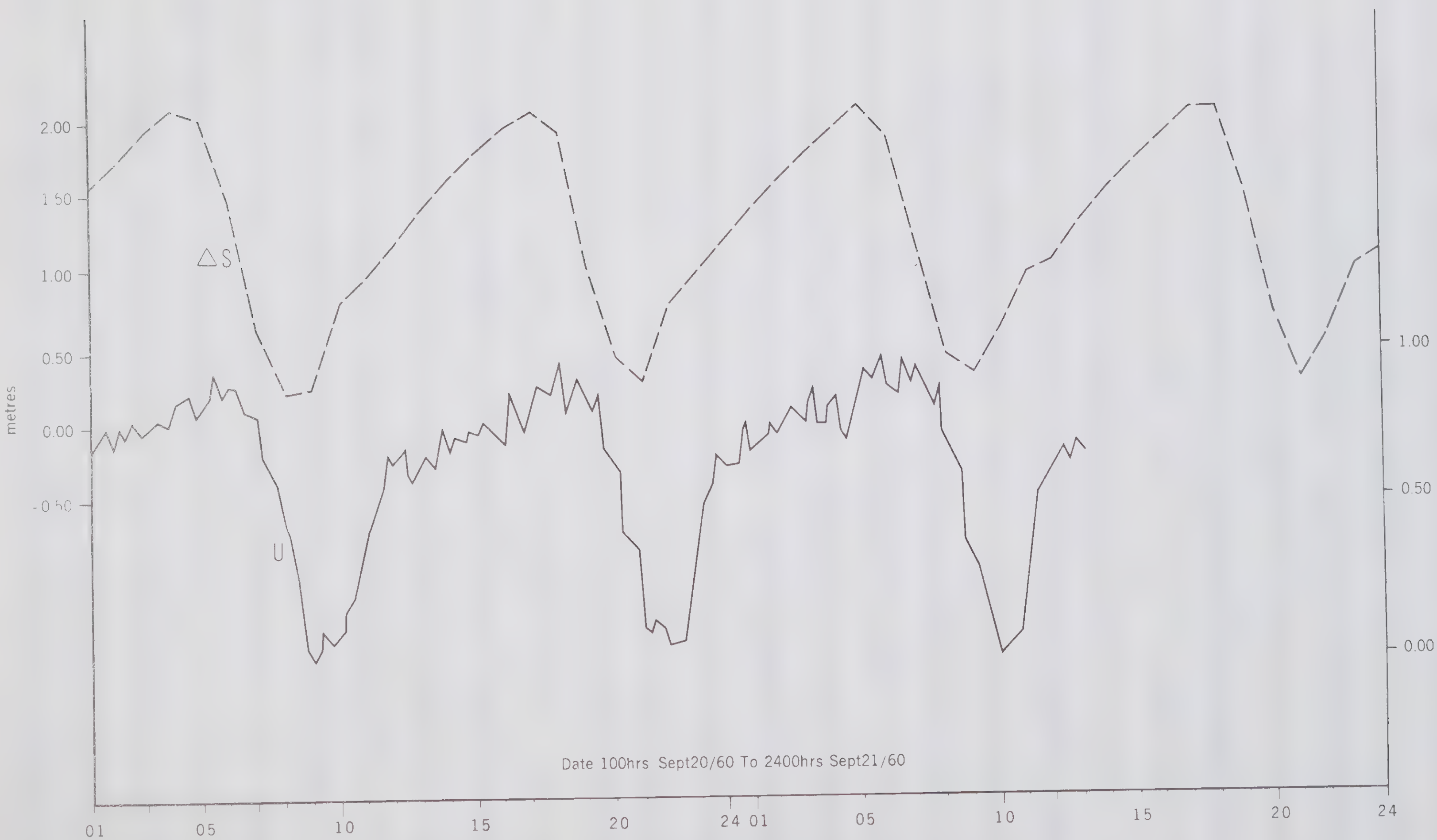


FIG.11 DIFFERENCE IN LEVEL BETWEEN TROIS RIVIERES AND CAP A LA ROCHE.

(Currents at Batiscan)

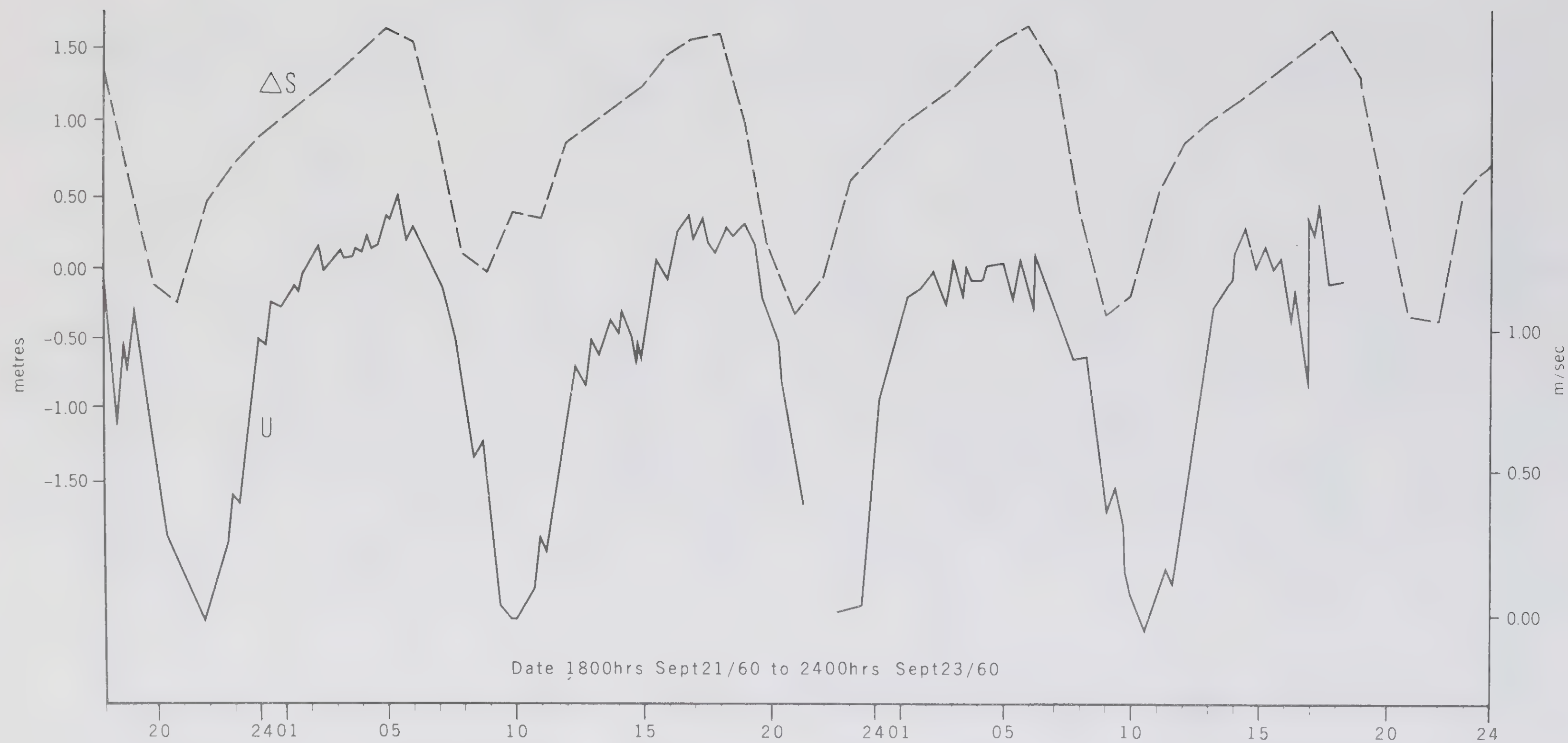


FIG 12 DIFFERENCE BETWEEN BATISCAN AND GRONDINES
(Current at Cap à la Roches)

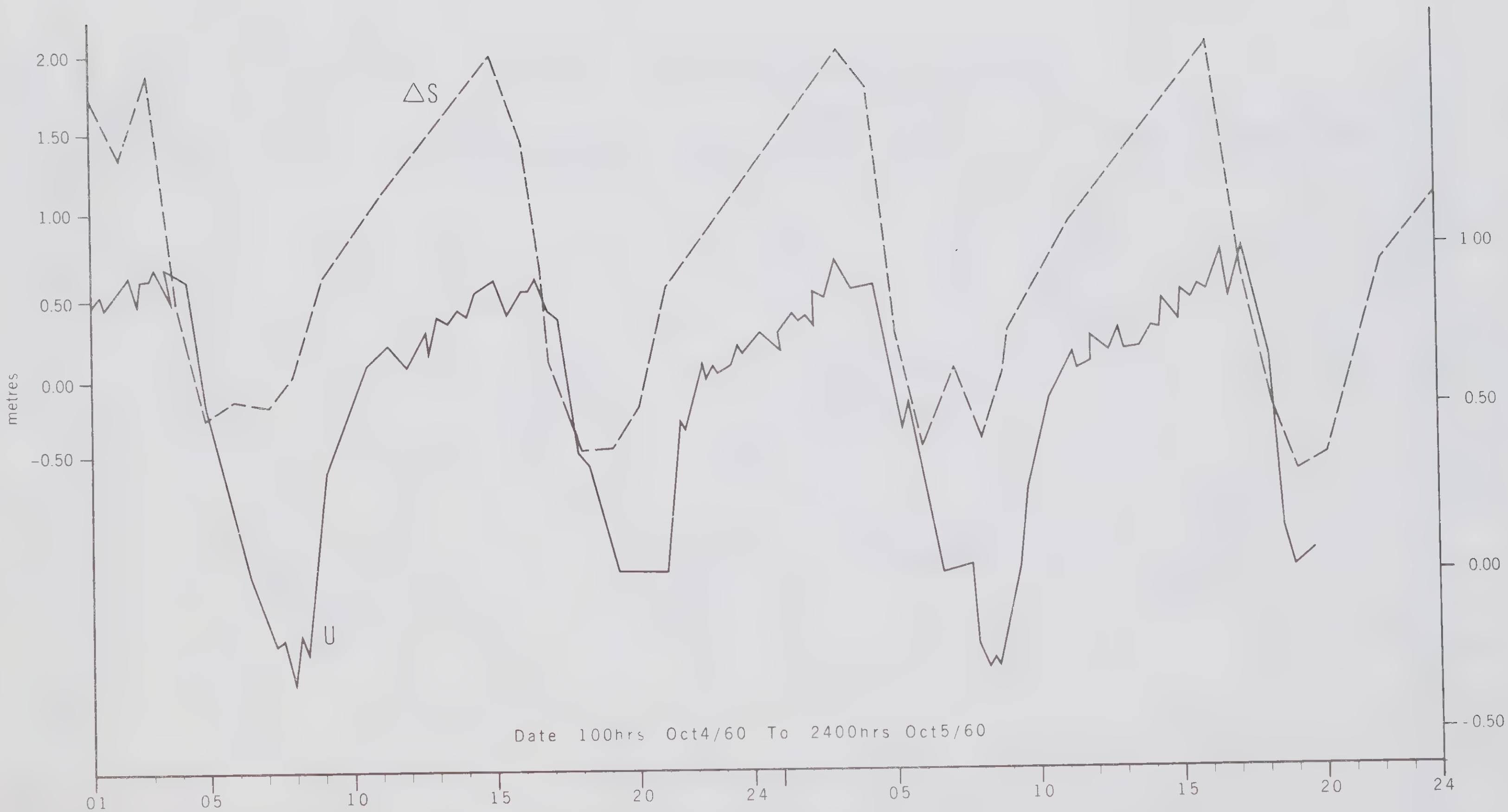


FIG. 13 DIFFERENCE IN LEVEL BETWEEN CAP A LA ROCHE AND PORTNEUF
(Currents at Grondines)

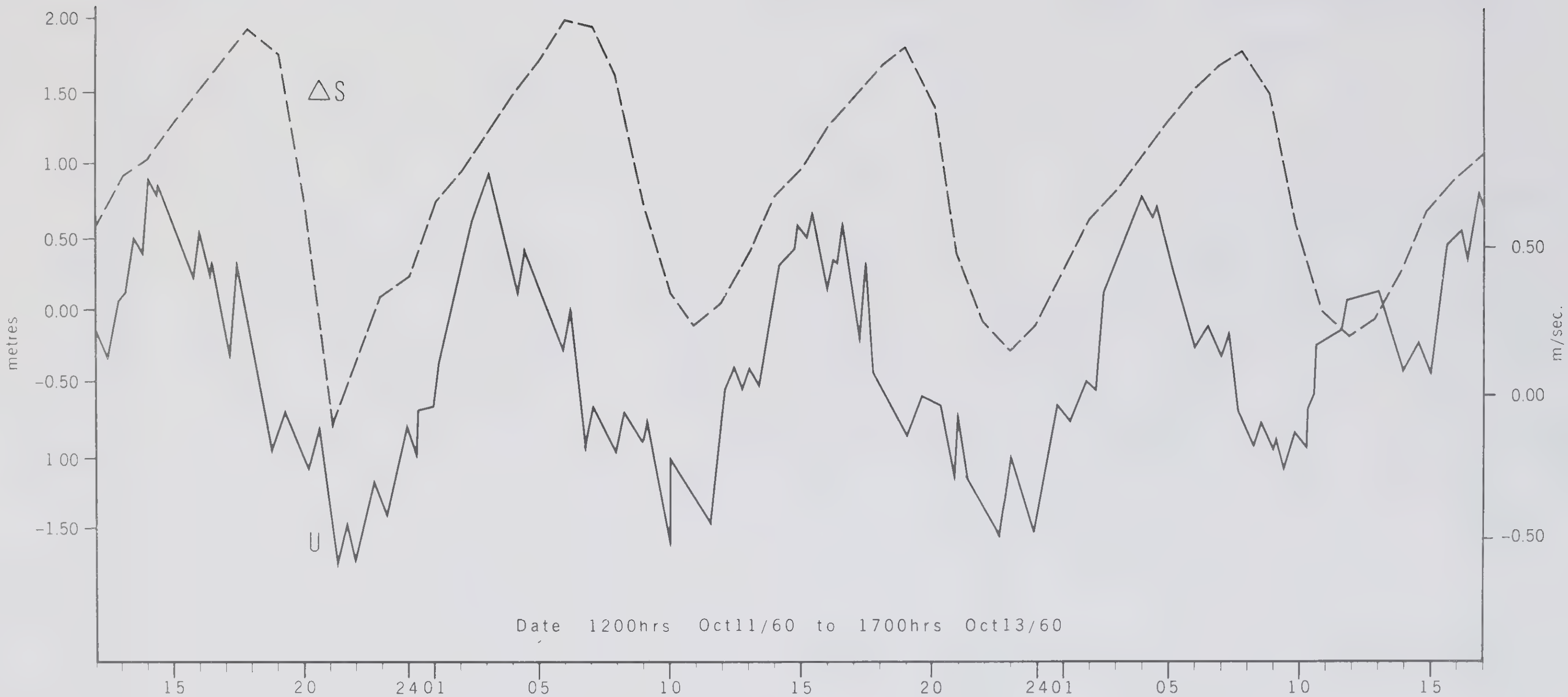


FIG. 14 DIFFERENCE BETWEEN GRONDINES AND NEUVILLE
(Currents at Portnuef)

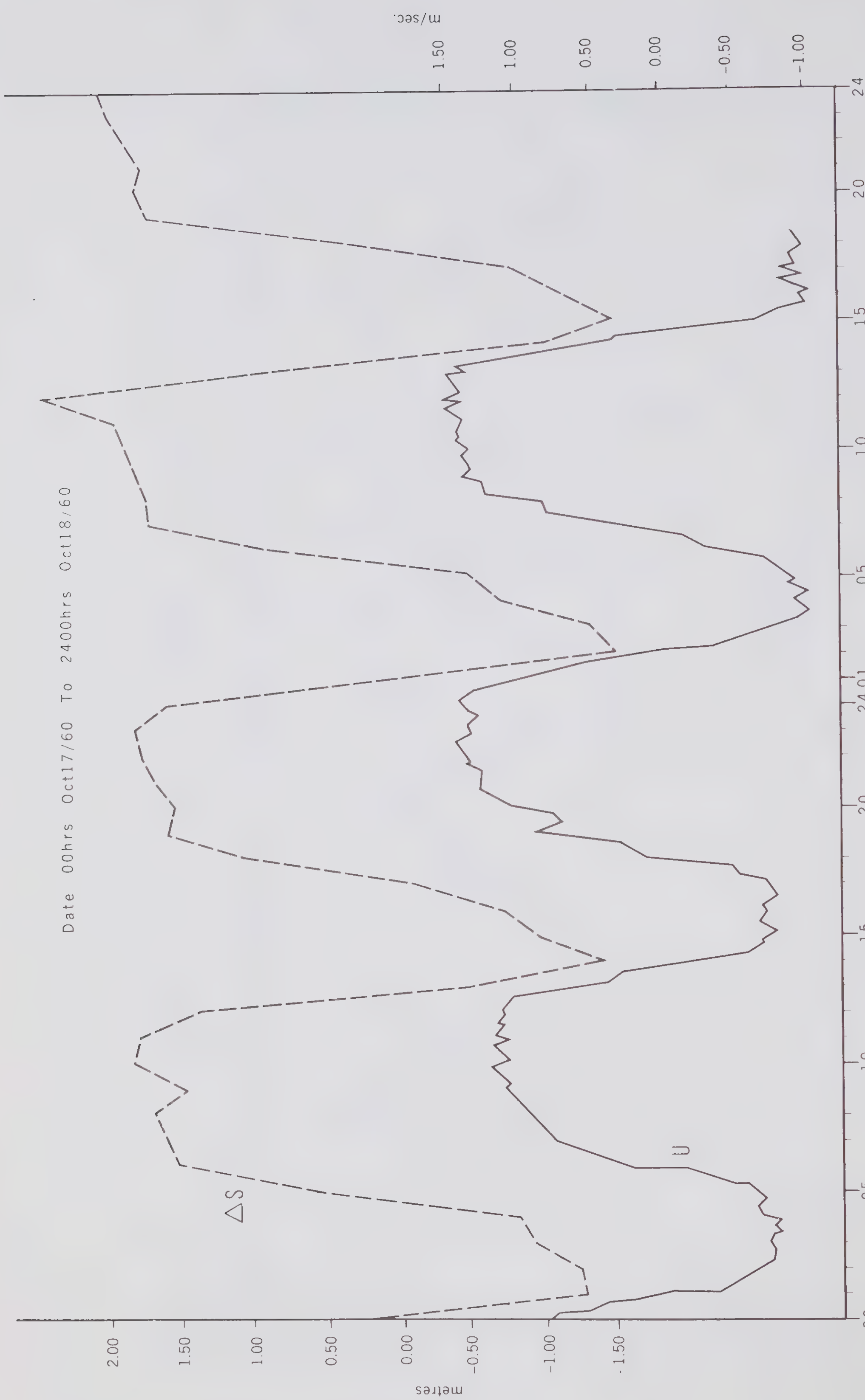


FIG. 15 DIFFERENCE IN LEVEL BETWEEN PORTNEUF AND QUEBEC
(Current at Neuville)

Tides and Water Levels section for the following stations:

Trois Rivières

Batiscan

Cap à la Roche

Grondines

Portneuf

Neuville

Québec (Lauzon)

The current plotted is the current measured by Dohler and his group at a point intermediate between the two stations. For instance in Fig. 11, we show the currents measured at Neuville on October 17 and 18, 1960 and we plot them along with the difference in level observed between Portneuf and Québec. Only at Neuville are the currents observations smooth enough to allow us to make some deductions from the case $\Delta S=0$. Table 3 lists the values of the Chézy coefficient we could deduce from the observations of Dohler.

The values of the Chézy coefficient which we deduce are rather consistent; the only exception is Grondines. Near Grondines, the river bed experiences a marked bend and constriction and there is a radical modification of the tidal regime in this area. Therefore, the high values of friction deduced for this area might be true.

3. Spectral Analysis

An analysis reveals the amplitude (and in tidal analysis, the phase as well) of all the oscillations contained in a sequence of observations over the range of frequencies

Table 3

Values of the Chézy Coefficient Deduced for the Section of the River Between Trois Rivières and Québec.

Stations	Current at	Δx km	Critical Point	C_{flood} $m^{\frac{1}{2}}/sec$	C_{ebb} $m^{\frac{1}{2}}/sec$
Portneuf-Québec	Neuville	62	$u_t=0$	56	62
			$\Delta S=0$	60	62
Grondines-Neuville	Portneuf	49	$u_t=0$	62	56
Cap à la Roche-Portneuf	Grondines	33	$u_t=0$	52	51
Batiscan-Grondines	Cap à la Roche	24	$u_t=0$		63
Trois-Rivières-Cap à la Roche	Batiscan	54	$u_t=0$		58

Average C_{mean} is 58 $m^{\frac{1}{2}}/sec$.

investigated. If the oscillations have approximately the same amplitude for any frequency, we are in presence of white noise which is of little interest to us. Whenever the tide is present in a record, some oscillations of given frequencies will dominate the spectrum, these frequencies lying in the semi-diurnal and diurnal bands. Along the river, the phase of such oscillations will vary gradually.

The low frequency band of current and water level observations is filled mainly with noise; it contains the slow fluctuations of the steady current or the weekly and semimonthly variations of the mean level. In this band the resolution between the various oscillations becomes increasingly difficult. I personally do not subject this part of the spectrum to a harmonic analysis. I prefer to have a visual inspection of the low pass first; a power spectrum afterwards will indicate which frequencies dominate in this part of the spectrum.

In the semidiurnal and diurnal bands of the spectrum the tidal constituents dominate. The main tidal constituent is M_2 which has a period of half a lunar day; the next constituent in importance is S_2 which has a period of half a solar day. Then come N_2, K_1 and O_1. N_2 is linked to the variation of the moon's distance from the earth which goes through a complete cycle each month. K_1 and O_1 are diurnal and they are relatively small in the St. Lawrence system.

The five constituents mentioned make up the bulk of the tide. They are of astronomical origin but they are felt in the St. Lawrence system through the cooscillation of the gulf

and the river with the tide present in the Atlantic ocean which imparts some of its energy through Cabot Strait and Belle Isle Strait. These constituents are modified by the friction and they interact amongst each other. The interference of M_2 with S_2 will create a new shallow water constituent, MSf, which has a period of two weeks, while M_2 and N_2 will interact to give Mm which has a period of one month. The interference of K_1 and O_1 creates Mf which also has a period of two weeks. Many more constituents are created by the interaction of M_2, S_2, N_2, K_1 and O_1 such as M_4, MS_4, S_4, etc., but in a river the slow constituents MSf, Mm and Mf have a particular importance as they are not as rapidly damped as the constituents with higher frequencies.

3.1 The data available for analysis in the St. Lawrence River

Observations on water levels in the St. Lawrence River have been carried out since the days of Bell Dawson in 1895. They are usually abstracted every hour as such a time step is appropriate to tidal analysis and they have been accumulated in numerous records. Since 1961 these records have been put on punched cards, microfilms and magnetic tape at the Tide and Water Levels section. As such they constitute a wealth of most precious information for the study of the mean level, the tide and its propagation, wind and storm surges, etc.

Current observations have been much more sporadic since these are so difficult to carry out. The observations of Dohler in 1960 although very useful in the study of the dynamics of the river are too short to be subjected to a meaningful analysis.

On the other hand the material accumulated by the Ship Channel Section in 1968 at Québec Bridge and Isle aux Couldres is extensive enough to yield to an analysis, and we give the results of our calculations in one of the paragraphs which follow. Dr. W.D. Forrester has made current measurements between Pointe au Père on the south shore and Pointe à Michel on the north shore in 1965 and the results of his work are contained in the publication "Currents and Geostrophic Currents in the St. Lawrence Estuary" BIO 67-5 1969. Cdr. W.I. Farquharson has carried out similar current observations in the vicinity of Pointe des Monts in 1963 (St. Lawrence Estuary Surveys. BIO 66-6).

3.2 Water levels

For some stations in the St. Lawrence we have observations on water levels which stretch almost continuously over many years. We may use such long series of observations to follow the slow variations in the mean level and to separate very sharply most of the tidal constituents. We consider first the low frequency spectrum of the following stations:

Pointe au Père

Rivière du Loup or Tadoussac

Québec

Trois Rivières

Cap à la Roche or Batiscan

Frontenac

These stations are located at strategic points along the river where the tidal regime is modified from an oceanic type to a

shallow water type to eventually become a straight river flow. Figures 16(a), (b) and (c) show the low pass of the hourly values of the water level at the stations mentioned for the years 1967-68-69. For every year we have put the stations in their proper sequence on the same time scale so that the common oscillations can be picked up at sight.

Out of the initial confusion, for instance at Pointe au Père, there are little but random oscillations, we may notice that during the calm summer months, semimonthly oscillations due mainly to MSf, start standing out sharply at Québec and reach their maximum amplitude in the vicinity of Cap à la Roche and Batiscan. A little bit of attention shows that these slow tidal oscillations appear even in Montréal during the summer months but greatly attenuated.

This qualitative assessment of the situation may be put on a more quantitative basis by computing the power spectrum of the oscillations of the low pass. Mr. K.B. Yuen of our organization, has kindly performed this calculation and his results are shown in Fig. 17 for Pointe au Père, Québec, Trois Rivières and Frontenac. The peaks indicate the predominant oscillations and they are labelled by the proper constituent when we happen to know it. We note an oscillation of 1 cycle/9.6 days, which is caused by the interference of S_2 with N_2 which we could call SN and an oscillation of 1 cycle/week which may be due to a further interference of the slow constituents MSf or Mf with themselves. This would explain as well the extra peak at 1 cycle/4.8 days which would depend on SN.

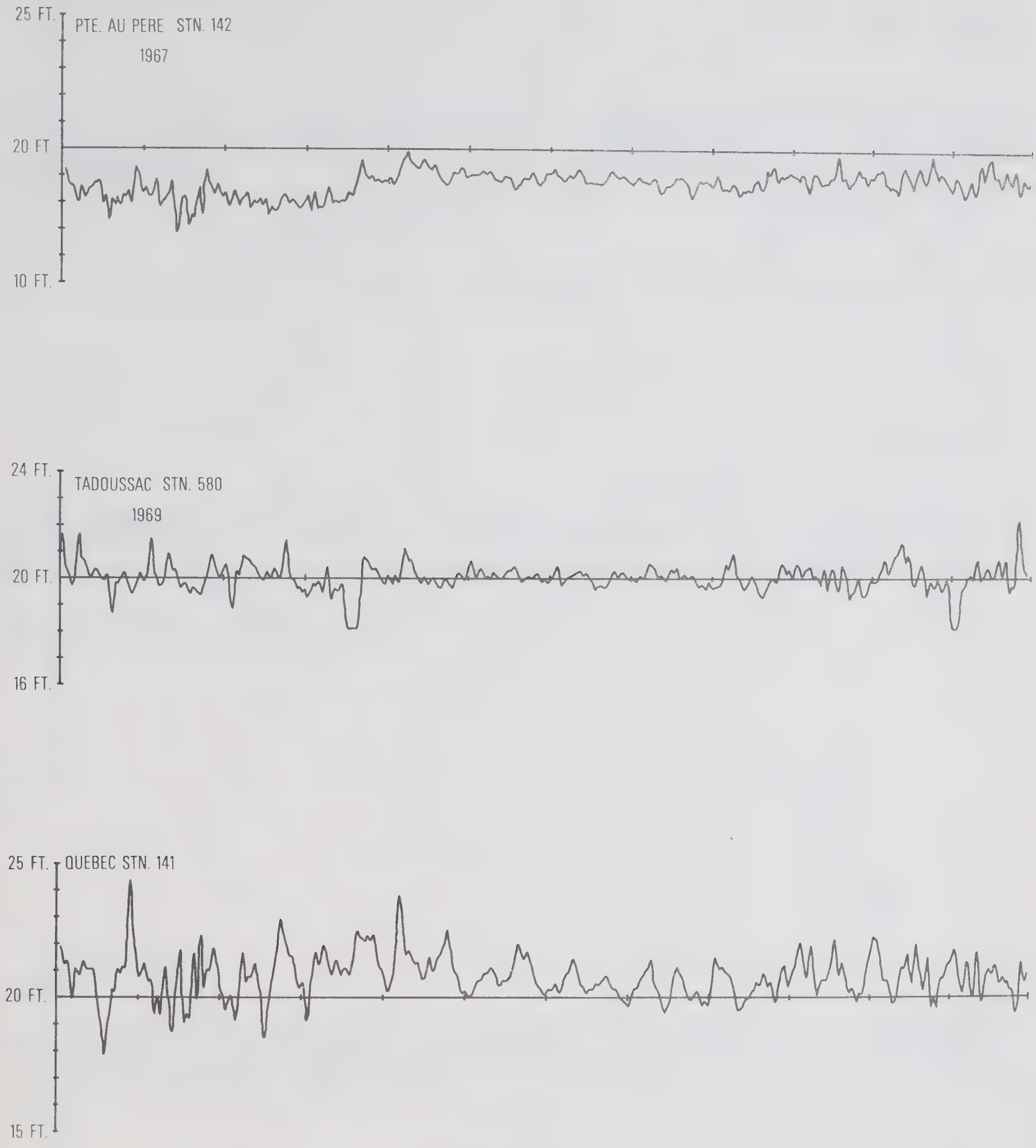


Fig. 16(a) Low Pass Filter of Hourly Heights, 1967.

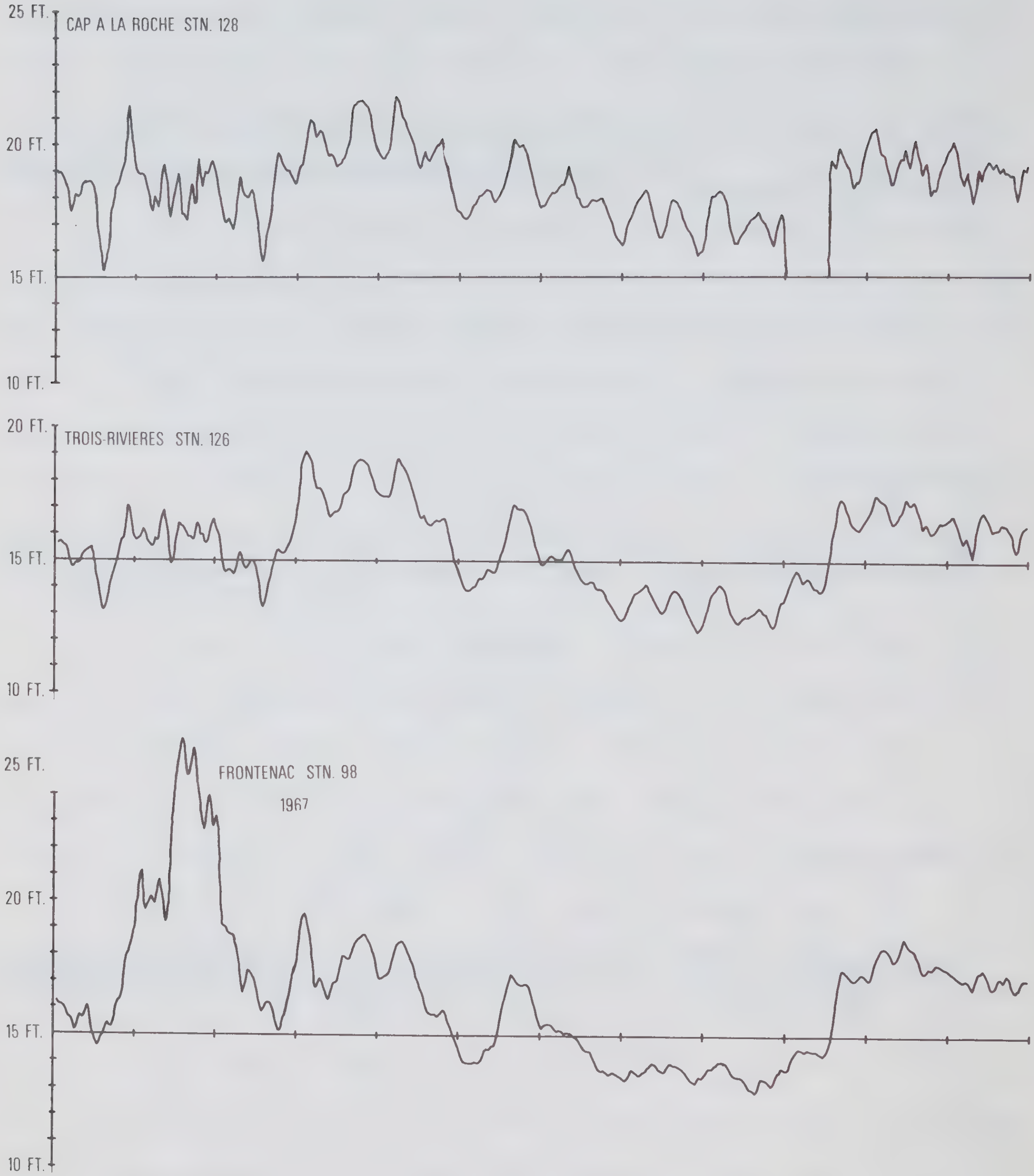


Fig. 16(a) (cont'd)

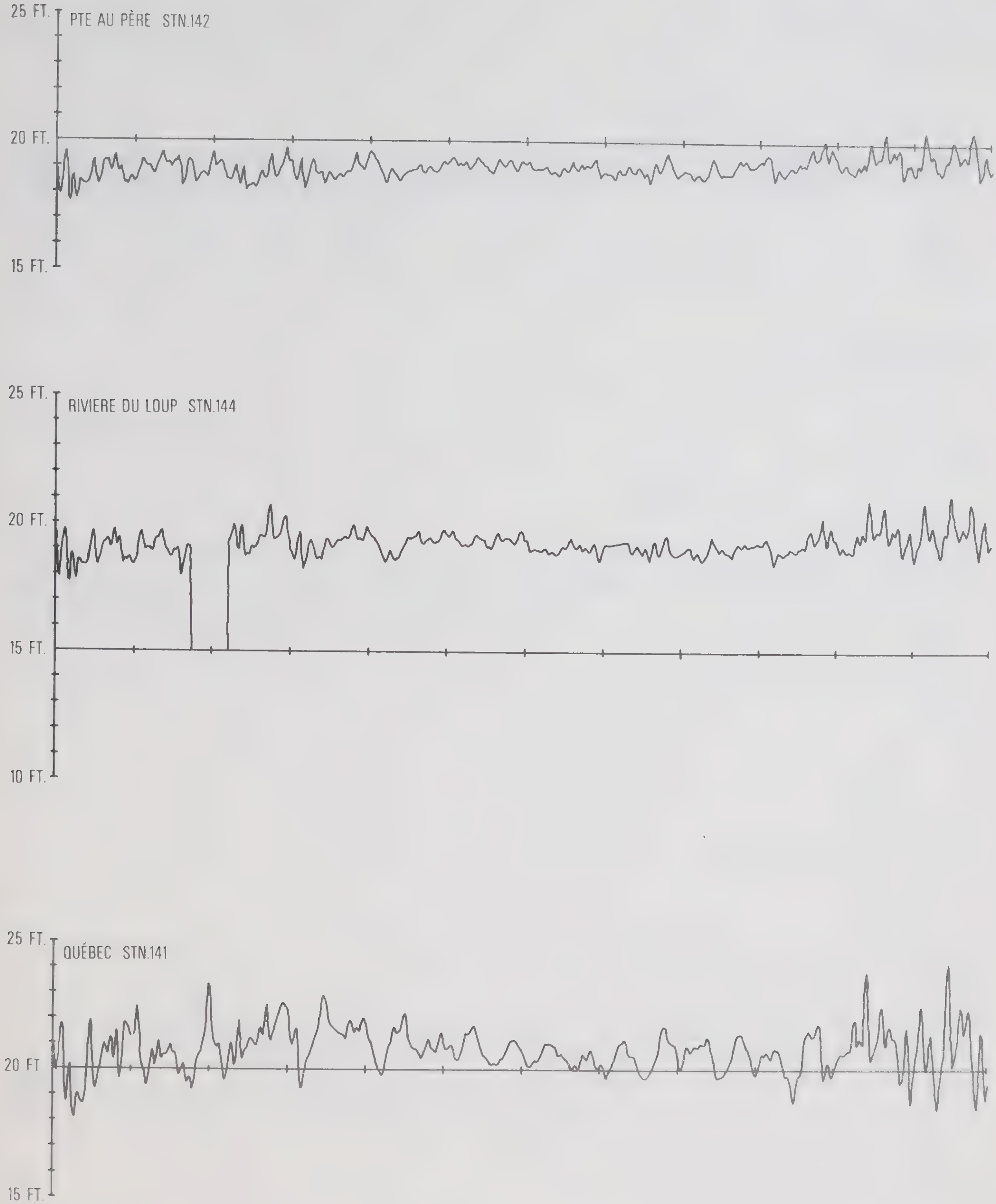


Fig. 16(b) Low Pass Filter of Hourly Heights, 1968.

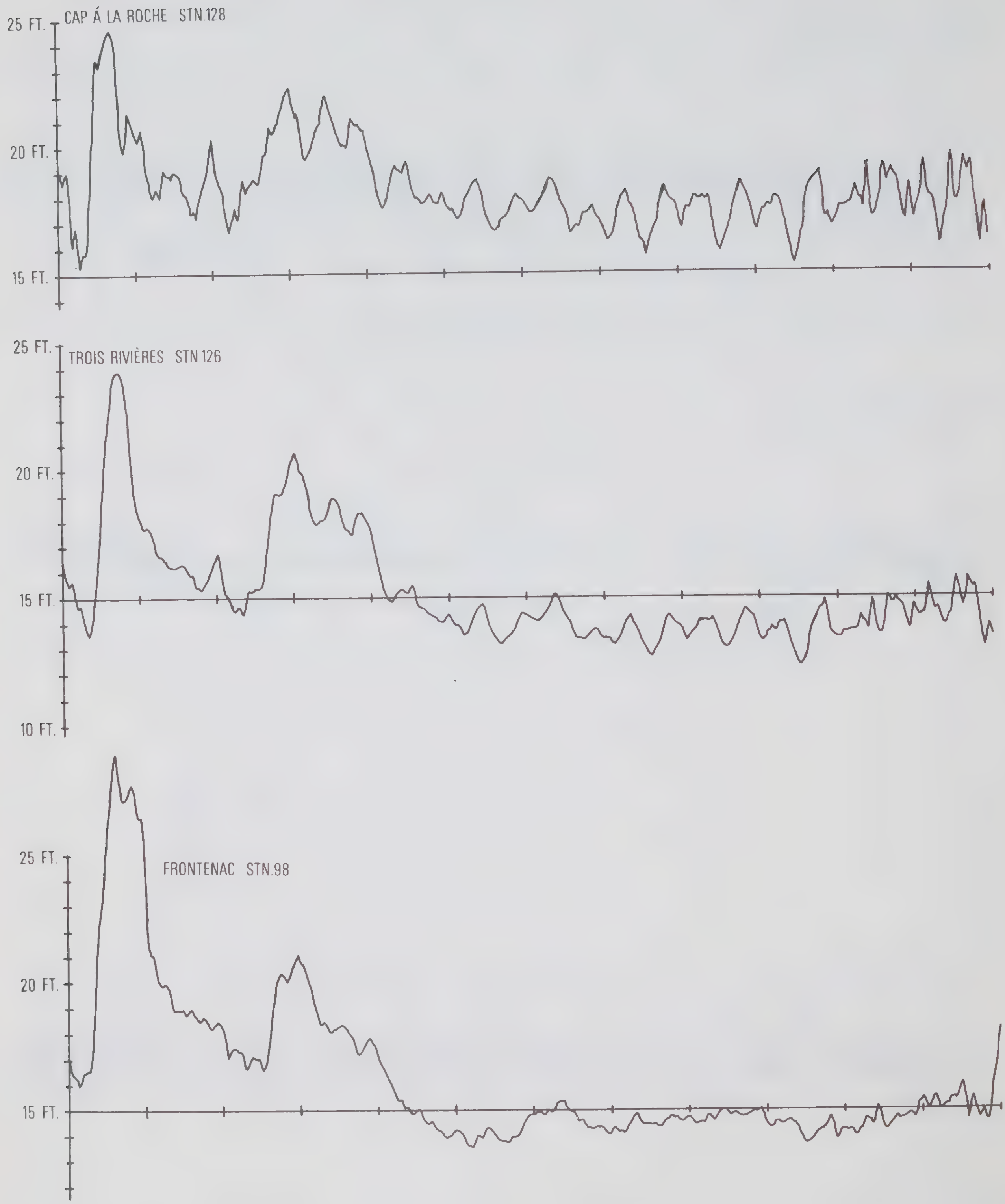


Fig. 16(b) (cont'd)

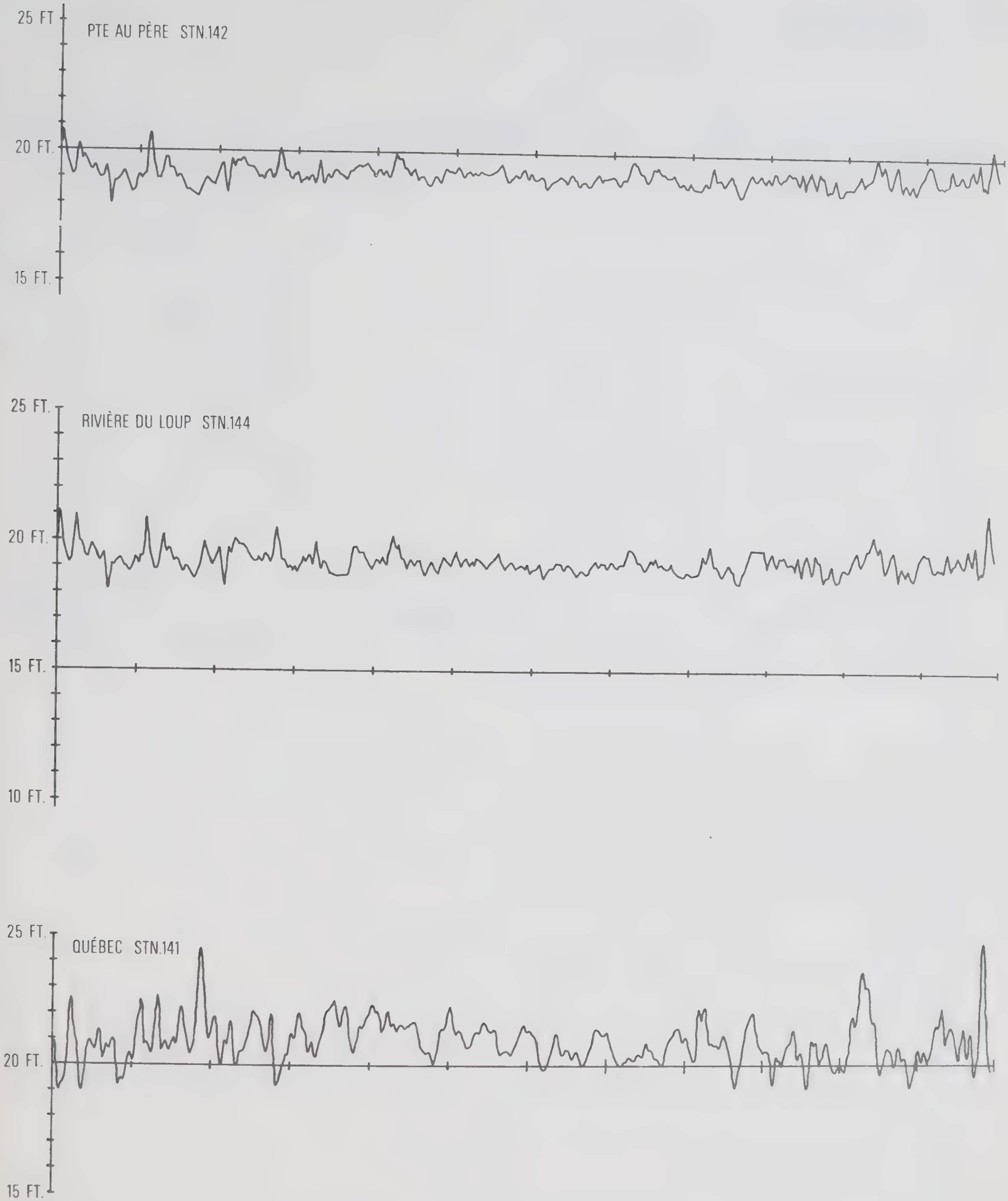


Fig. 16(c) Low Pass Filter of Hourly Heights, 1969.

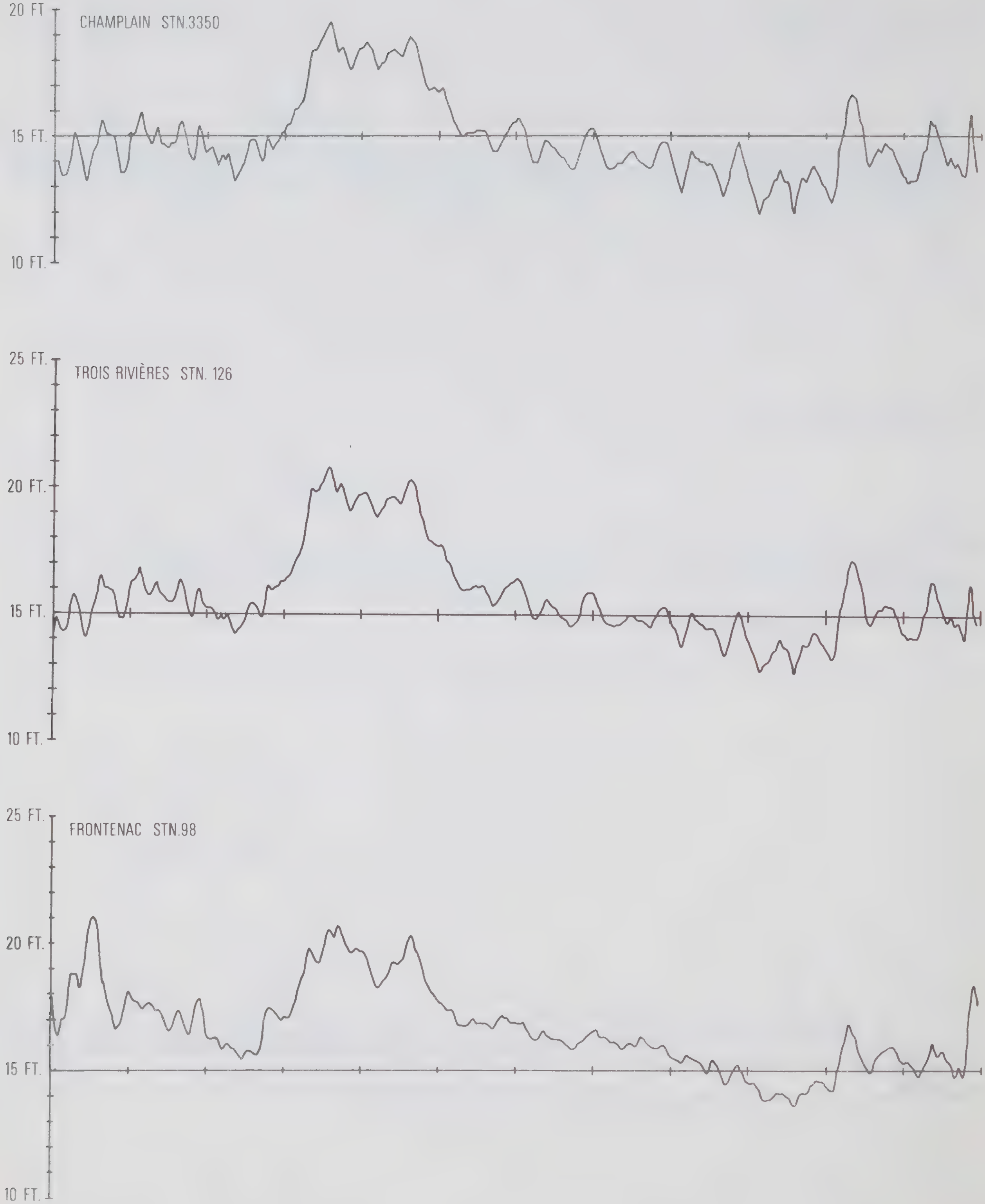


Fig. 16(c) (cont'd)

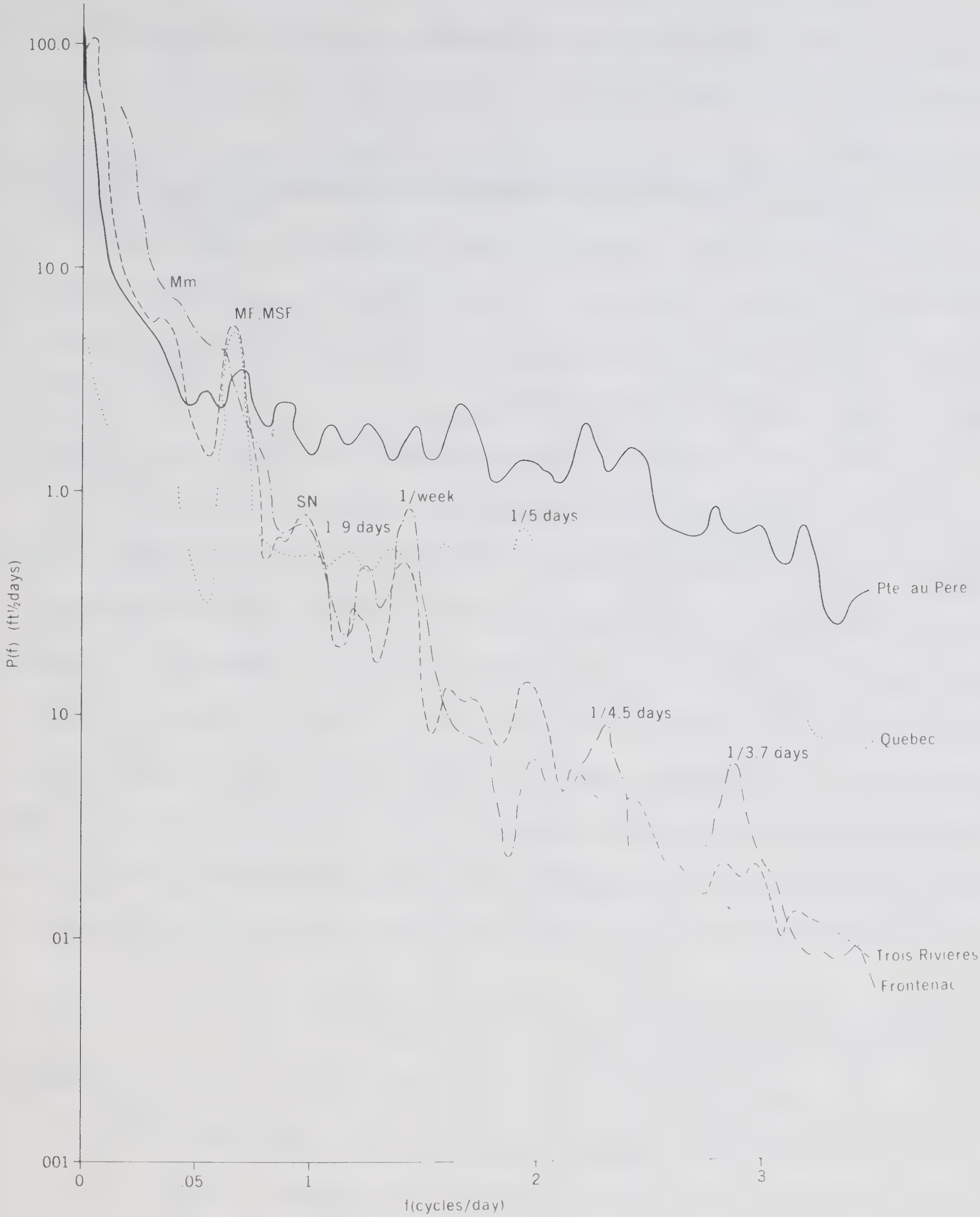


Fig. 17 Power Spectra of the Water Levels at Pointe au Père, Québec, Trois Rivières, and Frontenac. Low Frequency Band.

These oscillations, although of theoretical interest, are quite small compared to the diurnal and semidiurnal tidal oscillations and compared even to the random low frequency oscillations.

The bulk of the variations in the water levels is due to the semidiurnal and diurnal tides and Table 4 lists the amplitude in meters and phase in degrees of M_2, S_2, N_2, K_1, O_1, MSf, Mm and Mf for the stations along the river for which one year of observations is available.

In Figs. 18(a) to 18(h) we show the same constituents plotted on a distance scale along the river: we give the amplitude and the phase change. Over the points of phase change, we enter the actual Greenwich phase lag of the constituent at the given locality. Only for Mm and Mf is it impossible to give any value for the phase change as there is much scatter in the observations. We notice that the amplitude of the constituents increases upstream to a maximum and then decreases rapidly. The decrease takes place past Québec for the semidiurnal and diurnal constituents and past Cap à la Roche for the slow constituents.

3.3 The analysis of the current observations at Québec Bridge and Isle aux Couldres

We have described in a previous publication our method for analyzing current observations (Godin: The Analysis of Current Observations, International Hydrographic Review, Vol. 44, No. 1, 149-164, 1967). It consists first in smoothing the observations if they have been taken at intervals smaller than one hour and then of low passing them to have a look at the

Table 4 Tidal Constituents of the Water Levels at Various Stations Along the St. Lawrence River Deduced from a One Year Analysis

	M_2		S_2		N_2		K_1		O_1		MSf		Mm		Mf	
STATION	A	G	A	G	A	G	A	G	A	G	A	G	A	G	A	G
Ste. Anne Des Monts	.94	47.8	.29	84.5	.20	27.8	.21	204.7	.21	183.8	.03	102.6	.02	274.1	.01	144.1
Baie Comeau	1.19	48.1	.38	84.0	.26	25.7	.23	200.9	.21	181.0	.02	94.3	.02	246.1	.01	216.8
Pointe-au-Père	1.29	54.1	.41	90.4	.27	29.9	.23	203.6	.22	181.0	.01	310.0	.02	272.4	.02	9.1
Tadoussac	1.57	68.8	.50	105.3	.33	46.4	.24	207.1	.23	188.4	.04	273.9	.03	322.2	.04	213.5
Rivière Du Loup	1.52	72.3	.50	114.1	.32	54.5	.24	211.2	.22	190.8	.01	190.8	.02	292.2	.02	209.9
St. Jean Port Joli	1.88	136.4	.51	175.8	.34	111.2	.26	240.0	.24	219.6	.08	56.1	.04	340.8	.02	177.0
St François	2.02	161.1	.49	203.8	.36	137.1	.25	254.3	.24	234.0	.11	36.2	.06	57.5	.05	41.7
Québec	1.80	184.9	.42	228.1	.30	159.0	.23	270.8	.21	247.4	.11	48.9	.11	[illegible]	.07	[illegible]
Neuville	1.53	225.2	.31	274.3	.25	194.3	.19	293.1	.19	269.3	.17	54.8	.10	34.3	.06	29.0
Portneuf	1.44	244.4	.30	289.2	.22	217.1	.18	306.5	.17	282.7	.20	52.2	.12	53.4	.07	41.1
Grondines	.87	275.6	.18	317.8	.14	248.3	.14	328.7	.13	303.7	.23	54.7	.14	45.6	.05	50.2
Cap à la Roche	.70	284.6	.15	325.2	.11	253.5	.12	335.7	.12	310.3	.20	57.0	.13	52.1	.08	47.2
Batiscan	.33	320.6	.08	355.6	.06	285.4	.08	354.6	.08	331.5	.18	62.7	.12	60.9	.06	50.6
Champlain	.24	328.9	.06	2.8	.05	297.0	.06	3.0	.06	338.4	.16	62.1	.11	52.9	.05	58.7
Trois Rivières	.08	356.9	.02	28.0	.02	318.7	.03	27.6	.03	7.1	.14	75.4	.10	65.3	.05	48.3
Louiseville	.02	89.9	.01	119.6	.01	13.9	.02	88.9	.02	59.1	.12	88.8	.09	72.5	.04	39.4
Sorel	.01	128.2	.00	158.5	.00	81.8	.01	129.9	.01	89.3	.08	92.0	.07	60.8	.03	72.1
Lanoraie	.01	157.2	.00	225.9	.00	101.3	.01	129.5	.01	74.2	.10	326.0				
Lavaltrie	.01	211.7	.00	264.2	.00	118.0	.01	153.6	.00	140.6	.10	326.3				
Verchères	.01	215.0	.00	257.9	.00	136.8	.01	171.1	.00	250.1	.10	322.3				
Varennes	.01	209.8	.00	286.1	.00	112.3	.01	121.7	.00	342.4	.09	310.3				
Longue Pointe	.00	229.7	.00	301.4	.00	94.2	.00	113.2	.00	263.9	.08	310.3				
Frontenac Street	.00	225.2	.00	292.6	.00	106.2	.00	131.6	.00	[illegible]	.08	306.8				
King Edward Pier	.00	232.9	.00	289.0	.00	94.7	.00	93.1	.00	[illegible]	.08	304.5				

Fig. 18 Amplitude and Phase Lag Along the St. Lawrence

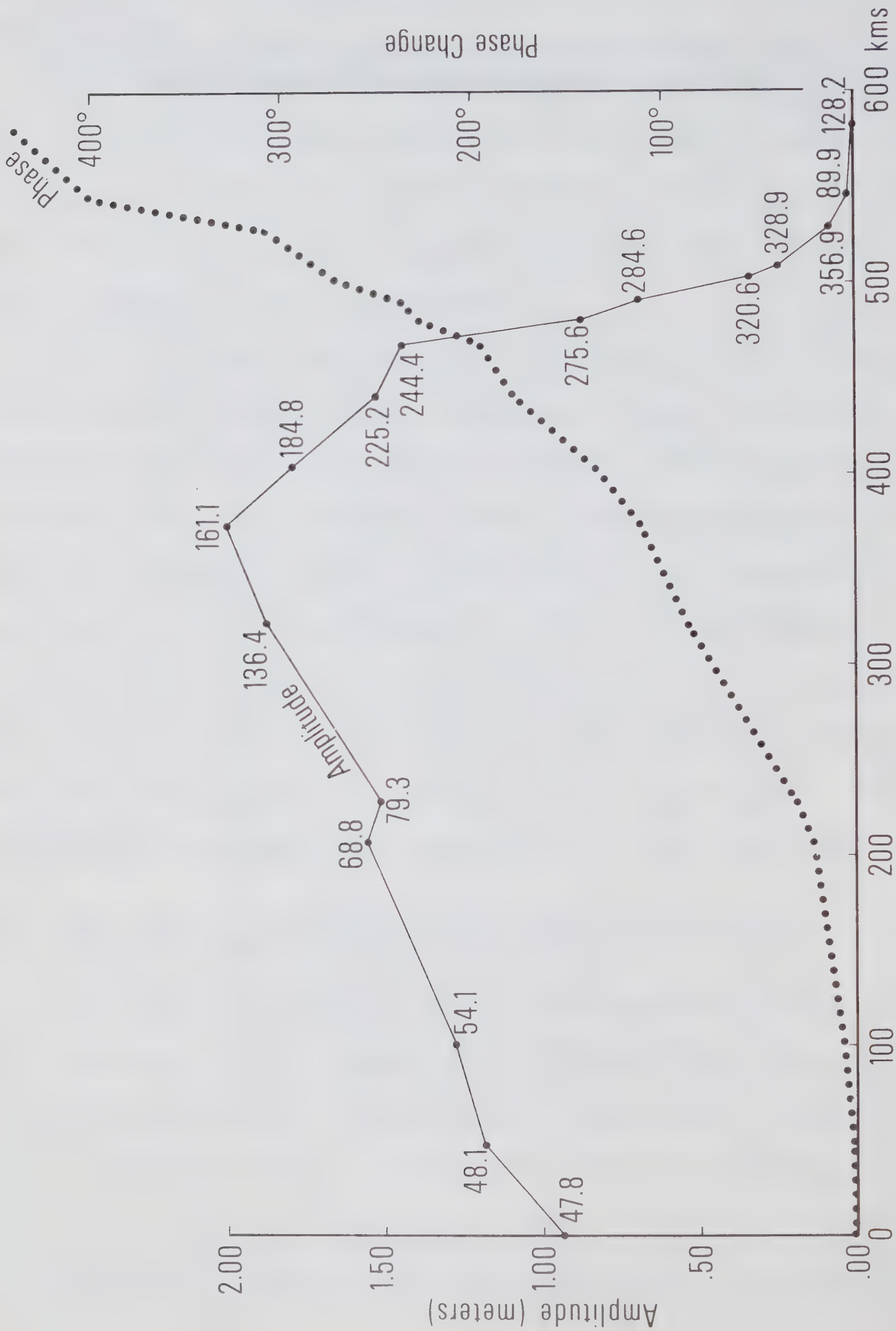


Fig. 18(a) M_2

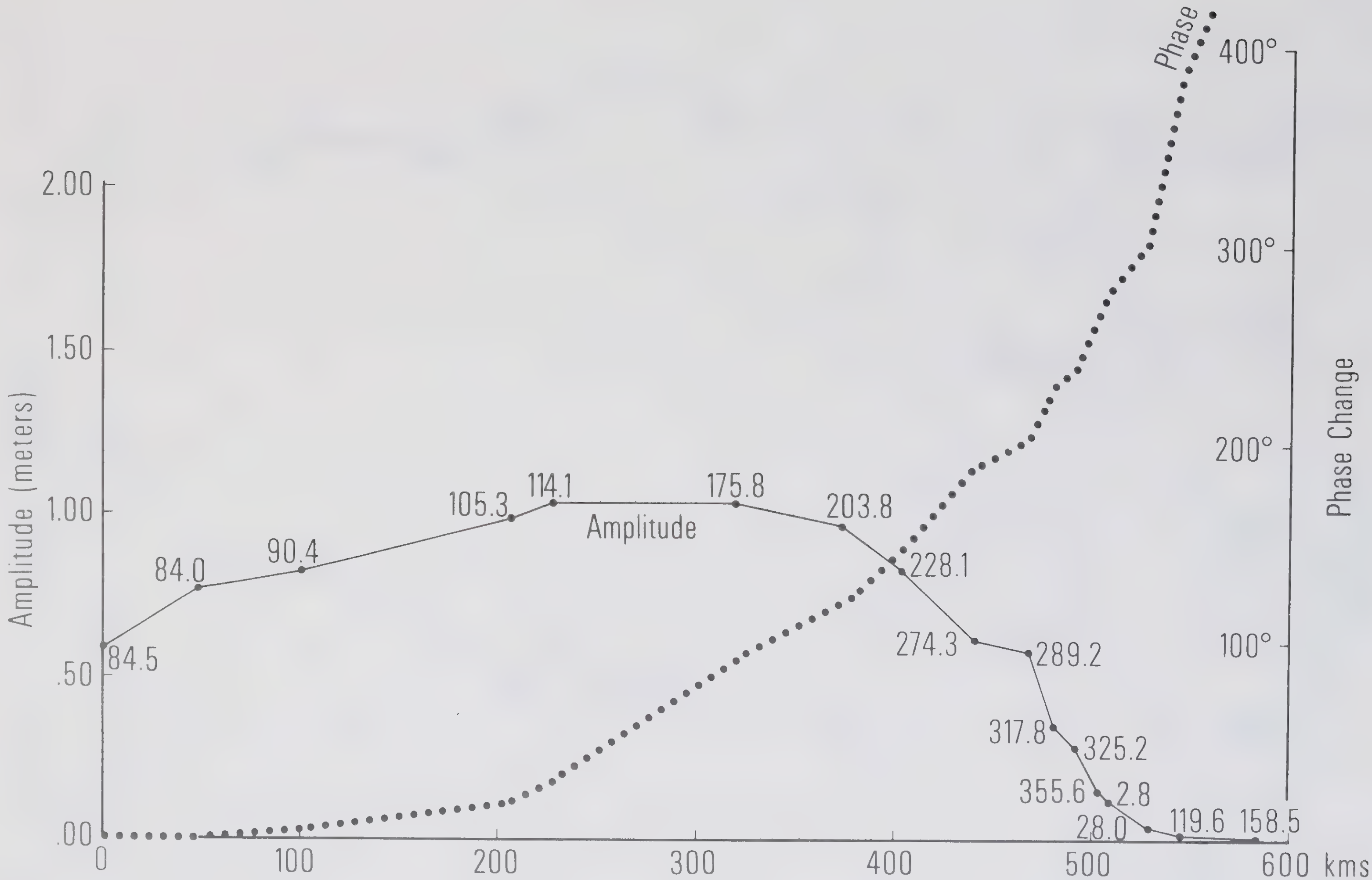

Phase
400°
300°
200°
100°
Phase Change
2.00
1.50
1.00
.50
.00
Amplitude (meters)
Amplitude
84.5
84.0
90.4
105.3
114.1
175.8
203.8
228.1
274.3
289.2
317.8
325.2
355.6
2.8
28.0
119.6
158.5
0
100
200
300
400
500
600 kms

Fig. 18(b) S_2

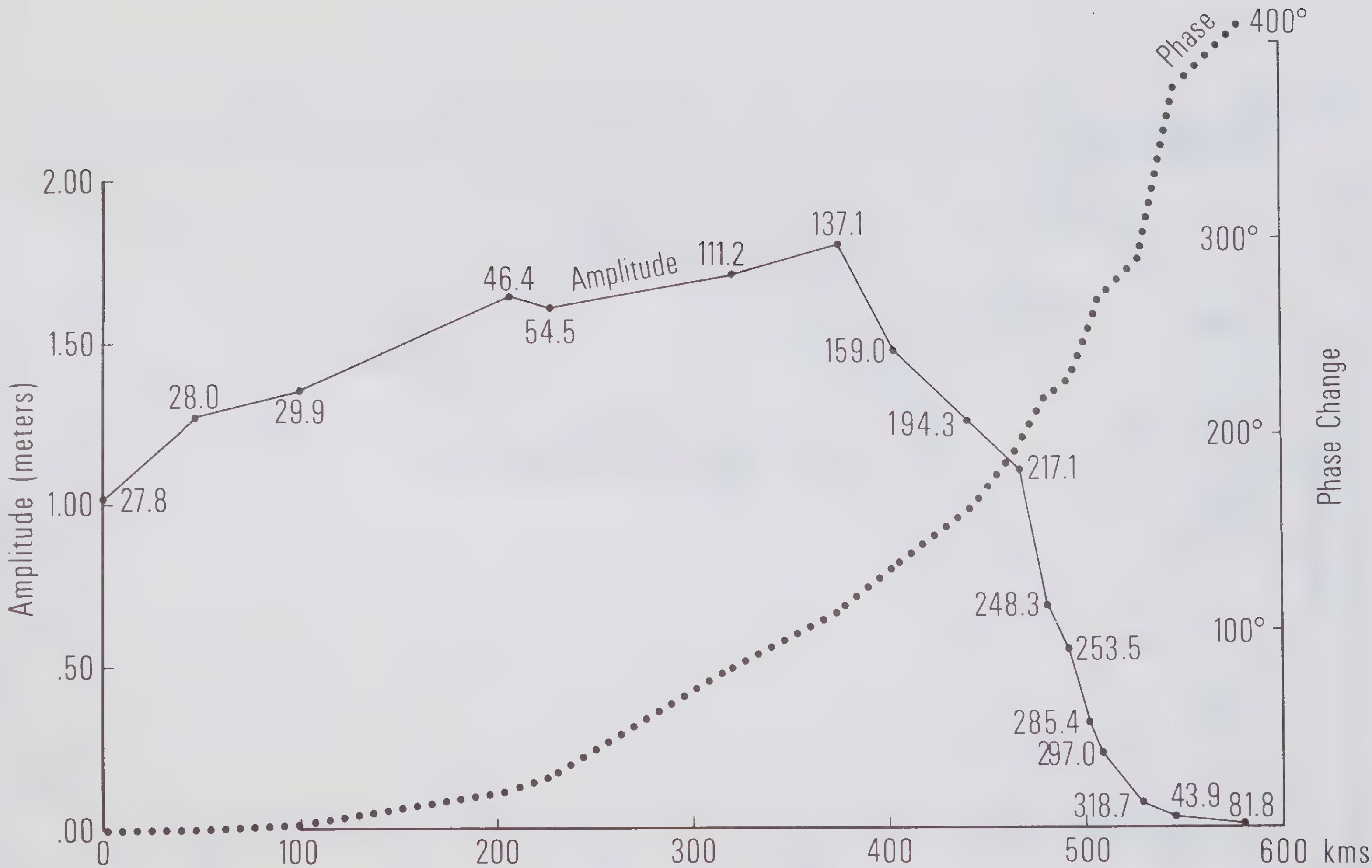

Phase
400°
300°
200°
100°
Phase Change
2.00
1.50
1.00
.50
.00
Amplitude (meters)
Amplitude
27.8
28.0
29.9
46.4
54.5
111.2
137.1
159.0
194.3
217.1
248.3
253.5
285.4
297.0
318.7
43.9
81.8
0
100
200
300
400
500
600 kms

Fig. 18(c) N_2

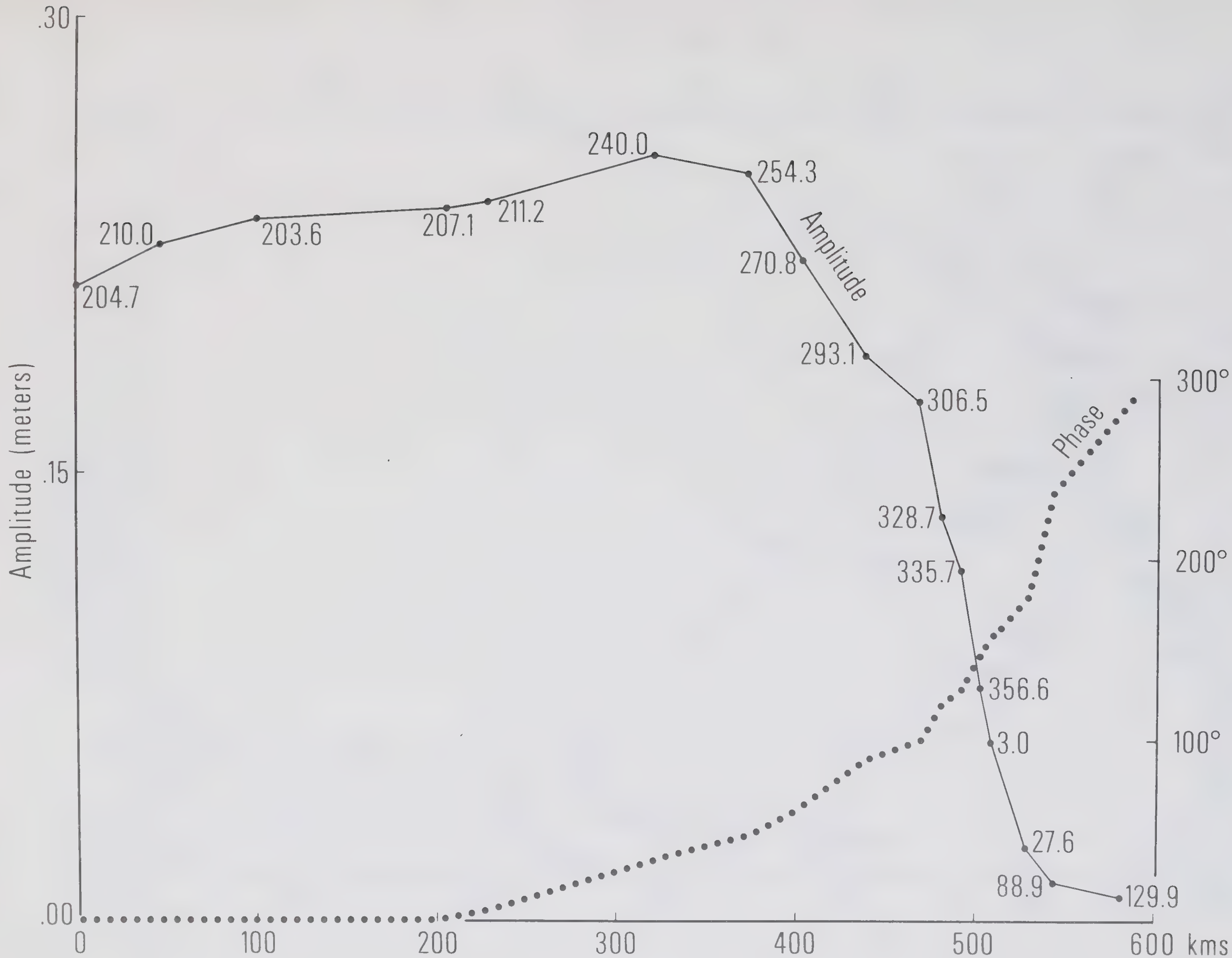

.30
.15
.00
Amplitude (meters)
210.0
204.7
203.6
207.1
211.2
240.0
254.3
270.8
Amplitude
293.1
306.5
328.7
335.7
356.6
3.0
27.6
88.9
129.9
Phase
300°
200°
100°
Phase Change
0
100
200
300
400
500
600 kms

Fig. 18(d) K_1

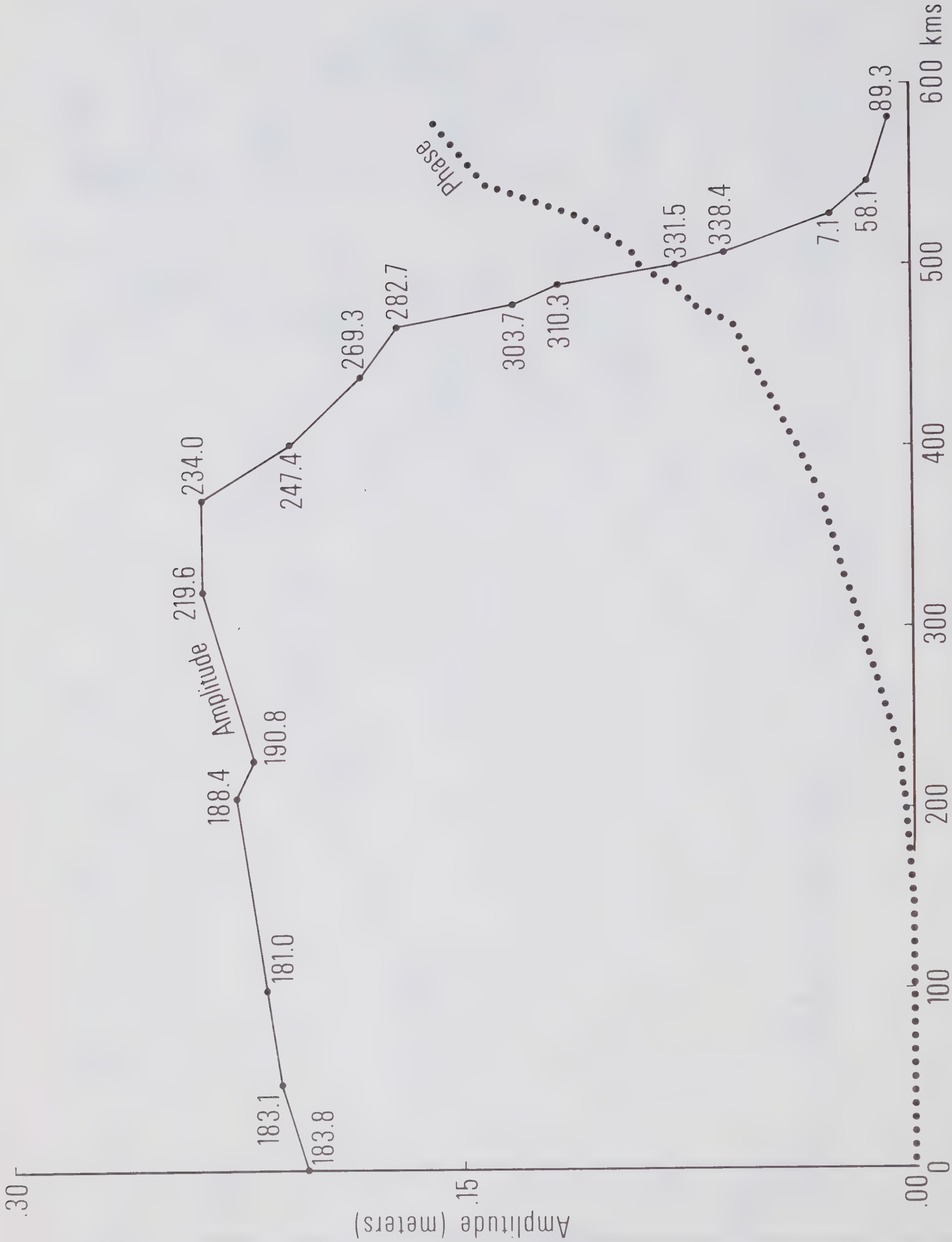
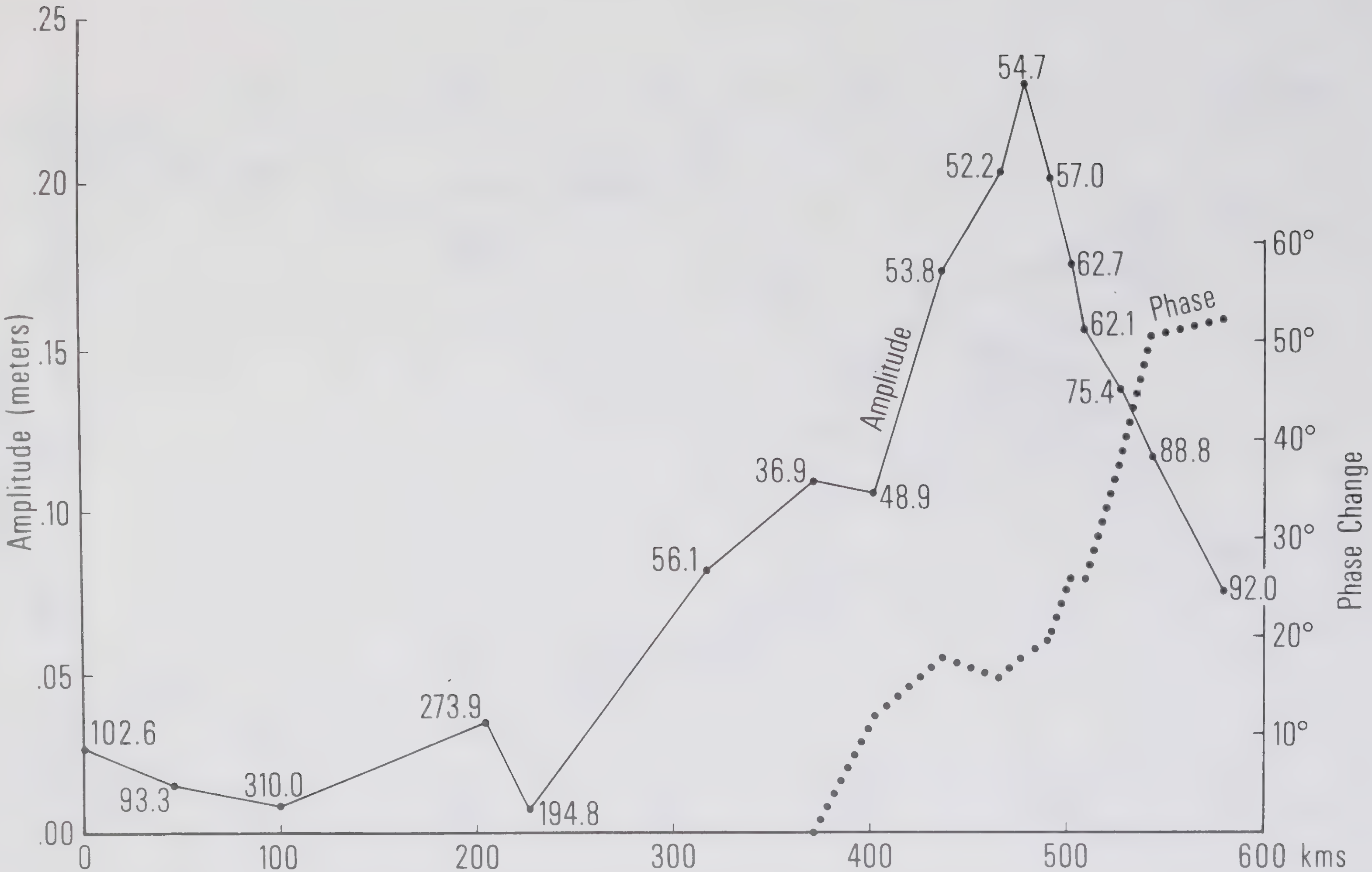

Amplitude (meters)
.30
.15
.00
0
100
200
300
400
500
600 kms
Amplitude
Phase
183.8
183.1
181.0
188.4
190.8
219.6
234.0
247.4
269.3
282.7
303.7
310.3
331.5
338.4
7.1
58.1
89.3

Fig. 18(e) O_1



.25
.20
.15
.10
.05
.00
Amplitude (meters)
0
100
200
300
400
500
600 kms
60°
50°
40°
30°
20°
10°
Phase Change
Amplitude
Phase
102.6
93.3
310.0
273.9
194.8
56.1
36.9
48.9
53.8
52.2
54.7
57.0
62.7
62.1
75.4
88.8
92.0

Fig. 18(f) MSf

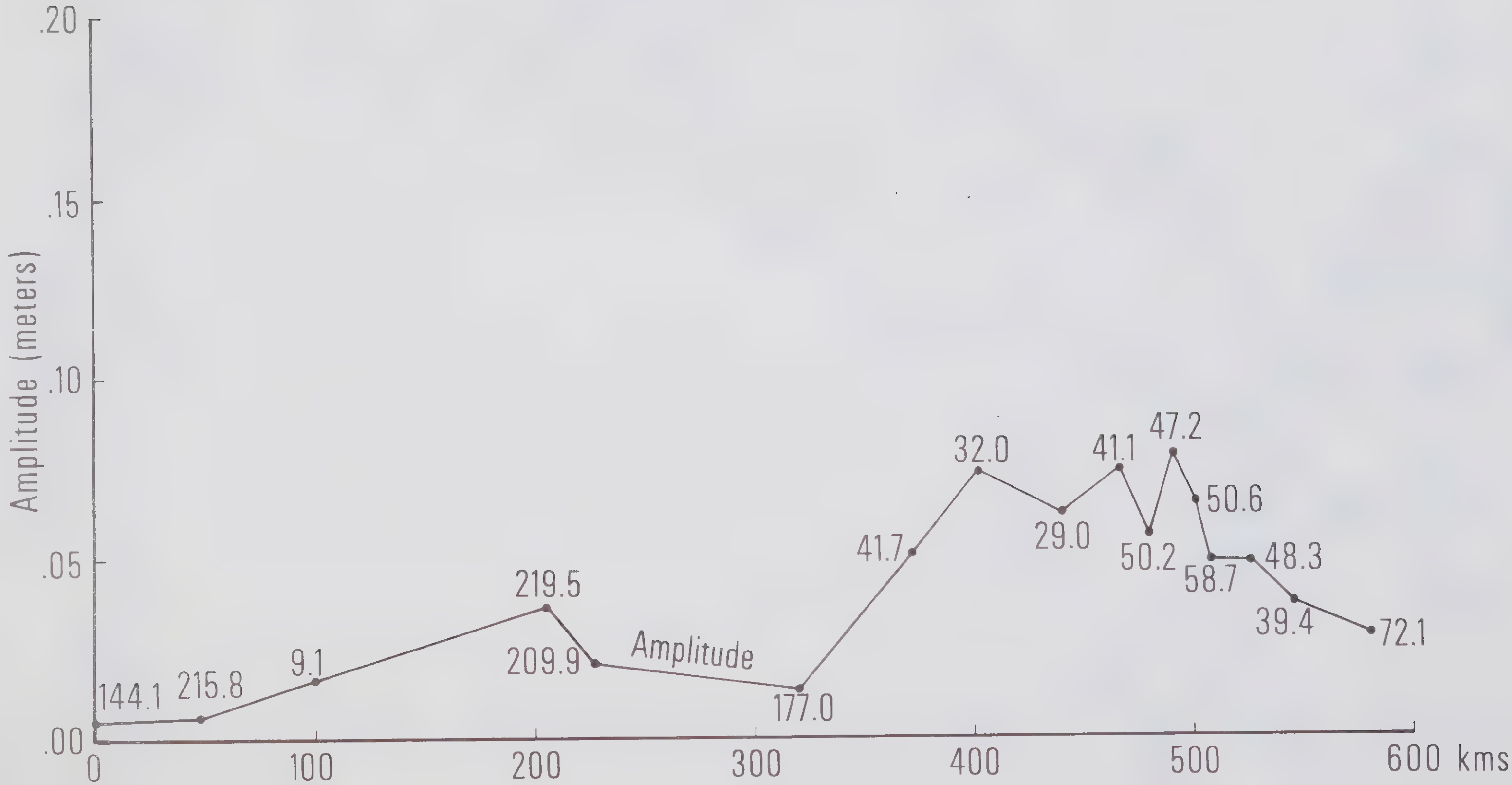
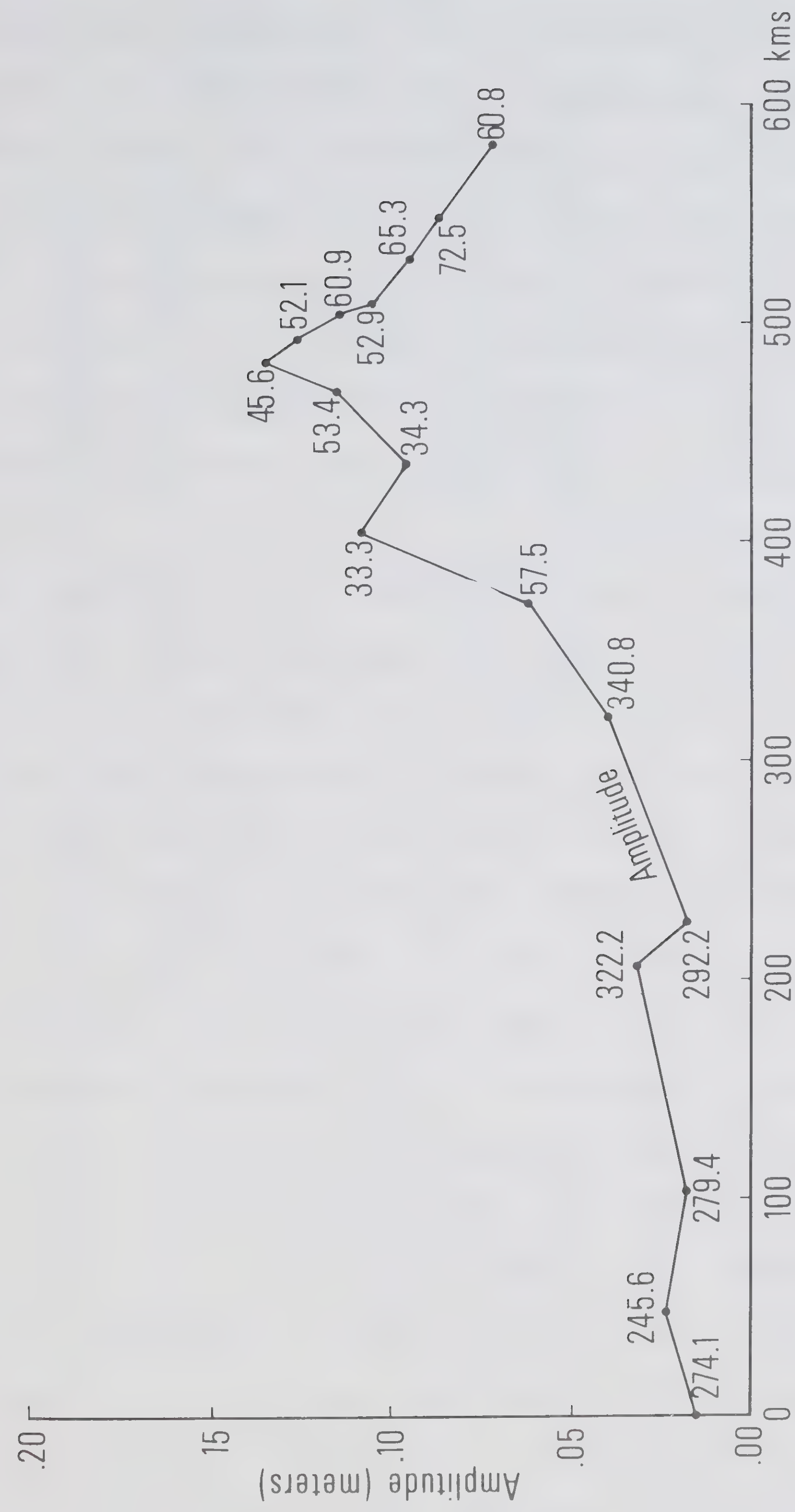

.20
.15
.10
.05
.00
Amplitude (meters)
0
100
200
300
400
500
600 kms
144.1
215.8
9.1
219.5
209.9
Amplitude
177.0
41.7
32.0
29.0
41.1
47.2
50.2
50.6
58.7
48.3
39.4
72.1

Fig. 18(g) Mf



Amplitude (meters)
.20
.15
.10
.05
.00
0
100
200
300
400
500
600 kms
Amplitude
274.1
245.6
279.4
322.2
292.2
340.8
57.5
33.3
34.3
53.4
45.6
52.1
60.9
52.9
65.3
72.5
60.8

Fig. 18(h) Mm

fluctuations of the residual current, a part of the spectrum which is not worth subjecting to a harmonic analysis. Next the east and north components of the semidiurnal and diurnal bands are analyzed harmonically. This method takes in any amount of data and can handle as well discontinuous records which are more the rule than the exception. All the steps of the analysis are plotted in order to afford a visual interpretation of the success or failure of the analysis; a side profit is the quick discovery of the errors in the original data which could easily have escaped a thorough inspection of the numerical records and which can then be corrected.

Table 5 lists the labels of the stations occupied by the Ship Channel Division at Québec Bridge and Isle aux Couldres during their survey of 1968. The table gives the code name of the station, the time interval at which the data were abstracted and the useful portion of the record which can be subjected to an analysis. Fig. 19 and 20 show the geographical location of the stations listed in Table 5. Stations B-29 and B-32 are located at nearly the same point of the river and their combined records may be analyzed as a single discontinuous record. The same thing holds for stations B-30 and B-31. The pair B-29 and B-32 is located exactly in the middle of the river while B-30 and B-31 are closer to the south shore.

For the Isle aux Couldres, station B-25 is located in the north arm of the river half way between St. Joseph de la Rive and Isle aux Couldres. The other stations are strung at approximately equal distances apart from the Isle aux Couldres to the south shore.

Table 5

Current Observations at Québec Bridge and Isle aux Couldres

Station Label	Time interval between Observations Δt minutes	Useable Portion of the Record
Québec Bridge		
Middle B-29	30	10.30 Oct. 2-15.30 Oct. 16 1968
B-32	30	12.30 Oct. 17-8.30 Nov. 3 1968
South B-30	30	12.00 Oct. 2-13.30 Oct. 16 1968
B-31	30	12.00 Oct. 17-7.00 Nov. 5 1968
Isle aux Couldres		
North B-25	30	21.00 June 10-4.00 June 27 1968
South N-20	10	17.30 June 18-17.10 July 10 1968
N-19	10	10.30 June 30-18.40 July 11 1968
B-26	30	0.00 June 21-0.30 July 4 1968
N-18	10	17.40 June 17-12.50 June 29 1968

Fig. 19 Location of the Current Meters and the Tide Gauges at Québec Bridge

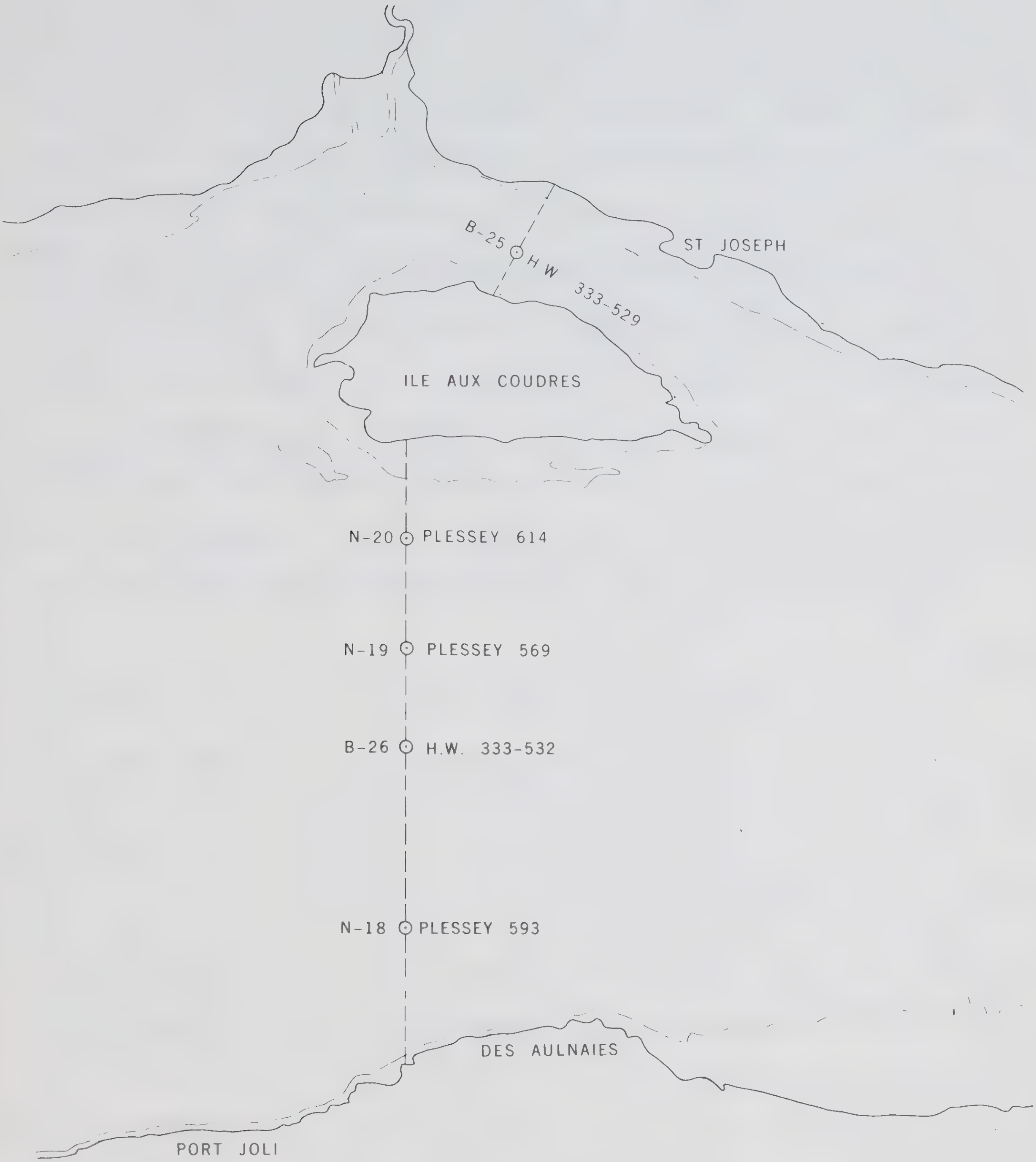


Fig. 20 Location of Current Meters at Isle aux Coudres

Figs. 21 and 22 show graphically the results for the analysis at Québec Bridge. X and Y represent respectively the east and north component of the current u in the raw. LP is the low pass and gives the residual current which here coincides with the steady current created by the river discharge. In the case under consideration the flow is remarkably constant. X-PR and Y-PR are the values of X and Y reconstituted from the analysis which is made up wholly of tidal harmonics. X-ER and Y-ER are the discrepancies between the observed current components and reconstituted components. The "error" graph shows more than random fluctuations: there are systematic high frequency oscillations which the harmonic analysis could not pin down. These are responsible for the failure of the reconstituted curve to reproduce the sharpness of the observed curve especially at neaps.

Fig. 23 shows the results for stations B-25, N-20, N-19, B-26 and N-18. The plots indicate that the analysis succeeds very poorly in reproducing the tide on account of the fact that the constituents N_2 and S_2 could not be separated from M_2 at most stations because of the short interval of observation. The only recourse left is to correct the analyzed values for these unknown constituents with the technique of inference, an unpleasant chore the results of which I have little faith in. For this purpose I have used the results of the analysis at the coastal station of St. Jean Port Joli which gives the following empirical relations within the subgroups of constituents:

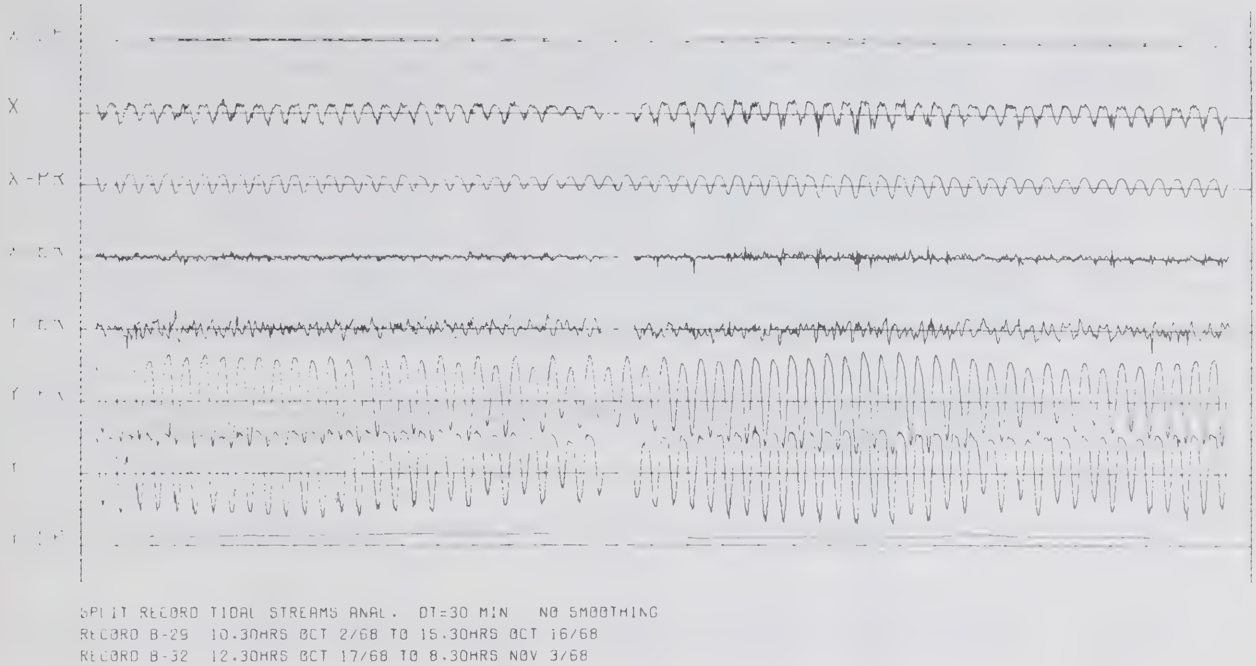


Fig. 21 Analysis of Stations B-29, B-32 at Québec Bridge

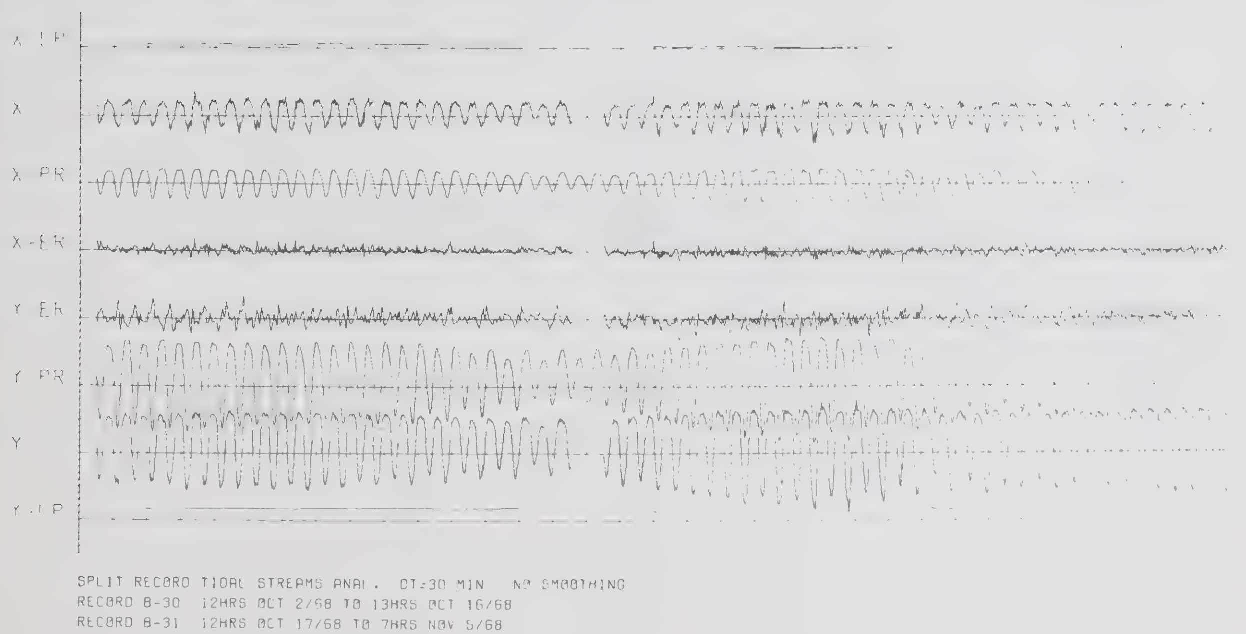


Fig. 22 Analysis of Stations B-30, B-31 at Québec Bridge

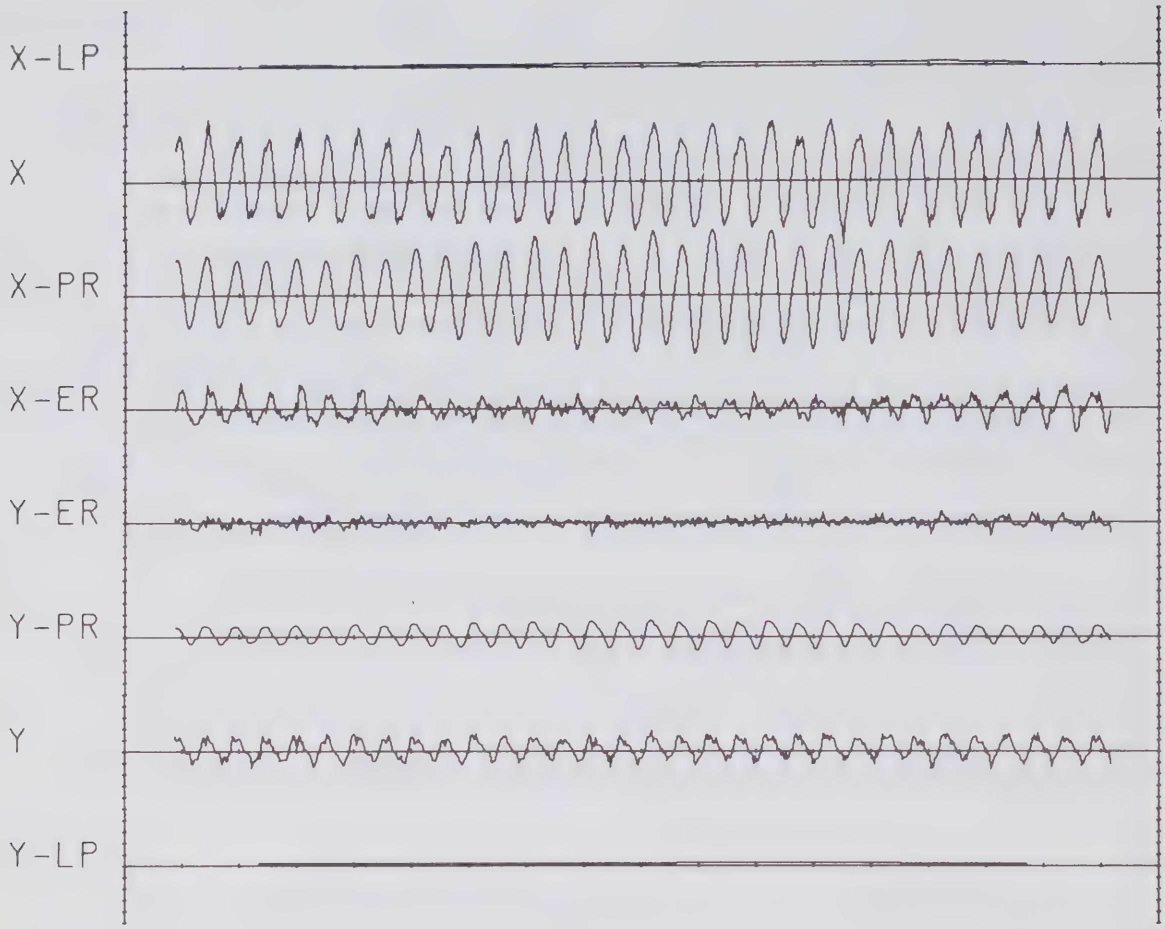


FIG. 23(a)#1. TIDAL STREAMS ANALYSIS DT=30 MIN NO SMOOTHING RC=.9 RECORD B-25 21HRS JUNE 10/68 TO 4HRS JUNE 27/68.

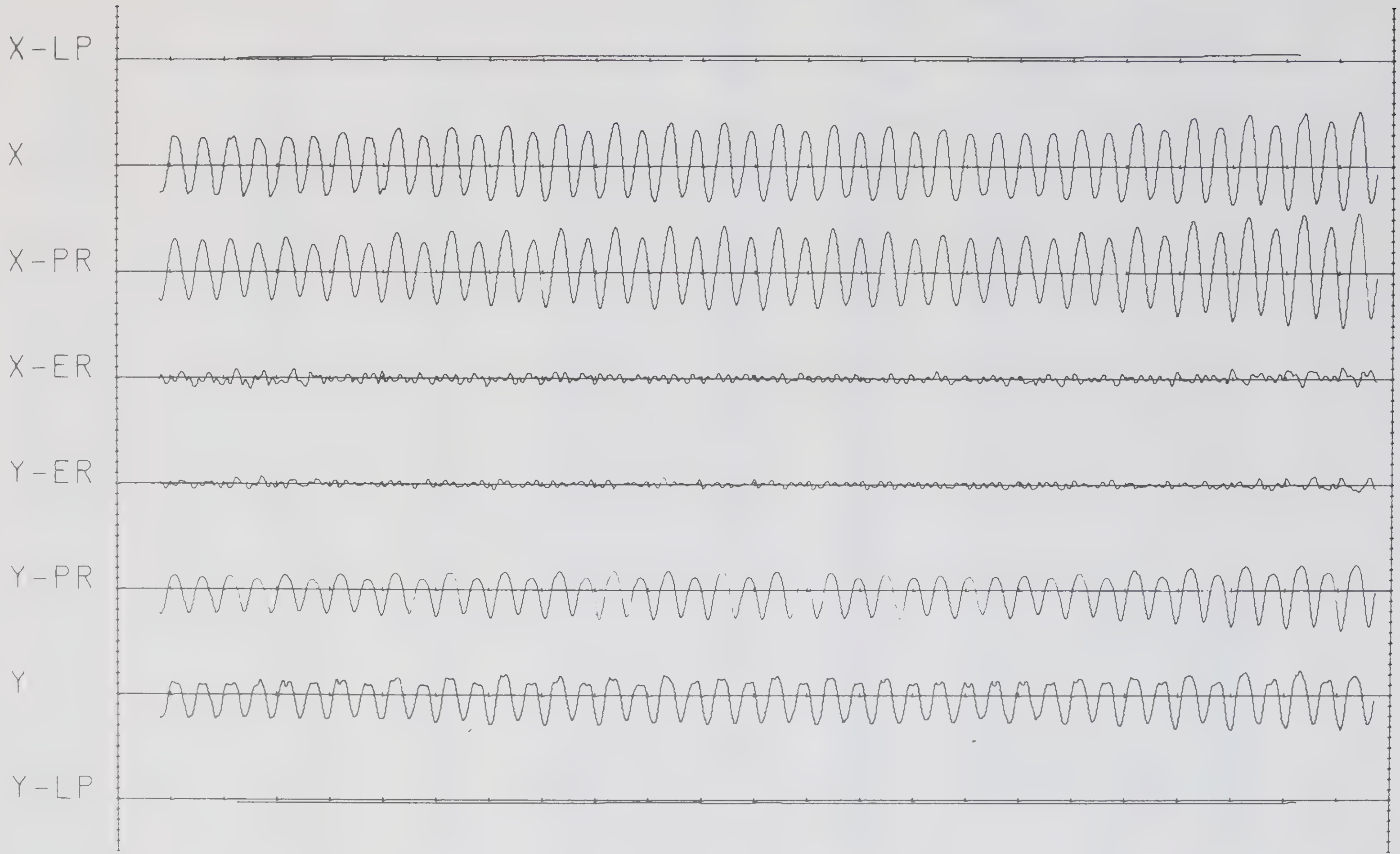


FIG. 23(a)#2. TIDAL STREAMS ANALYSIS DT=10 MIN SMOOTHING DT(ANAL)=30 MIN RC=0.8 RECORD N-20 17.40 JUNE 18/68 TO 18.50 JULY 11/68.

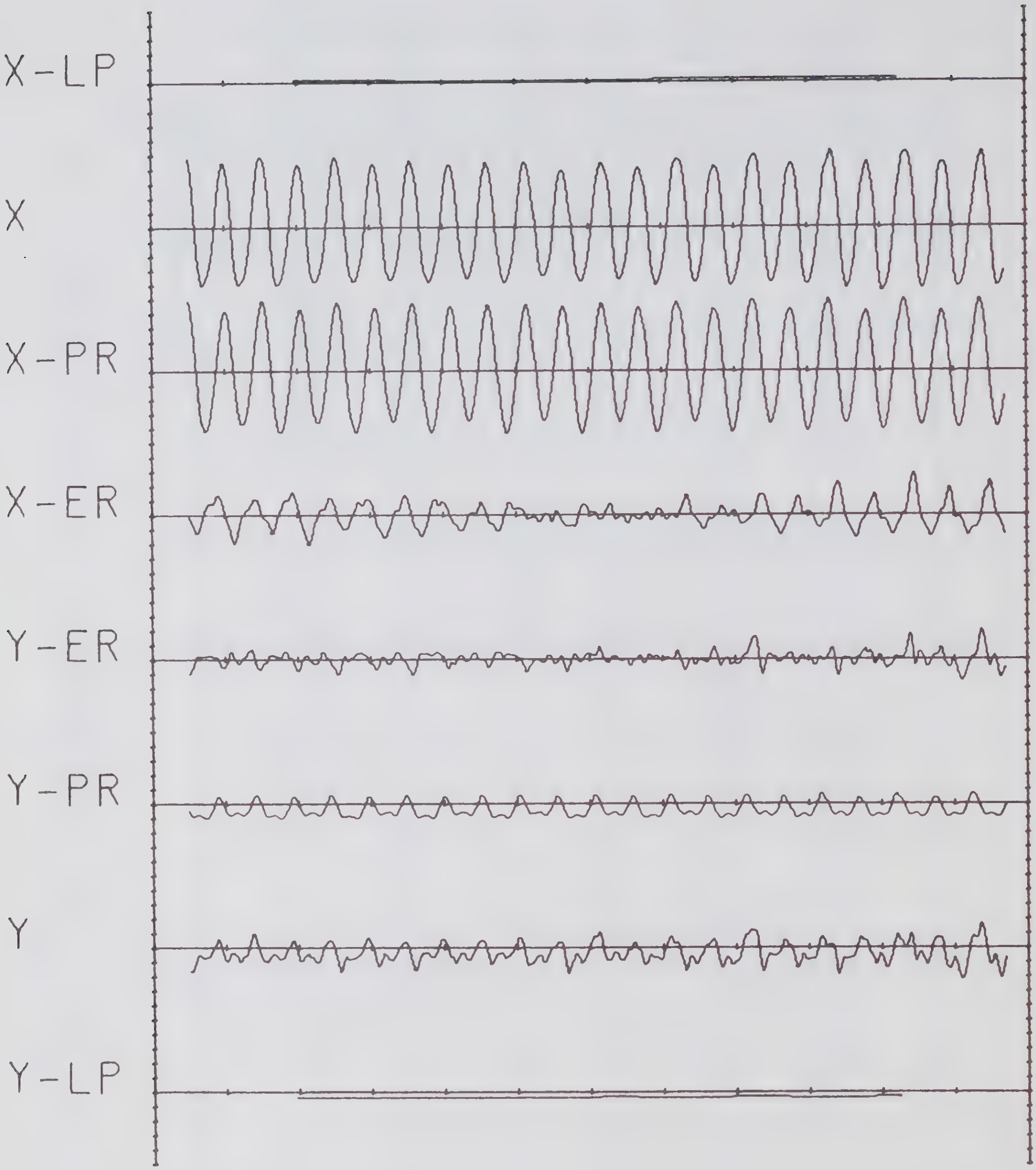


FIG. 23(a)#3. TIDAL STREAMS ANALYSIS DT=10 MIN SMOOTHING DT(ANAL)=30 MIN RC=0.9 RECORD N-19 10.30 JUNE 30/68 TO 18.40 JULY 11/68.

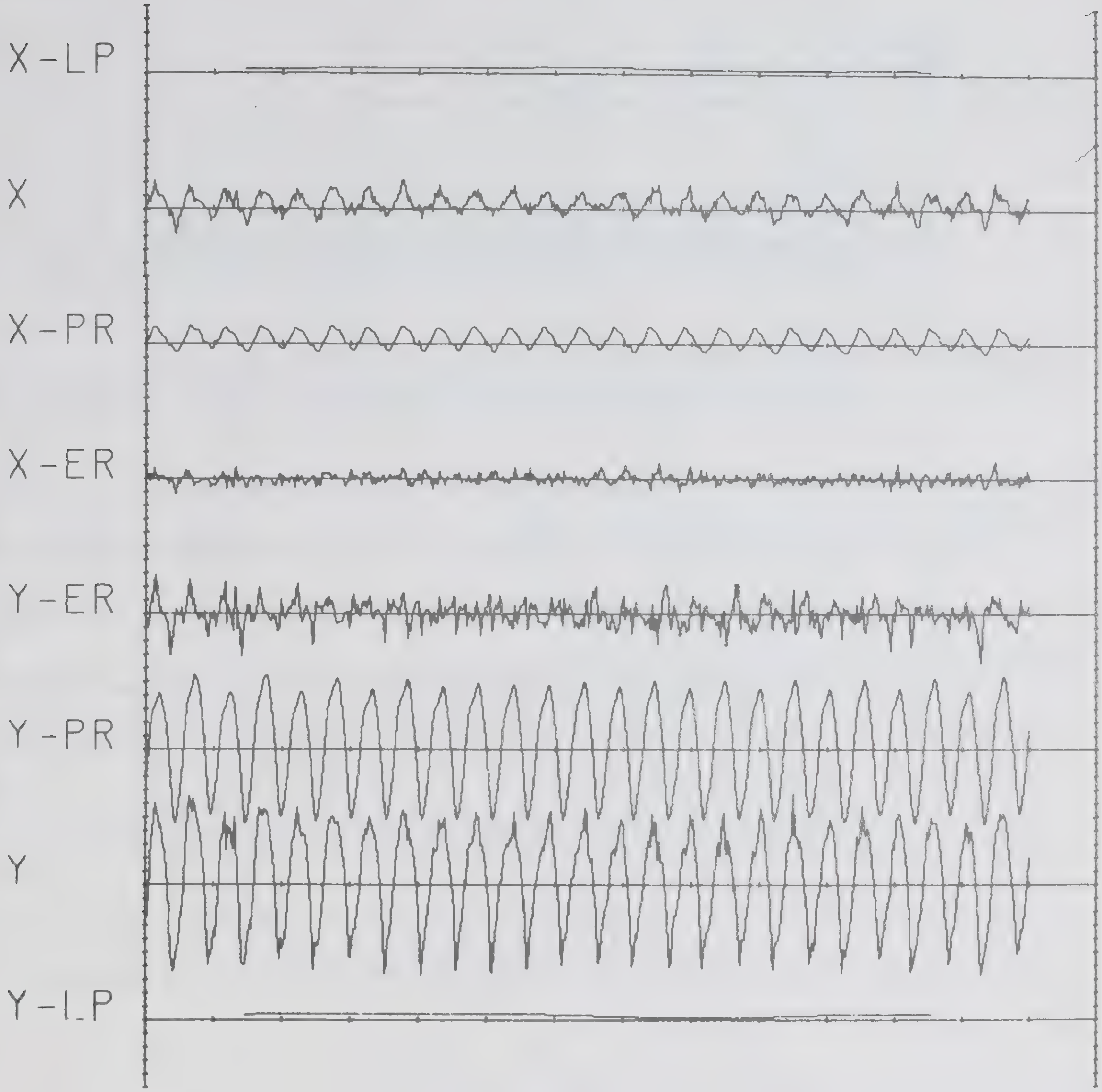


FIG. 23(a)#4. TIDAL STREAMS ANALYSIS DT=30 MIN NO SMOOTHING RC=.9 RECORD B-26 0HRS JUNE 21/68 TO 0HRS JULY 4/68.

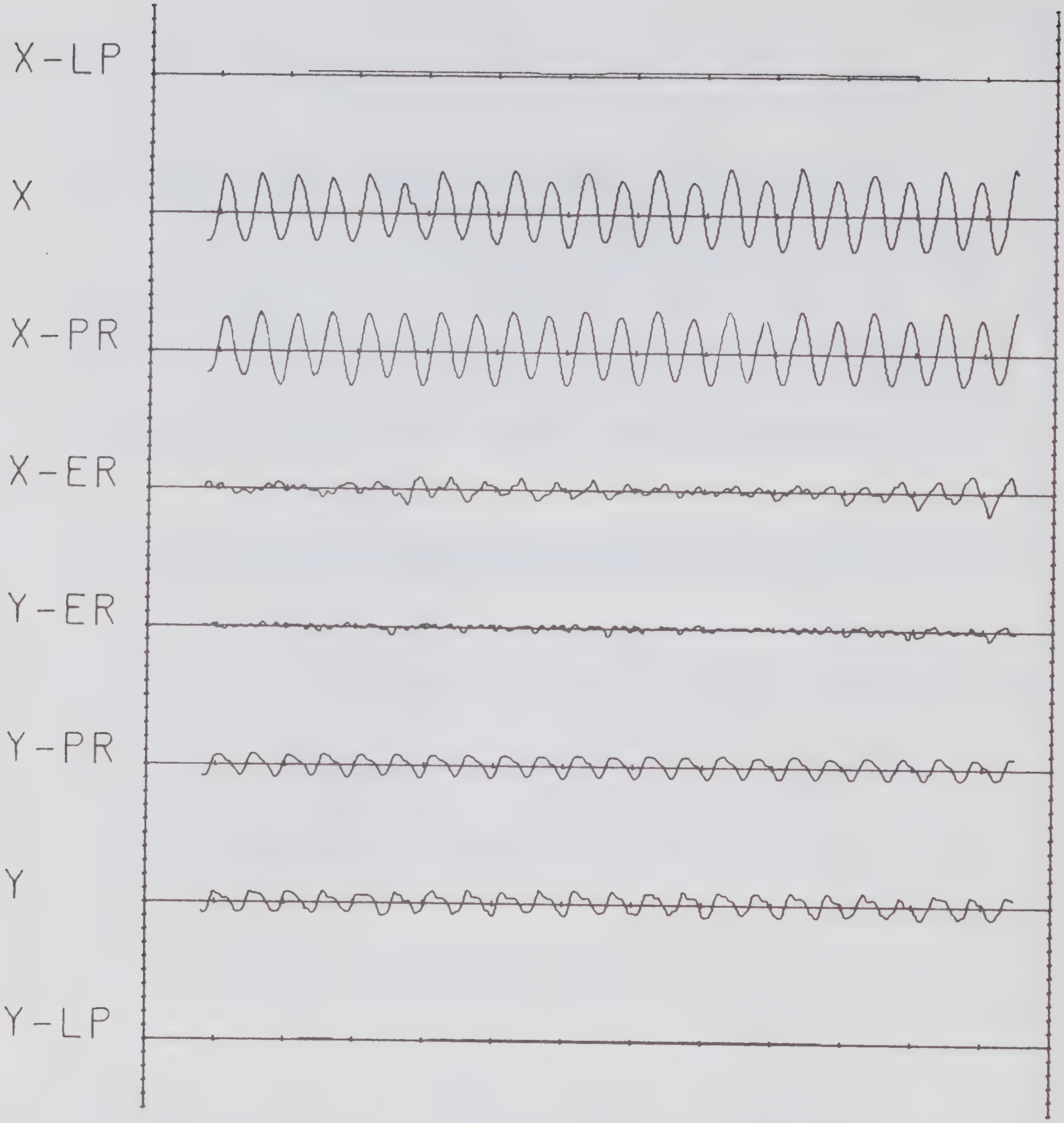


FIG. 23(a)#5. TIDAL STREAMS ANALYSIS DT=10 MIN SMOOTHING DT(ANAL)=30 MIN RC=0.9 RECORD N-18 17.40 JUNE 17/68 TO 12.50 JUNE 29/68.

Subgroup:	Constituents		
M_2	Name	Ratio	Phase Difference
	M_2	1.00	00
	S_2	.27	+40°
	N_2	.18	-25°
K_1	K_1	1.00	00
	O_1	.93	-20°
	P_1	.27	00

In only one instance, station N-20, was it possible to lower the Rayleigh criterion to .8 in order to separate directly the important constituents. Fig. 23(b) shows the result of the inference of the constituents for the observations at Isle aux Couldres and the reduction of the Rayleigh criterion at N-20. Table 6 lists the results of the analysis for the major constituents. From the table, we see that the tidal streams reach their maximum intensity first on the shallow portions of the river and then in the channel; the tide is at its latest in the north arm of the channel, lagging by nearly two hours. The results on the steady current in the main channel seem absurd at first sight: at N-20 and N-19 the current flows almost at a right angle to the channel. A little bit of reflexion however, indicates that there may be an eddy motion set up by the tide and the steady currents reflect this general situation. More prolonged observations might give entirely different steady currents on account of the stronger averaging, while photogrammetry would indicate the instantaneous eddies set up during a tidal cycle.

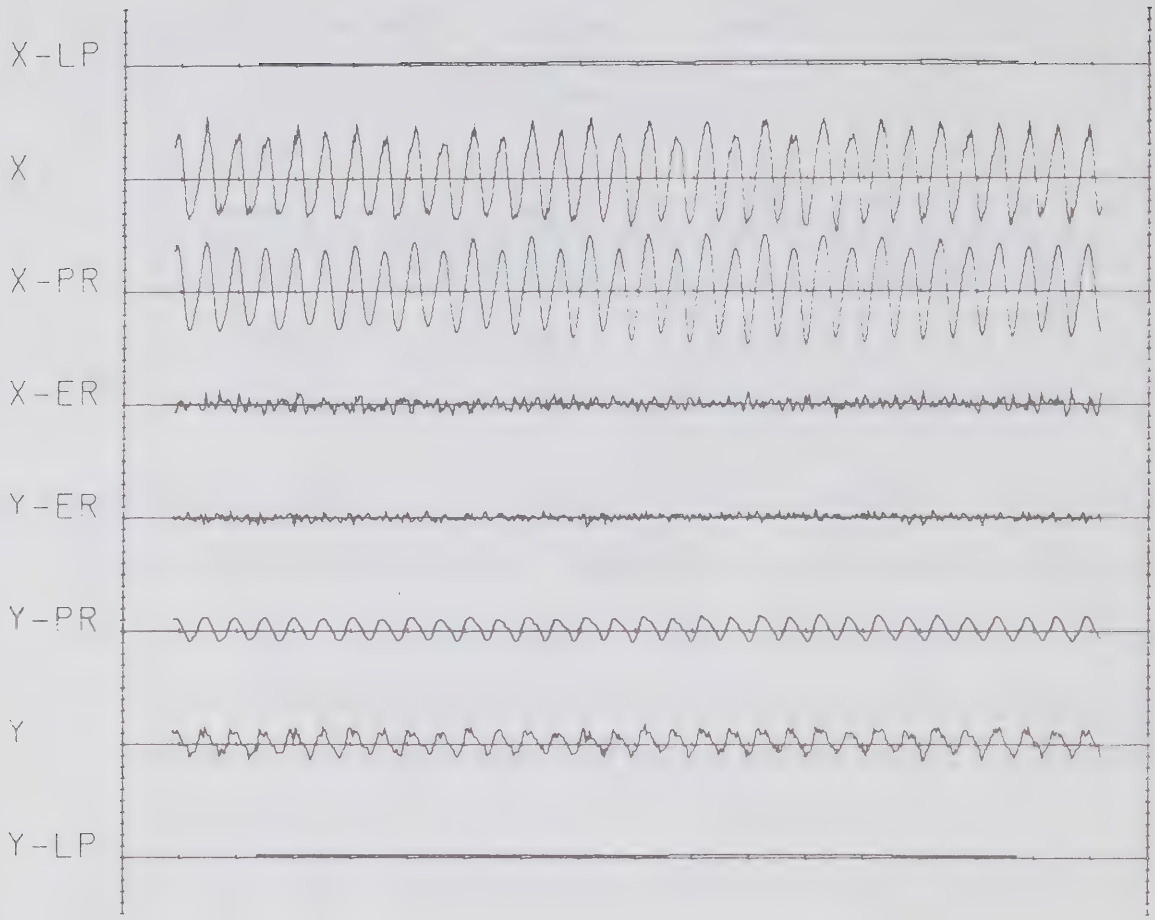


FIG. 23(b)#1. TIDAL STREAMS ANALYSIS DT=30 MIN
RECORD B-25 21HRS JUNE 10/68 TO 4HRS JUNE 27/68.

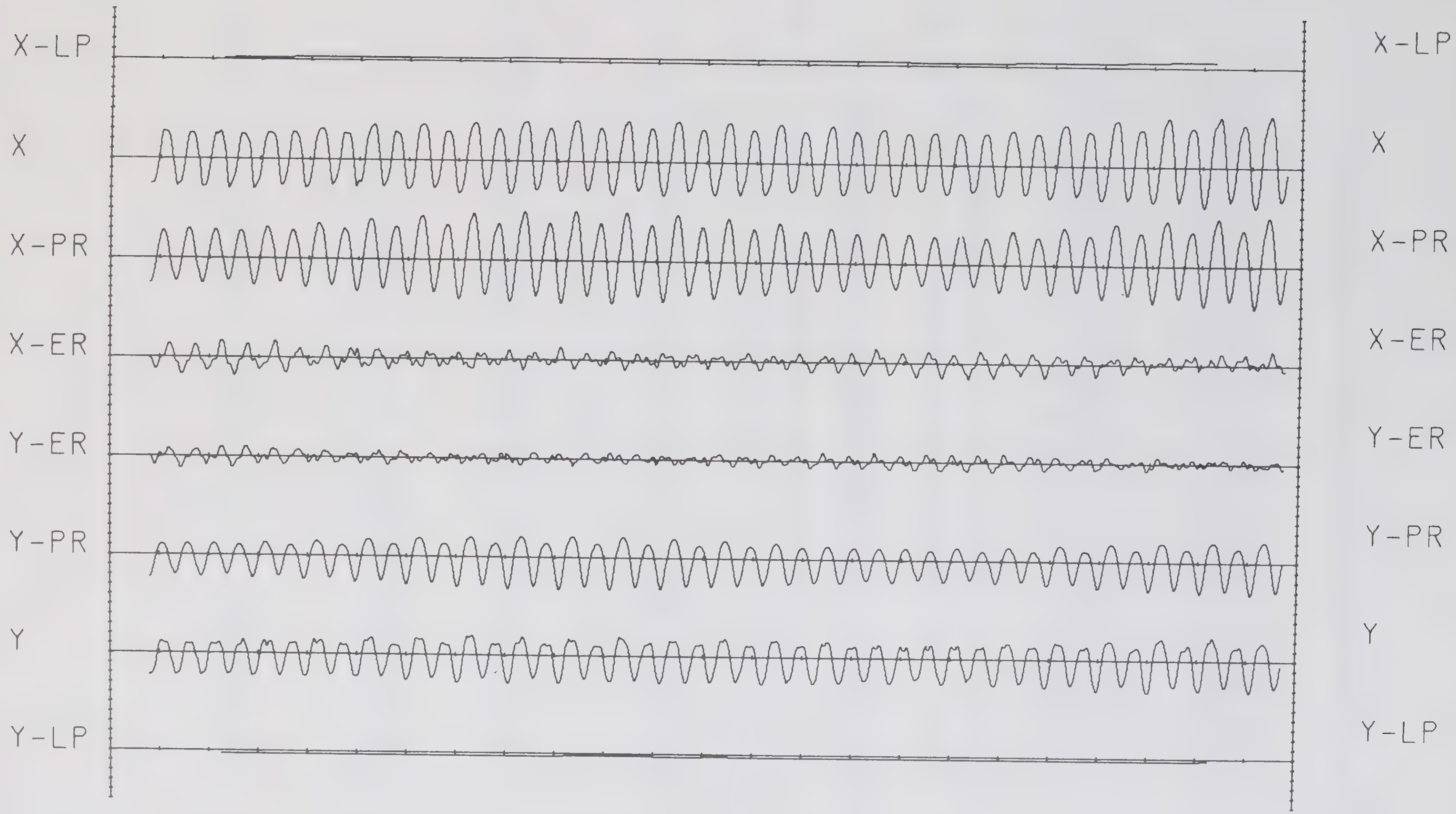


FIG. 23(b)#2. TIDAL STREAMS ANALYSIS DT=10 MIN SMOOTHING DT(ANAL)=30 MIN RC=0.9 RECORD N-20 17.40 JUNE 18/68 TO 18.50 JULY 11/68.

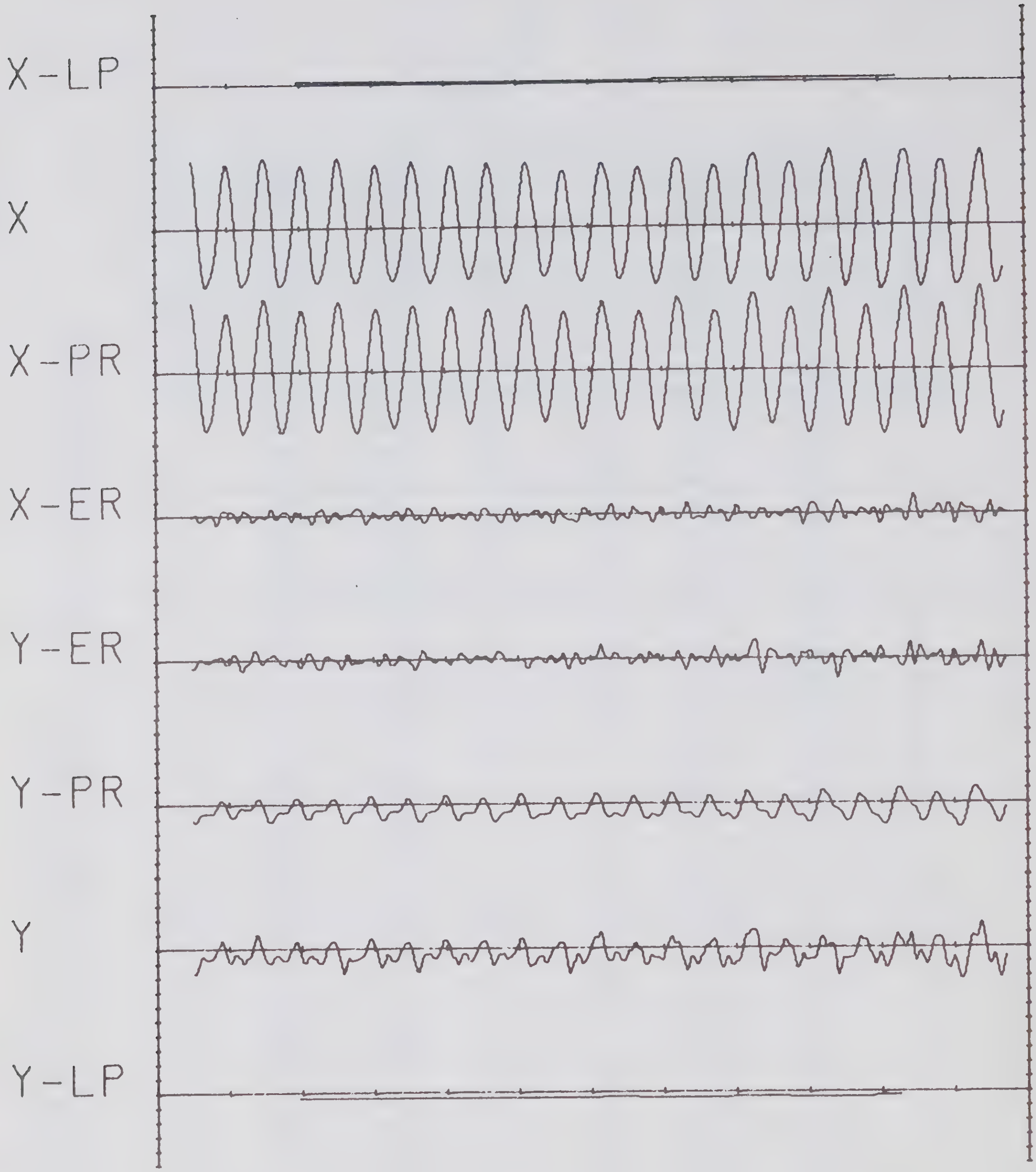


FIG. 23(b)#3. TIDAL STREAMS ANALYSIS DT=30 MIN
RECORD N-19 10.30 JUNE 30/68 TO 18.40 JULY 11/68.

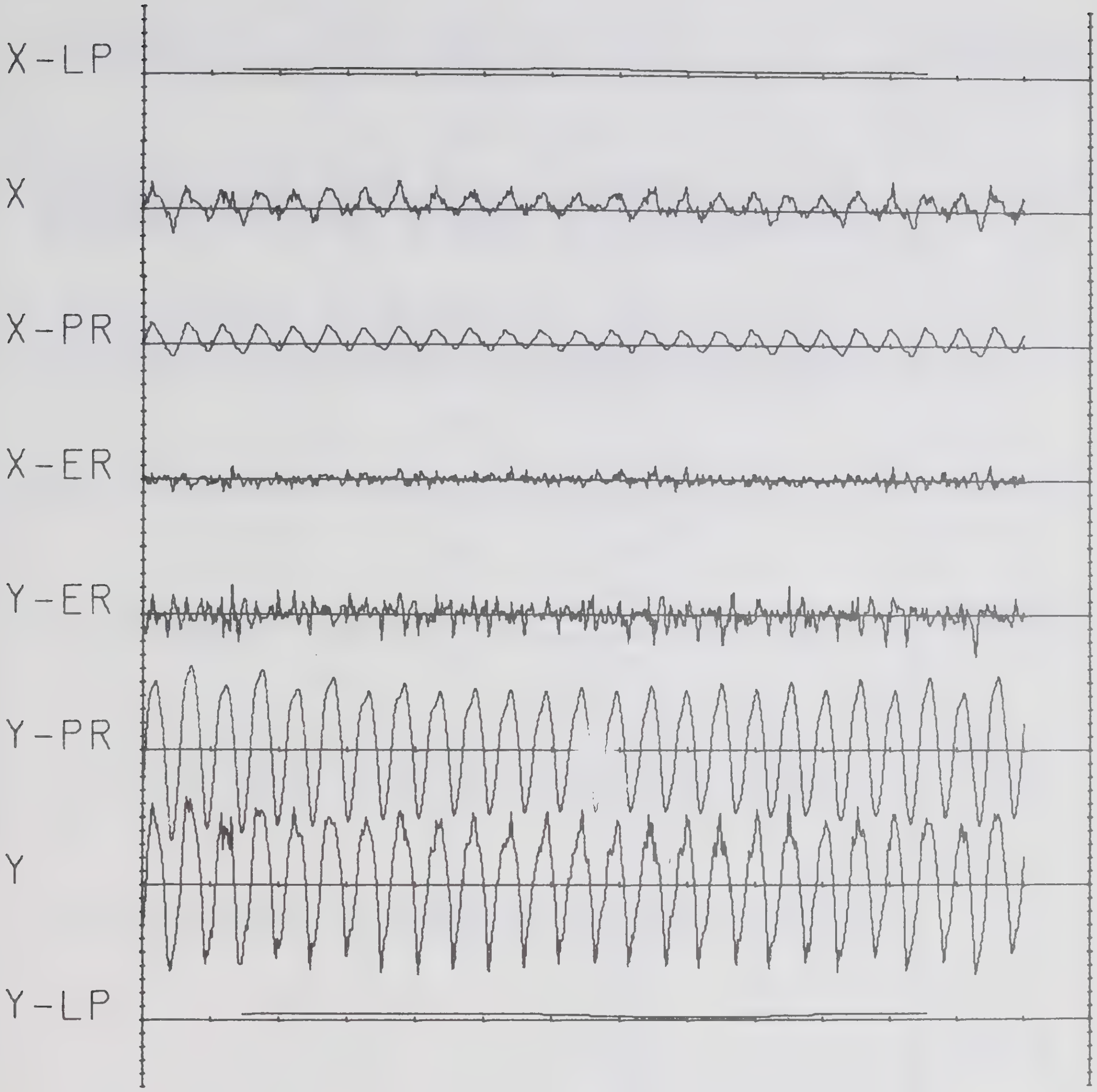


FIG. 23(b)#4. TIDAL STREAMS ANALYSIS DT=30 MIN
RECORD B-26 0HRS JUNE 21/68 to 0HRS JULY 4/68.

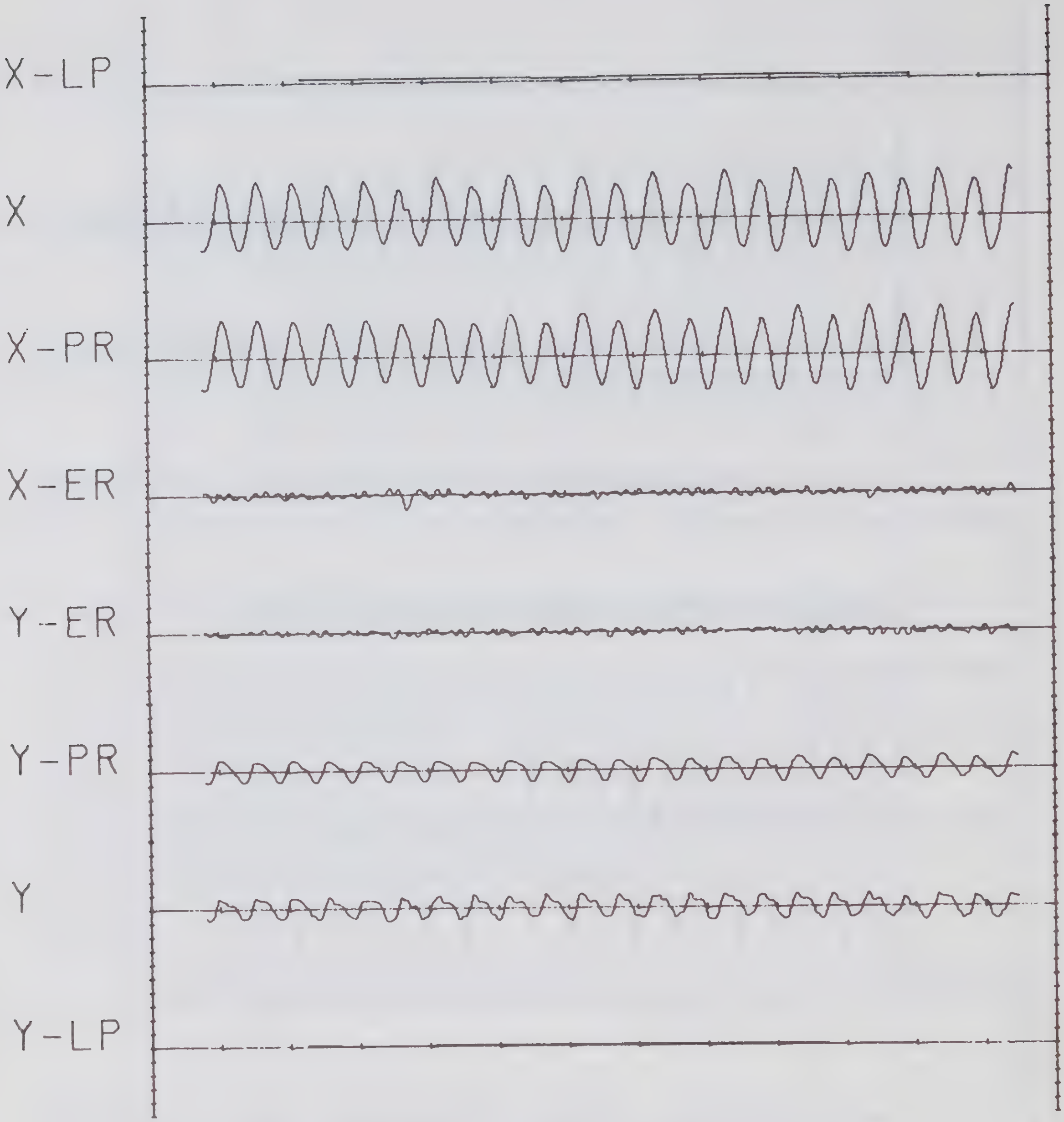


FIG. 23(b)#5. TIDAL STREAMS ANALYSIS DT=30 MIN
RECORD N-18 17.40 JUNE 17/68 TO 12.50 JUNE 29/68.

Table 6

Results of the Analyses of the current observations taken by the Ship Channel Division at Québec Bridge and Isle aux Coudres

Station	u m/sec	Inc.[2] deg	M_2 M m/sec	M_2 m* m/sec	M_2 g deg	M_2 Inc deg	S_2 M m/sec	S_2 m m/sec	S_2 g deg	S_2 Inc deg	N_2 M	N_2 m	N_2 g	N_2 Inc	K_1 M	K_1 m	K_1 g	K_1 Inc	O_1 M	O_1 m	O_1 g	O_1 Inc	M_4 M	M_4 m	M_4 g	M_4 Inc	MS_4 M	MS_4 m	MS_4 g	MS_4 Inc
Québec Bridge																														
B-29,B-32	.41	74	1.67	-	153	78	.49[3]	-	184[3]	78	.26[4]	-	148[4]	76	.12	-	227	80	.13	-	196	79	.34	-	318	87	.22	-	349	86
B-30,B-31	.51	75	1.73	.01	147	72	.59[†]	-	172[†]	71	.40	-.02	162	73	.14	-	220	73	.13	-	199	72	.38	-	306	70	.28	.01	329	71
Isle aux Couldres																														
North Shore																														
B-25	.10	43	1.27	.03	98	14	.30	-	147	12	.23@	-	73@	14@	.12	-	265	10	.08	-	268	14	.16	-	314	342	.04	-	184	
Main Channel																														
N-20	.14	323	1.30	-.04	35	31	.24	-.01	83	30	.26	-	351	30	.12	-	153	29	.09	-	126	28	.12	.03	235	62	.03	-	290	75
N-19	.13	294	1.51	-.19	53	3	.41@	-.06@	93@	3@	.27@	-.05@	28@	3@	.10	-	188	11	.10@	-	168@	11@	.13	-	147	302	.08	-	231	304
B-26	.17	48	1.39	.06	67	81	.38@	.02@	107@	81@	.25@	.02@	42@	81@	.07	-	268	72	.08	-	223	85	.25	-	97	93	-	-	-	-
N-18	.09	11	.95	-.03	46	16	.26@	-	86@	16@	.17@	-	21@	16@	.08	-	178	12	.07@	-	158@	12@	.07	-	173	104	-	-	-	-
Average	.06	45[1]	1.26		051		.32		94		.22		021		.07		186		.07		163		.09		140		.05		248	

M=semimajor axis
m=semiminor axis
*when positive, indicates that the current vector rotates counterclockwise around the ellipse; otherwise, it is clockwise

[1] Vector average of the component along the river bed which we assume orientated at 45°
[2] The inclination is in trigonometric degrees, not in geographical degrees
[3] inferred [4] =uncorrected for ν_2
[†] =uncorrected for K_2

The minor component of the tidal stream ellipse is very small; an indication that the tidal streams are very nearly rectilinear everywhere.

3.4 Comparison of the values of the tidal streams observed with those predicted by W.D. Forrester using the Equation of Continuity

W.D. Forrester (B.I. 69-2 and B.I. 67-5) has used the equation of continuity

$$Q_x = -BD_t \qquad (6)$$

in order to estimate the tidal streams in the St. Lawrence River from the observations derived from the tide gauges strung along the river, with the assumption that the tidal oscillation vanished around Lake St. Peter.

For a given harmonic, (6) becomes

$$Q_x = -i\sigma BD \qquad (21)$$

where σ is the frequency of the harmonic. In finite difference form (21) becomes

$$\frac{\Delta Q}{\Delta t} = -i\sigma BD \qquad (22)$$

If the river is divided into a succession of channels of length Δx_j, width B_j, j labelling each channel, (22) becomes for a given channel:

$$Q_{j+1}-Q_j=-i\sigma B_j D_j \quad \Delta x_j=-i\sigma A_j D_j \tag{23}$$

A_j being the horizontal area of the jth channel. (23) along with the boundary condition:

$$Q_o=0 \tag{24}$$

allows the value of the flow Q_j at section j to be evaluated for the vertical displacements upstream:

$$Q_{j+1}=-i\sigma \sum_{k=o}^{j} A_k D_k \tag{25}$$

For a harmonic constituent with amplitude $|D_k|=d_k$ and phase lag δ_k, (25) may be written as

$$Q_{j+1}=\sigma \sum_{k=o}^{j} A_k d_k e^{\,i(\delta_k-\frac{1}{2}\pi)} \tag{26}$$

from which the magnitude and phase of the flow Q at the entrance of the j+1th channel may be computed. We refer to the publication "Tidal Transports and Streams in the St. Lawrence River and Estuary" B.I. 69-2 by Dr. Forrester for specific values of Q and u for the various tidal constituents along the river.

In Table 7 we entered the value observed at Pointe des Monts by Farquharson (BIO 66-6), at Pointe au Père by Forrester

Table 7

Value of the Harmonic Constituents of the Currents Observed Compared with the Value Calculated by Forrester Using the Equation of Continuity

	Steady Current m/sec	M_2 m/sec	°	S_2 m/sec	°	N_2 m/sec	°	K_1 m/sec	°	O_1 m/sec	°	M_4 m/sec	°	MS_4 m/sec	°
Pointe des Monts															
calc	.001	.18	339	.06	015	.04	316	.018	118	.016	101	.005	353	.001	089
obs		.16	341	.06	022	.04	318	.015	120	.015	115	.008	332	.001	005
Pointe au Pére															
calc	.001	.14	356	.04	031	.03	330	.014	124	.012	107	.005	354	.001	058
obs		.12	343	.03	023	.03	330	.013	150	.010	107	.007	021	.003	079
Isle aux Couldres															
calc	.03	1.16	058	.30	100	.21	033	.08	157	.07	141	.07	159	.04	214
obs	.06	1.26	051	.32	94	.22	021	.07	186	.07	167	.09	140	.05	248
Québec Bridge															
calc	.39	1.55	154	.35	205	.25	117	.11	224	.11	209	.40	312	.19	008
obs	.41	1.67	153	.38	199*	.30	140*	.12	227	.13	196	.34	318	.22	349

*S_2 has been corrected for the influence of K_2. N_2 has been corrected for ν_2.

(BIO 67-5), Isle aux Couldres and Québec Bridge by the Ship Channel Division (1969) and we compare them with the values deduced by Forrester using (26) which are based directly on the equation of continuity.

The values predicted agree to within 10 percent of the value observed; it must also be realized that the "observed" values are uncertain by this amount as well on account of the relatively short duration of the observations and the highly variable character of the currents.

This brings to the fore the question of the wisdom of taking current observations in a river, an undertaking which costs hundreds of thousands of dollars, when quite adequate values of the mean flow may be deduced from the tide gauge network if the latter is dense enough. The fine structure of the current flow across a given vertical section cannot be predicted in this way. To find out about it, one needs to set up a network of current meters at various depths and locations across the river during one or two tidal cycles at neap and spring tide. The fine structure is of no use in a one dimensional study of the river and has to be integrated away in the first place. It can only be required on rare occasions to solve specialized problems of hydrodynamics or of hydraulics.

4. A mathematical model of the St. Lawrence River

If the equations (6) and (7) hold everywhere in the St. Lawrence there is good reason to believe that their integration over the whole river bed using the actual widths B and depths D would yield all the possible modes of motion.

There is much wishful thinking in this statement since (6) and (7) are only approximations to the more exact Equations of Motion and there is no a priori reason why the river should "feel" a width B and a depth D obtained by schematizing its channel into parallepipeds of rectangular cross section. There is also the added complication that the discharge is not wholly present at the head in Montréal but that it is gradually but irregularly augmented all along by the contribituon of the tributaries like the Richelieu, the St. Maurice, the Saguenay and the Manicouagan. There is the problem of the advection of salt water into the fresh water régime which forces a vertical distribution in the densities and the velocities, with which (6) and (7) cannot cope. Finally, as the river widens there are definite gradients in the elevations and currents across the width of the channel.

Still it is obvious that we do not want to face all the complications in a first approach study and that it is desirable to integrate (6) and (7) over a rectangular schematization of the channel, neglecting the gradual changes in the discharge. The results of such calculations should give us an overall view of the first order type of motion occurring. Comparison with observations will show that such calculations model adequately the main features of the tidal motion upstream and its interference with the discharge.

Also, we will then be in a position to obtain definite information on the influence of some of the basic parameters of the river on the propagation of the tide upstream. We will be

able to investigate the effect of an increase of the discharge on the tide moving upstream and to investigate the effect of closing the river at various crucial points like at Montréal, at the mouth of Lake St. Peter or at the constriction of the site of Québec Bridge. The conclusions which we will reach cannot be essentially modified by the increased complications in the river flow, which we have mentioned at the beginning of this paragraph.

4.1 The schematization

The bed of the river has been subdivided into 53 subchannels each of which is schematized as a channel of rectangular section of width B and depth D. The width B is the average width of the section while D is the overall average of the depth over the section. The criterion for initiating or terminating a subchannel is the occurrence of a geographical accident, an island, a constriction or widening or a change of depth. Working in this way from King Edward Pier in Montréal to Pointe des Monts, we ended up with 53 sections of various lengths each of these sections being relatively homogeneous in depth and width, as shown in Figs. 24 and 25.

There exists no single map showing the whole of the river bed between Montréal and Pointe des Monts; it is therefore not practical to attempt to show our subdivisions. However, Table 8 lists, besides the section number, the name of the coastal stations which are located within a section; this may give to the reader an idea of the location of the subdivision.

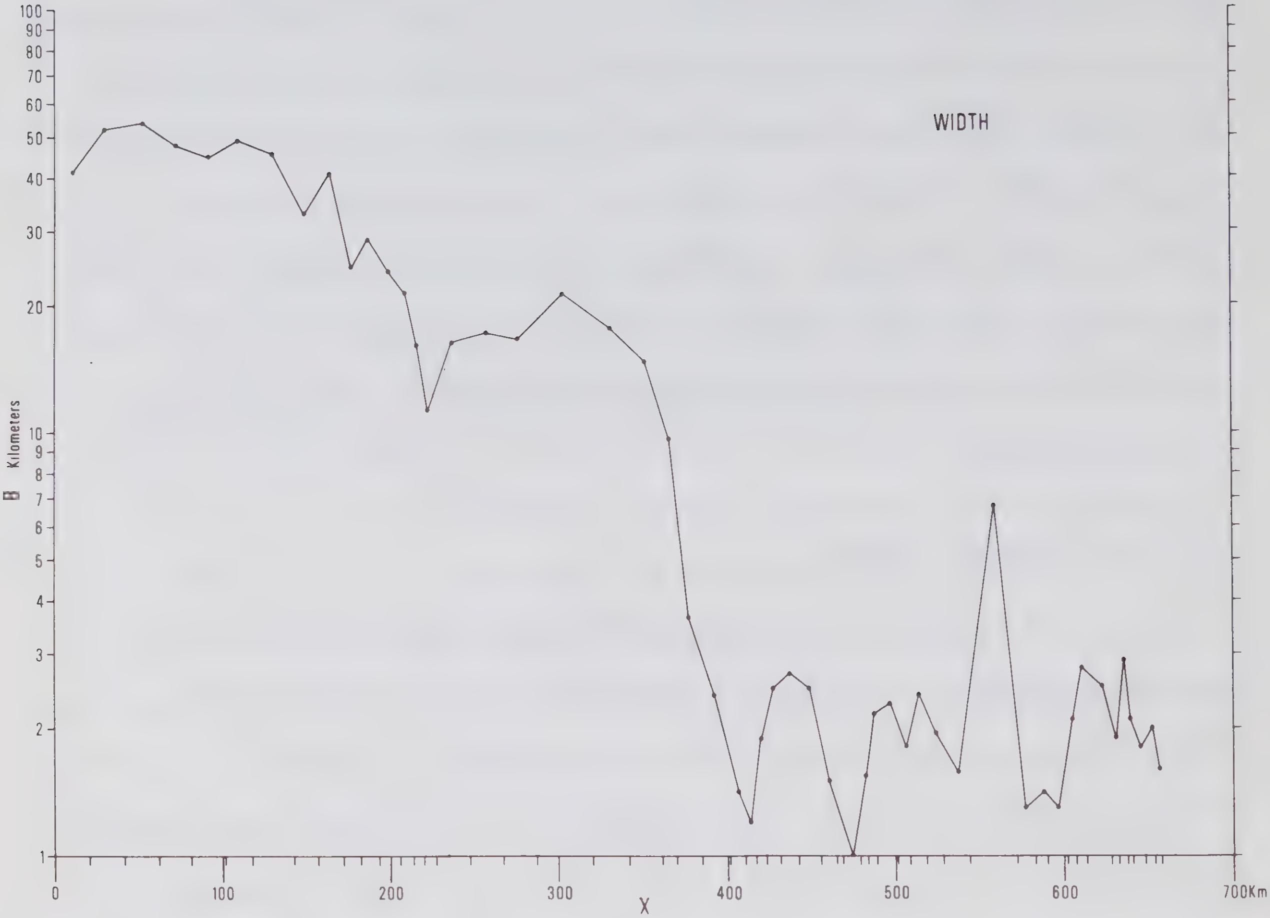


Fig. 24 Width B of the Schematized St. Lawrence River

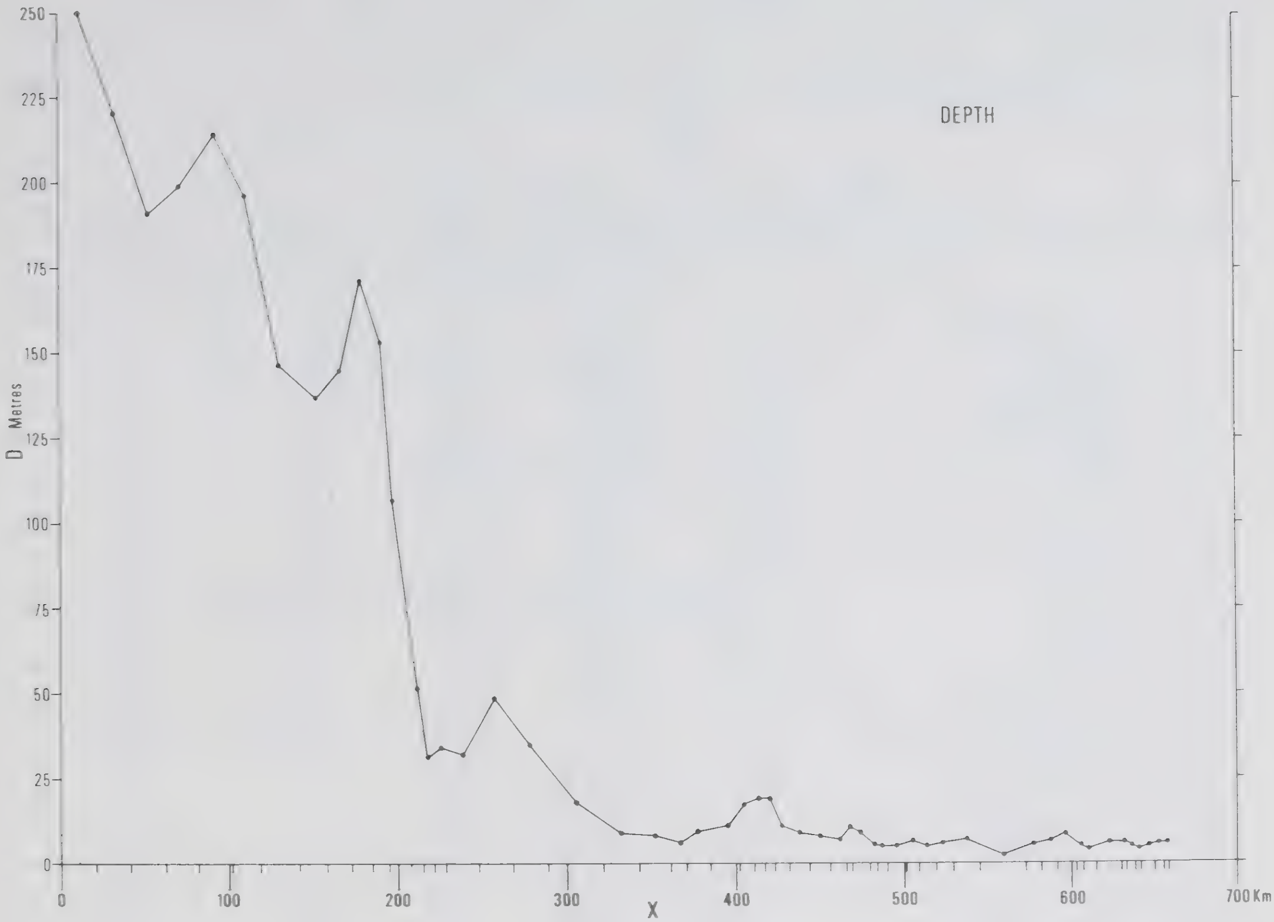


Fig. 25 Depth D of the Schematized St. Lawrence River

Table 8

Schematization of the St. Lawrence River

Section		Horizontal Area $x10^6$ meter^2	Volume $x10^6$ meter^3	Width B meters	Mean Depth D meters	Accumulated distance x $x10^3$ meters
0-1	Pointe des Monts Petite Matane	883	220353	41520	249.5	21.2
1-2	Matane	1077	237424	51788	220.3	42.1
2-3	Baie Comeau	1141	218171	53740	191.2	63.3
3-4	Manicouagan	859	169882	48054	197.7	81.2
4-5	Mont-Joli	914	195475	45166	213.8	101.4
5-6	Pointe au Père	896	175774	49314	196.2	119.6
6-7	Rimouski	997	146326	46024	146.8	141.3
7-8	Bic	558	76508	33131	137.2	158.1
8-9		600	86751	40693	144.7	172.8
9-10	Escoumain	289	49563	24272	171.6	184.7
10-11	Trois-Pistoles	308	47041	29059	152.8	195.3
11-12		282	30358	23723	107.6	207.2
12-13	Tadoussac	171	8828.0	21305	51.8	215.2
13-14		82.6	2657.0	15615	32.2	220.5
14-15	Cacouna	148	4987.5	11378	33.8	228.2
15-16	Rivière du Loup	413	13424	20030	32.5	248.8
16-17		310	14872	16455	48.0	267.6
17-18	Pointe au Pic	349	12375	17160	35.5	288.0
18-19	St. Joseph de la Rive	543	8969.8	16628	16.5	320.6
19-20	St. Jean Port Joli	474	4470.3	21438	9.4	342.7
20-21	Montmagny Isle aux Couldres	337	2612.4	17633	7.8	361.9
21-22		154	1002.4	14565	6.5	372.4
22-23	St. François IO	117	872.10	9601	7.5	384.6
23-24	St. Laurent IO	74.6	829.82	3641	11.1	405.1
24-25	Lauzon	13.8	241.91	2390	17.5	410.9
25-26	Wolfe Cove	9.29	175.74	1445	18.9	417.3
26-27	Pont de Québec	7.77	149.91	1194	19.3	423.8
27-28		15.7	191.81	1879	12.2	432.1
28-29	Neuville	30.1	280.33	2456	9.3	444.4
29-30		32.5	254.62	2741	7.8	456.3
30-31		23.3	168.77	2509	7.2	465.5
31-32	Portneuf	5.41	59.016	1492	10.9	468.2
32-33	Deschambault	8.98	82.147	803	9.1	480.4
33-34	Grondines	5.84	33.551	1555	5.7	484.1
34-35		13.7	64.069	2217	4.7	490.3
35-36	Cap à la Roche	25.6	123.34	2336	4.8	501.3
36-37	Batiscan	14.7	95.413	1841	6.5	509.2
37-38	Champlain	23.6	128.39	2439	5.4	518.9
38-39	Trois-Rivières	20.1	131.56	1943	6.5	529.3
39-40	Port St. François	25.1	205.45	1591	8.2	545.1
40-41	Louiseville	188	541.10	6794	2.9	572.8
41-42		11.7	71.639	1304	6.2	581.7
42-43	Sorel	14.1	104.33	1429	7.4	591.6
43-44	Lanoraie	14.6	123.94	1320	8.5	602.6
44-45		10.1	52.539	2078	5.2	607.5
45-46	Lavaltrie Contrecoeur	21.6	92.242	2852	4.3	615.1
46-47	Verchères	30.0	175.03	2047	5.8	629.7
47-48	Répentigny	6.16	35.131	1932	5.7	632.9
48-49	Varennes	11.6	55.736	2093	4.8	638.5
49-50	Pointe aux Trembles	6.35	24.850	1757	3.9	642.1
50-51	Longue Pointe	13.7	70.631	1992	5.2	649.0
51-52	Laurier	9.11	54.947	1579	6.0	654.7
52-53	King Edward Pier	3.03	18.797	1015	6.2	657.7

Table 8 lists the various sections along with their relevant parameters such as the horizontal area, the volume, the average width, the average depth and the accumulated length of the schematized sections.

4.2 The rewriting of the Equations of Hydrodynamics into systems of finite differences

The equations (6) and (7)

$$Q_x+Bh_t=0$$

$$-h_x-Q|Q|/C^2B^2D^3 = Q_t/gBD + 2QQ_x/gB^2D^2$$

have to be rewritten as

$$h_t=-Q_x/B \qquad (27)$$

$$Q_t=-gBD\ (h_x + Q|Q|/C^2B^2D^3) + 2QQ_x/gB^2D^2 \qquad (28)$$

in order to be able to go from past to future values of the variables h and Q.

The river may be cut up into subsections which we label as

h	Q	h	h	Q	h	Q	h	Q	h	Q
•	•	•	•	•	•	•	•	•	•	•
1	2	3	n-2	n-1	n	n+1	n+2	n+3	53	54

At one point we compute h and the next one we compute Q. At an interior point (27) and (28) can be written out in terms of finite differences as

$$h(n,t+2\Delta t) = h(n,t)-(2\Delta t/B_n)(Q(n+1,t)-Q(n-1,t))/(\Delta x_{n-1}+\Delta x_n) \quad (29)$$

$$Q(n+1,t+2\Delta t)=Q(n+1,t) -$$

$$2g\Delta t B_{n+1}D_{n+1}[(h(n+2,t)-h(n,t))/(\Delta x_{n+1}+\Delta x_{n+2}))$$

$$+Q(n+1,t)|Q(n+1,t)|/C^2B^2_{n+1}D^3_{n+1}))$$

$$+2Q(n+1,t) \quad (Qn+3,t)-Q(n+1,t))/(\Delta x_{n+2}+\Delta x_{n+3})] \quad (30)$$

These relations hold for an inner point. The values of B and D are averages of the values they have in the two sections delimited by the three points.

At the mouth we impose an h boundary condition so that h(1,t) is specified at all times. At the head we impose a condition on Q. The differential for Q_x in the convective term is computed in a biased fashion but the term is quite small anyway. The boundary condition on h at the mouth is

$$h(1,t)=1.03\cos(28.98°t+89°)+.33\cos(30.00°t+273°)+.23\cos(28.44°t +107°)+.22\cos(15.04°t+38°)+.20\cos(13.94°t+79°) \text{ meters}$$

The Q boundary condition at the head is

$$Q(54,t)=0,8.5,11.3,14.2\text{x}10^3 m^3/sec$$

The h boundary condition embodies the approximately observed values of M_2, S_2, N_2, K_1 and O_1 at section 1 and the phase has been chosen to correspond to the instant 00 hours, September 1, 1964. t is measured in hours. At Montréal the discharge is taken to be a constant at all times. $Q=8.5\text{x}10^3$/sec corresponds

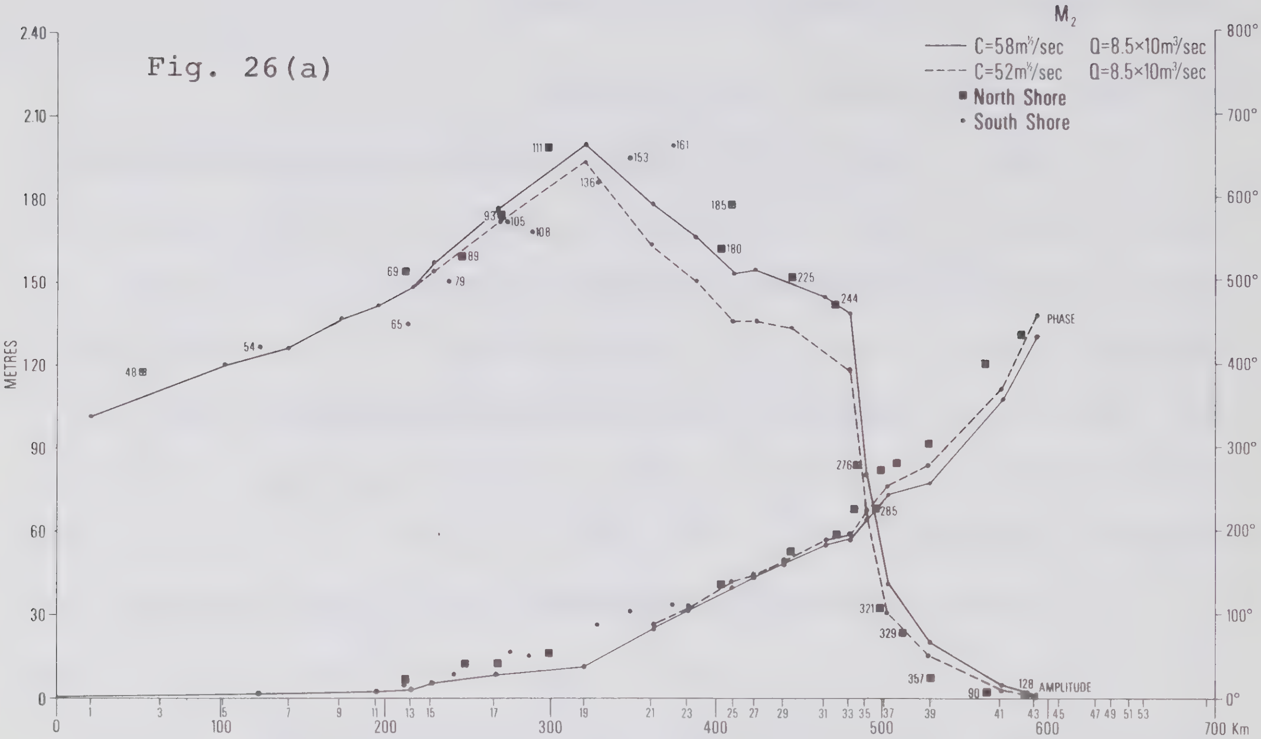
to a little bit more than the average discharge in Montréal. The higher values may prevail during the spring run off. Q=0 corresponds to a closure of the river.

4.3 The results of the calculations

Plots 26(a) to 30 show the computed amplitudes and phase changes of some constituents compared to their observed values derived from the analysis of observations in the St. Lawrence River. Most of the calculations have been performed for C=58 $m^{\frac{1}{2}}$/sec since our previous investigations in par. 2.6 indicated that such a value of the friction may hold in the lower St. Lawrence.

M_2 being the most important constituent, we shall look at it first. Fig. 26(a) like all the others which follow, shows as continuous curves the results of the calculations using our schematized model of the river. One curve shows the variation of the amplitude up the river while the other shows the phase change upstream of the particular constituent. The analyzed values based on observations are shown by dots; those which are squared were derived from stations located on the north shore. The Greenwich phase lag of the constituents is written explicitely over the point and allows the location of the observed phase lag points over the phase curve. For M_2, all the stations observed have been entered even if the observations covered less than one year. For the other constituents we have entered only those based on a one year analysis since the shorter observations bring in much scatter. Fig. 26(a) shows the calculated amplitude and phase change of M_2 for C=58 $m^{\frac{1}{2}}$/sec

Fig. 26 Amplitude and Phase Change for Two Values of the Chézy Coefficients



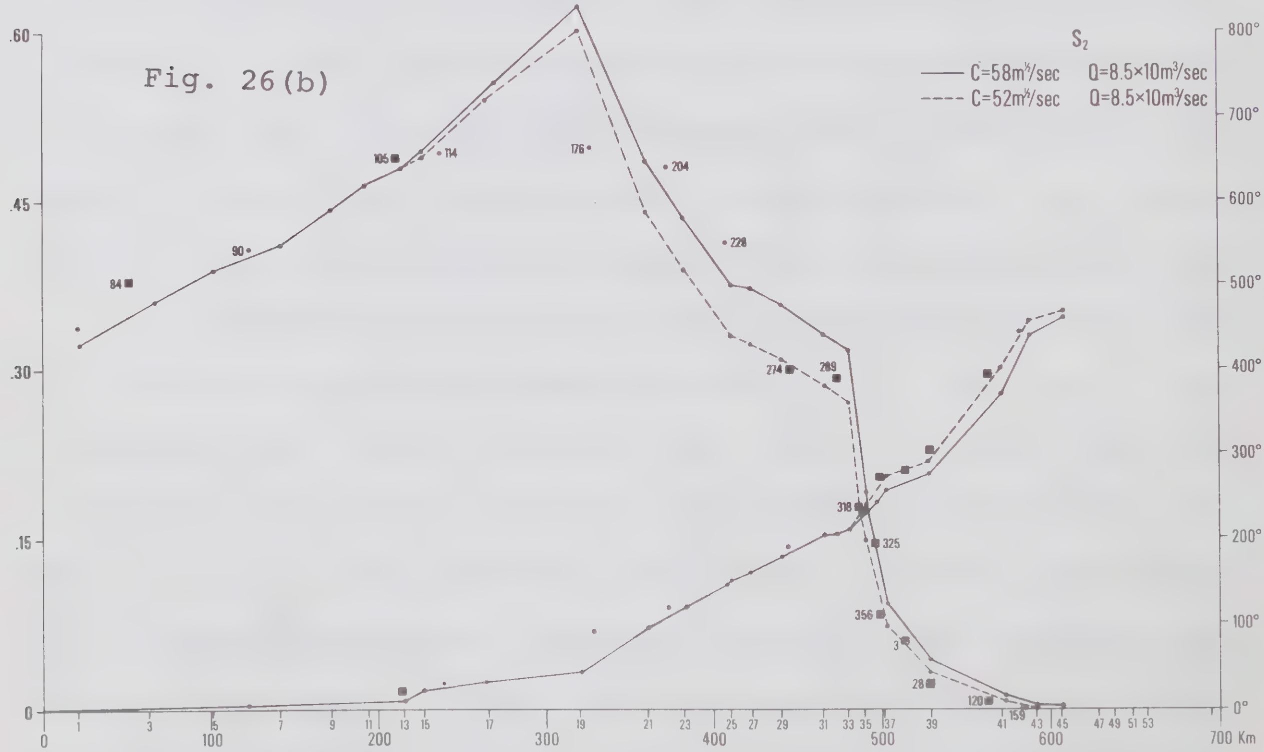


Fig. 26(c)

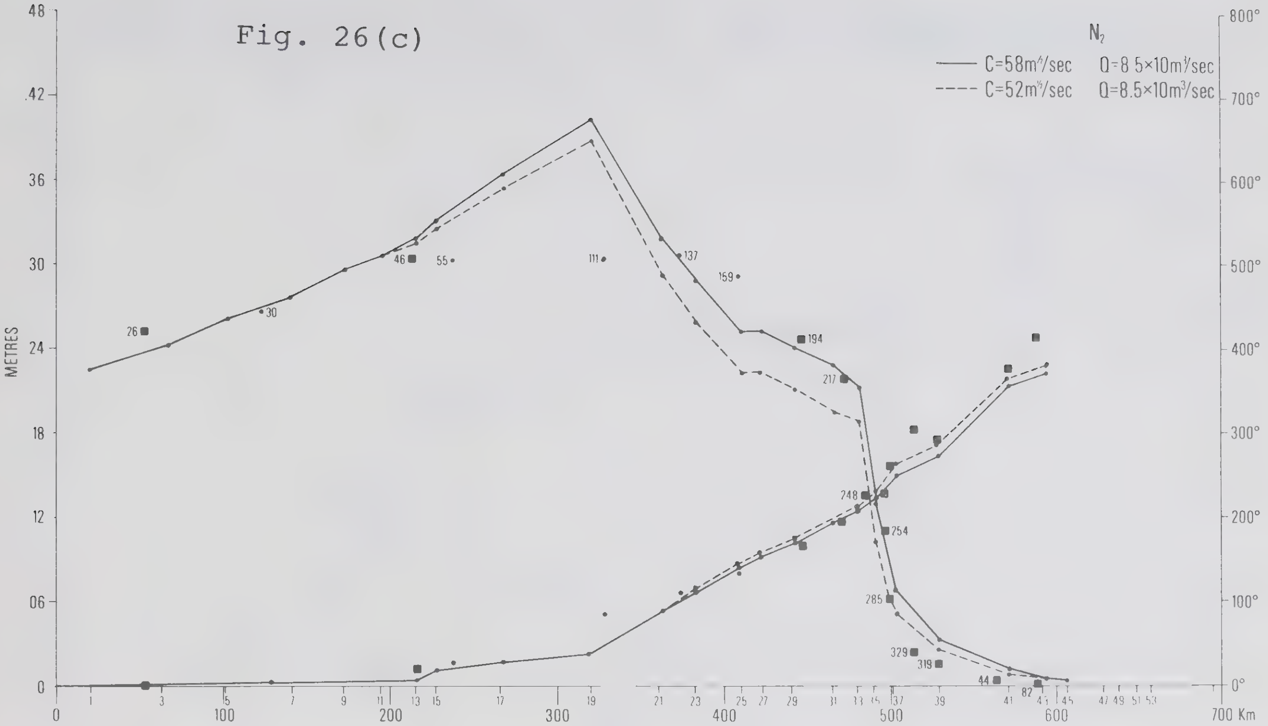
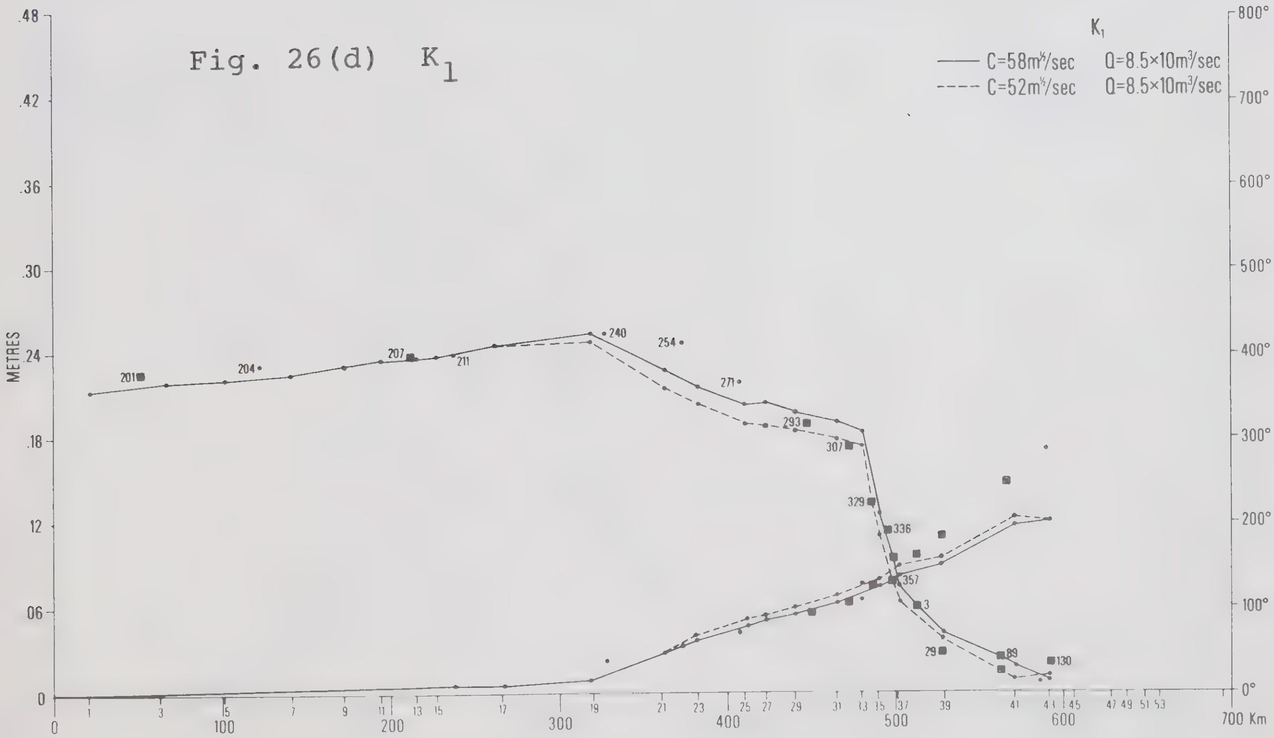


Fig. 26(d) K_1



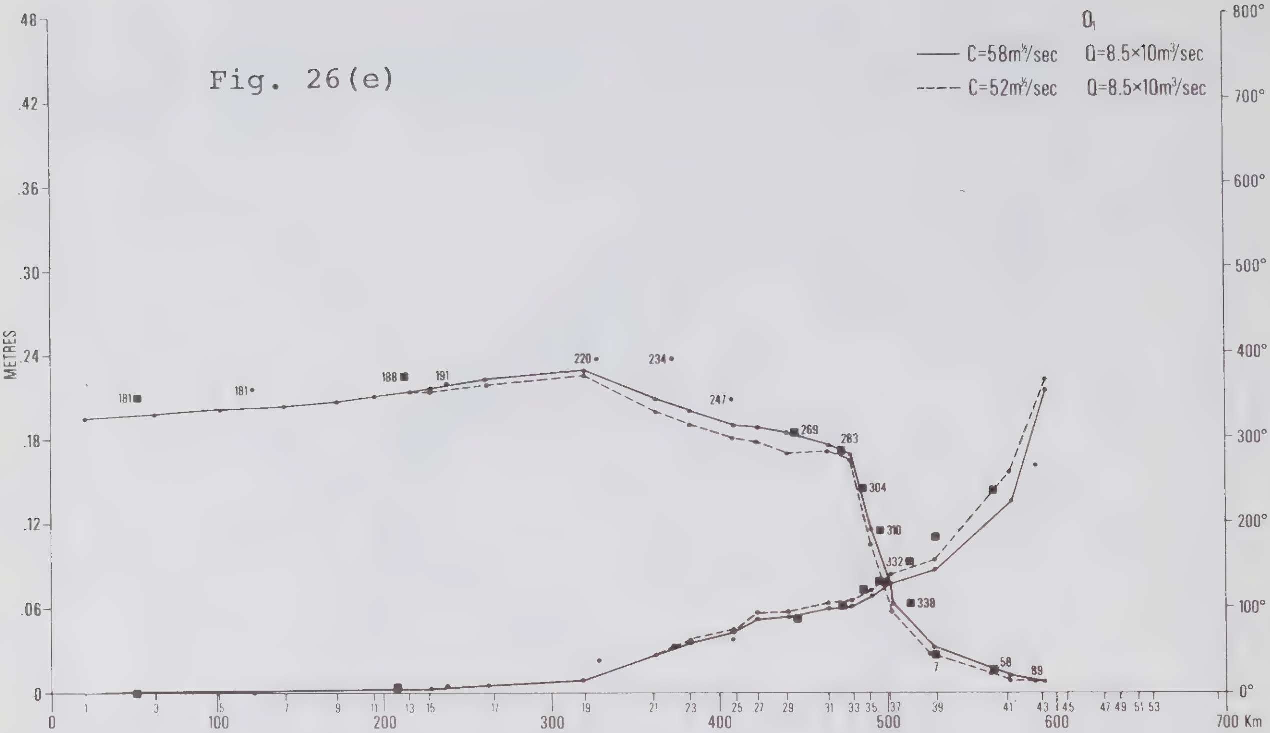


Fig. 27 The Fast Shallow Water Constituent M_4 for Three Values of the Discharge

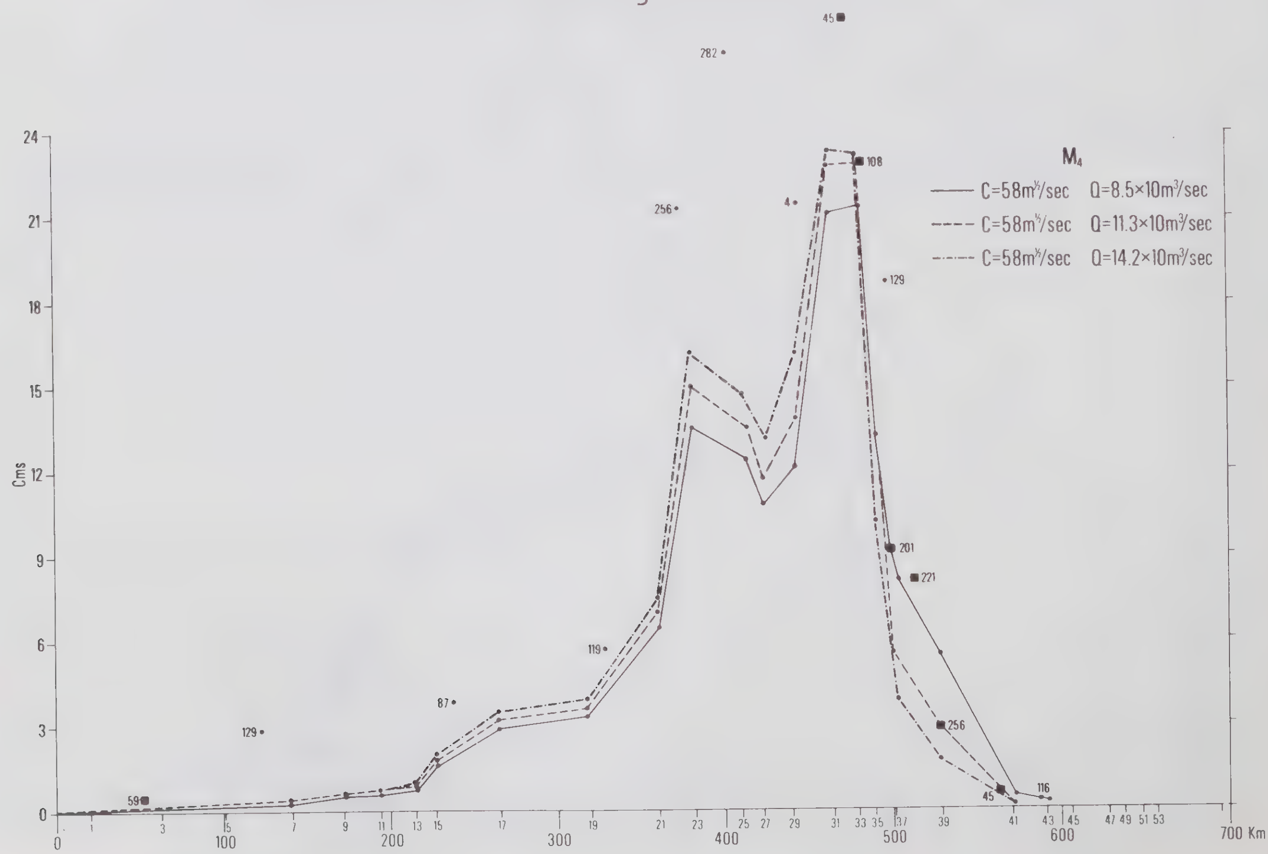


Fig. 28 The Slow Shallow Water Constituents MSf for Three Values of the Discharge and Two Values of the Chézy Coefficient

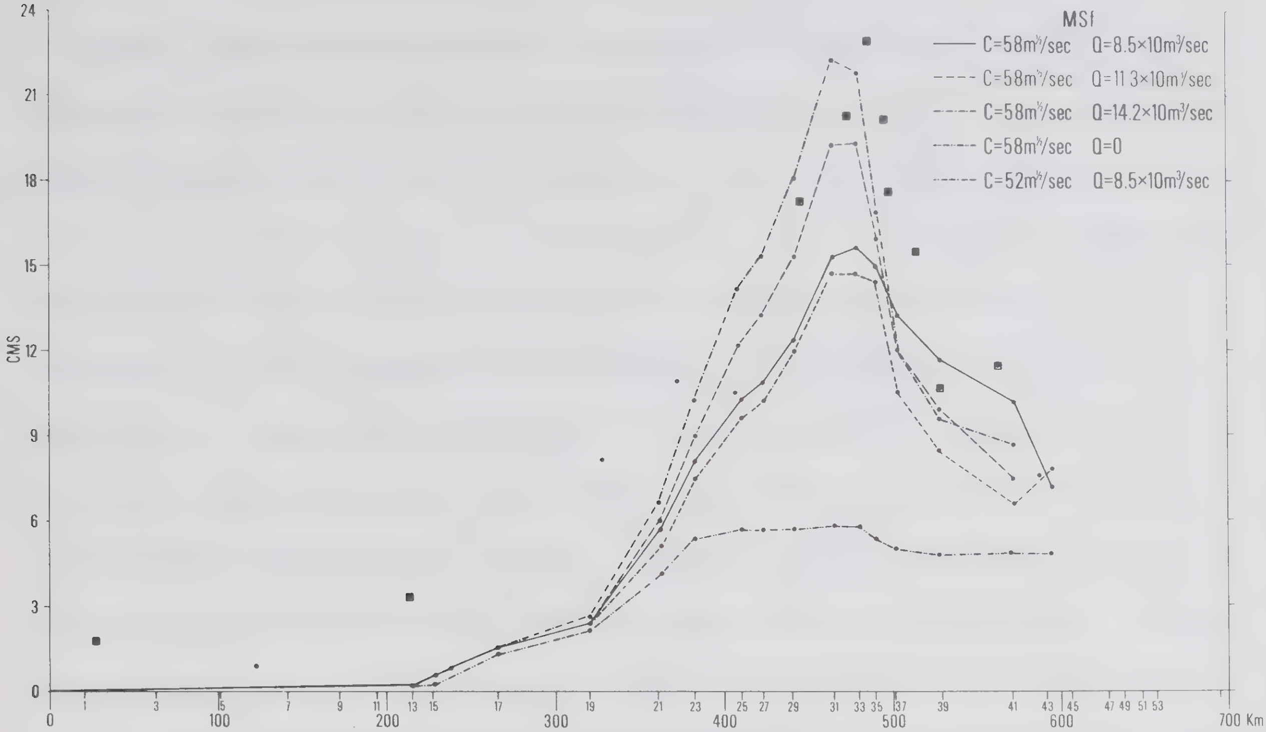
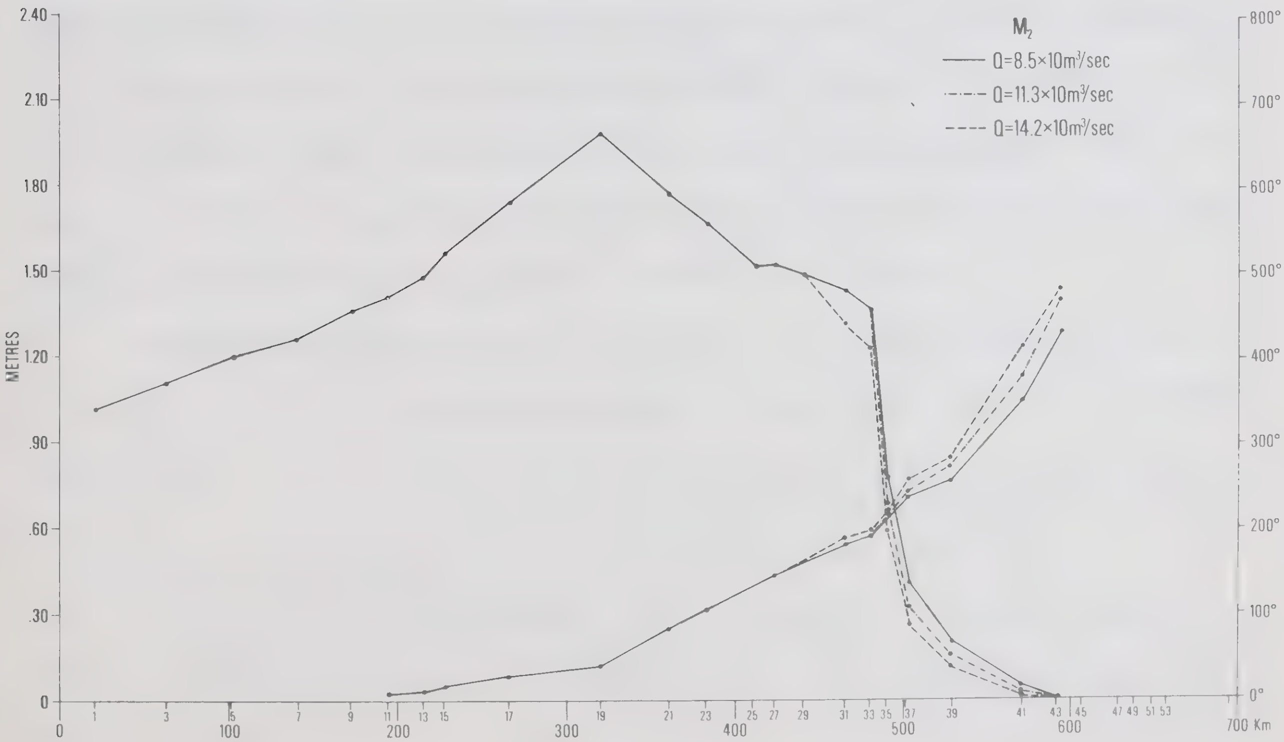


Fig. 29 The Constituent M_2 for Three Values of the Discharge



at a head discharge of 8.5×10^3 m^3/sec; it underlines the fundamental weaknesses of our model which we have discussed in the introductory paragraph. We had to leave out of our schematization the Saguenay River whose mouth stretches over sections 11 to 13 and we schematized the double channel around by the Ile d'Orléans by a single channel which stretches between sections 22 and 25.

At Saguenay, some of the tidal energy is diverted into the fiord and we notice that upstream of Saguenay our calculated phases of M_2 fall consistently below those observed. Similarly our analyzed amplitude does not model the observed amplitude adequately over sections 21 to 24. Still the overall agreement between the values derived from our model and those observed is adequate and suffices for a study of the fundamental parameters which influence the propagation of the tide upstream.

It is obvious that we get better agreement for $C=58m^{\frac{1}{2}}/sec$ with the observed amplitudes, while the difference in phase between C=52 and $C=58m^{\frac{1}{2}}/sec$ is slight. The value of $C=58m^{\frac{1}{2}}/sec$ corresponds to the value of C we deduced from the observations of the Ship Channel Division.

The subsequent plots of S_2, N_2, K_1 and O_1 show as well the overall adequacy of our model for describing the general features of the tidal motion upstream.

4.3.1 The shallow water constituents

One of the attractive features of our model is that it creates as well, shallow water constituents through the non-linear friction term $Q|Q|/C^2B^2D^3$. Out of the interaction

of M_2, S_2, N_2, K_1 and O_1 we should get new constituents such as M_4, MS_4, S_4, $2MS_6$, M_8, MSf, SN, Mm and Mf amongst others. Our model does yield some values for these quantities which may be compared with observations. In Figs. 27 and 28 we show plots of M_4 and MSf. The three curves in the diagram for M_4 show the calculated M_4 for various values of the discharge; M_4 increases first for larger values of the discharge up to Québec, then the relation is reversed. On the way to Montréal it falls off sharply as our rudimentary algebraical calculations had indicated. Our calculated M_4 is significantly smaller than the observed values; there is a discrepancy of up to 12 cm in the Isle d'Orléans. MSf has been calculated for three values of the discharge and two of friction, which we know are possible in the river. For $Q=8.5 \times 10^3 m^3/sec$ which corresponds to the mean discharge, our calculated MSf falls quite short of the observed values; increasing the discharge does enhance MSf very appreciably though. A further indication that MSf is fundamentally dependent on the value of the discharge is that for $Q=0$, MSf is very slight throughout. The calculated values for the other shallow constituents, fast and slow, are similar to those we have shown for M_4 and MSf.

We conclude that our model provides a qualitative agreement with the observed values of the shallow water constituents but certainly not a quantitative agreement. It is my personal feeling that such a quantitative agreement would be very difficult to obtain even with a very sophisticated model as these constituents are created through a very subtle and sensitive interaction of the fundamental parameters of the river.

4.3.2 The influence of the discharge on the propagation of the tide

An inspection of Figs. 27, 28 and 29 shows the influence of the discharge on the propagation of the tide upstream. In Fig. 29 we see that the fundamental constituent M_2 decreases in amplitude and is slowed down in its progress upstream for increasing values of the discharge. The effect of the change in discharge is not felt till beyond the constriction of Québec City. The modification in amplitude amounts to at most 10 to 20 percent of the tidal amplitude; the time of arrival of the M_2 tide however may be retarded by up to two or three hours. The shallow water constituents created by the interaction of the tide with the discharge and bottom friction are significantly increased in the downstream portion of the river for increased values of the discharge but they become more rapidly extinguished upstream of Québec City, basically because the original tidal constituents are themselves more rapidly damped.

4.3.3 The influence of quadratic friction

Figs. 26(a) and 28 show the effect of the quadratic friction. In Fig. 26(a) we notice that for lesser values of the friction, the tide is damped less rapidly and it suffers less phase retardation, a fact which we could demonstrate algebraically in par. 2.2 in the case of linearized friction. Now we see that this holds as well in the case of quadratic friction.

The effect of quadratic friction on the shallow water constituents is not that apparent; the use of linearized friction does not even give us a clue as to their existence. Fig. 28 indicates that MSf is lessened for increased values of friction;

the same holds for the fast shallow water constituents such as M_4 and MS_4 although we have not shown this explicity in diagrams. The use of our intuition would lead us to believe that the stronger the friction the sharper the shallow water constituents. The facts show that this is not the case; an indication once again that non-linearities escape the grasp of the human mind. I believe that if in our case the non-linear friction were to be varied slowly from zero, the shallow water constituents would increase initially but beyond an optimum value they would start diminishing as the primary wave starts wasting itself into more complicated additional modes of motion. Most likely between the values of $C=58\ m^{\frac{1}{2}}/sec$ and $C=52\ m^{\frac{1}{2}}/sec$, we must have passed beyond this optimum value of C.

4.3.4 The effect of the closure of the river

Our model allows us to investigate the effect of closing the river wherever we please; it seems interesting to see what would happen to the propagation of the tide if we closed the river at the constriction where the Québec Bridge has been laid, or at the mouth of Lake St. Peter or at Montréal itself.

Even before attempting any calculations, a little bit of reasoning indicates that the tide originates from the Atlantic Ocean and that it becomes progressively wasted in the Saguenay River and up the St. Lawrence. The erection of any barrier would allow more tidal energy to stay downstream. Our calculations support this intuitive conclusion; they also delineate the portion of the river where the dyke would modify

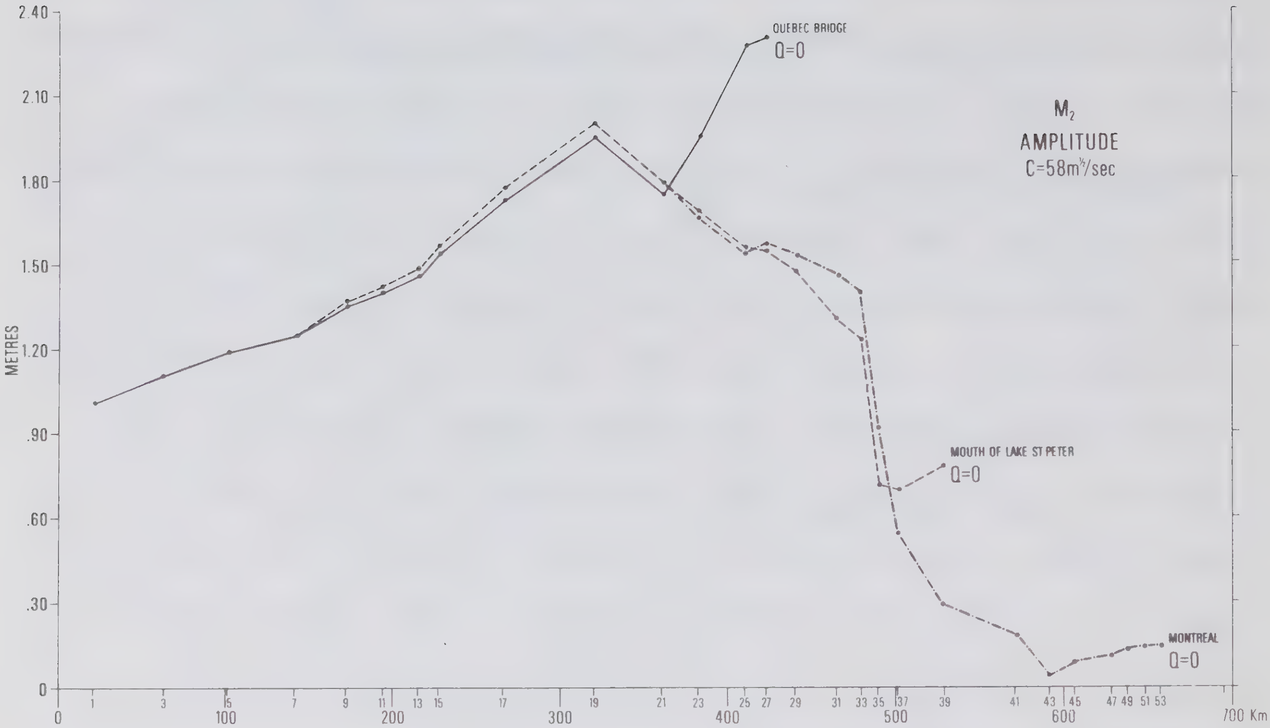


Fig. 30 The Amplification of M_2 due to the Closure of the River at Québec Bridge, Trois-Rivières and King Edward Pier

appreciably the tidal régime. The results of such calculations are shown for M_2 in Fig. 30.

If we blocked the river at Québec Bridge, the tide between Isle aux Couldres and Québec Bridge would be significantly enhanced by values which would reach up to 50 percent of the present value of M_2 at the dyke site. At Québec Bridge M_2 would have an amplitude of 2.31 meters compared to its actual value of 1.53 meters.

Blocking the river at section 39 (Trois Rivières) has the same effect. In terms of absolute magnitude, the enhancement of the tide is not as dramatic. M_2 would take an amplitude of .78 meters compared to its value of .08 meters. Finally setting up an opaque barrier at King Edward Pier would once again enhance the tide. In fact, Montréal as it is presents an effective wall against the tide but the discharge present in the river has a marked damping effect as we notice in Fig. 26.

This brings to the fore the fact that we cannot build opaque barriers in a river where the mean discharge hovers around 8.5×10^3 m^3/sec; this water has to be accommodated out. If it were at all possible in practice to let the water out at descending tide while impeding the flow of the tide upstream, the actual amplitude of the tide downstream of the dyke would be significantly less than the values predicted by our model. An added problem in the St. Lawrence is that water is not always in its liquid form and that the outflow of ice would create major problems in the design of semi-opaque dykes. A natural solution would be to take advantage of the increasing effects of pollution.

It would perhaps be possible in the near future to raise the temperature of the water to a point where it would never freeze in winter.

CONCLUSIONS

Steady state conditions prevail on the average between Montréal and Sorel. The parameters present in the Equations of Hydrodynamics may be uniquely defined and derived in a physically significant fashion from the hydrographic charts, gauges and current meter readings. The gauge readings on their own supply, when averaged, the mean inclination of the water surface. The mean levels derived from gauge readings and hydrographic soundings supply data on the mean water depth at a station; this depth varies throughout the year but with lesser amplitude as one moves downstream. The motion everywhere in the river is subcritical and the tidal wave may move upstream at all times.

The tide moving up the St. Lawrence reaches its maximum amplitude between Isle aux Couldres and Isle d'Orléans. Shallow water constituents become significant between Isle d'Orléans and Sorel. The tide behaves as a mixture of standing and travelling waves between the mouth at Pointe des Monts and Isle d'Orléans; from there on it has the character of a travelling wave.

Use of the Equation of Hydrodynamics for time dependent motion in conjunction with current observation water level readings helps to delineate the mean value of the Chézy coefficient which holds in the area lying between Trois Rivières and Québec Bridge. The analysis of water level observations gives the values of the major constituents which should be reproduced in a mathematical model of the river. An analysis of the current observations taken at Isle aux Couldres and Québec Bridge by the Ship Channel Division reveals that these quantities could have been deduced theoretically with adequate accuracy using the equation of continuity as suggested by W.D. Forrester.

A one dimensional model of the river, neglecting the variation in discharge, the existence of the Saguenay River and the bifurcation of the channel at Isle d'Orléans reproduces adequately the major tidal constituents for a value of the Chézy constituent of $C=58m^{\frac{1}{2}}/sec$ which was deduced from the current observations of the Ship Channel Division and of G.C. Dohler. The model also gives values for the shallow water constituents which are created along the river but these values cannot be considered as accurate. Increases in the discharge do not significantly alter the tide between Pointe des Monts and Isle d'Orléans; beyond, the tidal amplitude is slightly reduced and the progress of the tide is retarded by up to 3 hours. The closure of the river by opaque dykes at any point would always enhance the tide downstream; semi-opaque dykes would increase the tidal amplitude as well, but to a lesser degree.

ACKNOWLEDGEMENTS

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No. 19

The restoration of beaches contaminated by oil in Chedabucto Bay, Nova Scotia

E.H. Owens

1971

Marine Sciences Branch
Department of Energy, Mines and Resources, Ottawa

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THE RESTORATION OF BEACHES CONTAMINATED BY OIL IN CHEDABUCTO BAY, NOVA SCOTIA

E.H. Owens

1971

ABSTRACT

Following the wreck of the "Arrow" in Chedabucto Bay, Nova Scotia, more than 150 miles of shoreline were polluted with Bunker C oil and a beach restoration programme was established which involved 30 miles of coastline. The beaches of this region vary from low-energy marsh environments to complex shingle spit systems, as well as many areas of eroding rock and till deposits. Only a few miles of beach are made up of sand-size material.

The contaminated sand beaches were cleaned relatively easily but although various manual and mechanical methods were implemented, no effective or efficient method of removing oil from shingle beaches was found. Oil on active shingle beaches was often buried up to a depth of 5 feet with clean and contaminated sediments having been thoroughly mixed by wave action. The amount of oil in these sediments was as low as 10 ppm and restoration of this type of shore involved the removal of large volumes of beach material. Where oil remained as a surface layer on the beach, a front-end loader proved to be effective in removing the contaminated layer.

In active beach environments normal wave processes will clean the beaches naturally unless there is so much oil or wave energy is so low, that the movement of sediments is prevented. On these paralysed beaches some attempt should be made to break up the surface so that wave action is able to rework and clean the beach sediments.

CONTENTS

LIST OF FIGURES, TABLES AND PHOTOGRAPHS

1. INTRODUCTION

Following the wreck of the tanker "Arrow" on Cerberus Rock, February 4, 1970, the oil spill of Bunker C affected more than 150 miles of shoreline in Chedaducto Bay, which has a coast of 380 miles from Point Michaud in the north-east to Cape Canso in the south-east (Figures 1 and 2). The Project Oil Task Force initiated a beach restoration operation and a total of 24 miles of coast was initially selected for the work programme which was carried out by the Department of Public Works during the period April to October, 1970. In April, a coastal geomorphologist was assigned to aid the restoration project and this paper reports the results of the observations and investigations of that part of the programme.

This operation was the first attempt to restore non-sand beaches without the use of dispersants. On certain beaches the work was followed very closely in order to gain knowledge which would be of value to future projects of this nature. These instances are reported in detail along with a broader discussion of the more general aspects of the operation. A full account of the entire beach restoration programme which deals with each area where work was undertaken and the techniques employed has been prepared by the Department of Public Works site engineer (MacKay, 1970). This report is complimentary to that of MacKay but is concerned primarily with the geological problems rather than the engineering, economic, or social factors, although these are discussed in the final section. Most of the investigations were of a qualitative nature, though data collection was undertaken wherever possible to supplement the subjective observations.

The use of earth-moving equipment for beach restoration was not recommended in this area and was found to be only partially adequate in removing all contaminated material. Sand beaches were cleaned by manual and mechanical methods with success but no satisfactory mechanical methods were available for the restoration of cobble and shingle beaches. This lack of effectiveness can be attributed to the fact that it was not possible to remove all the contaminated material from the polluted cobble beaches and to the movement of oil alongshore and offshore which led to the recontamination of several of the restored beaches.

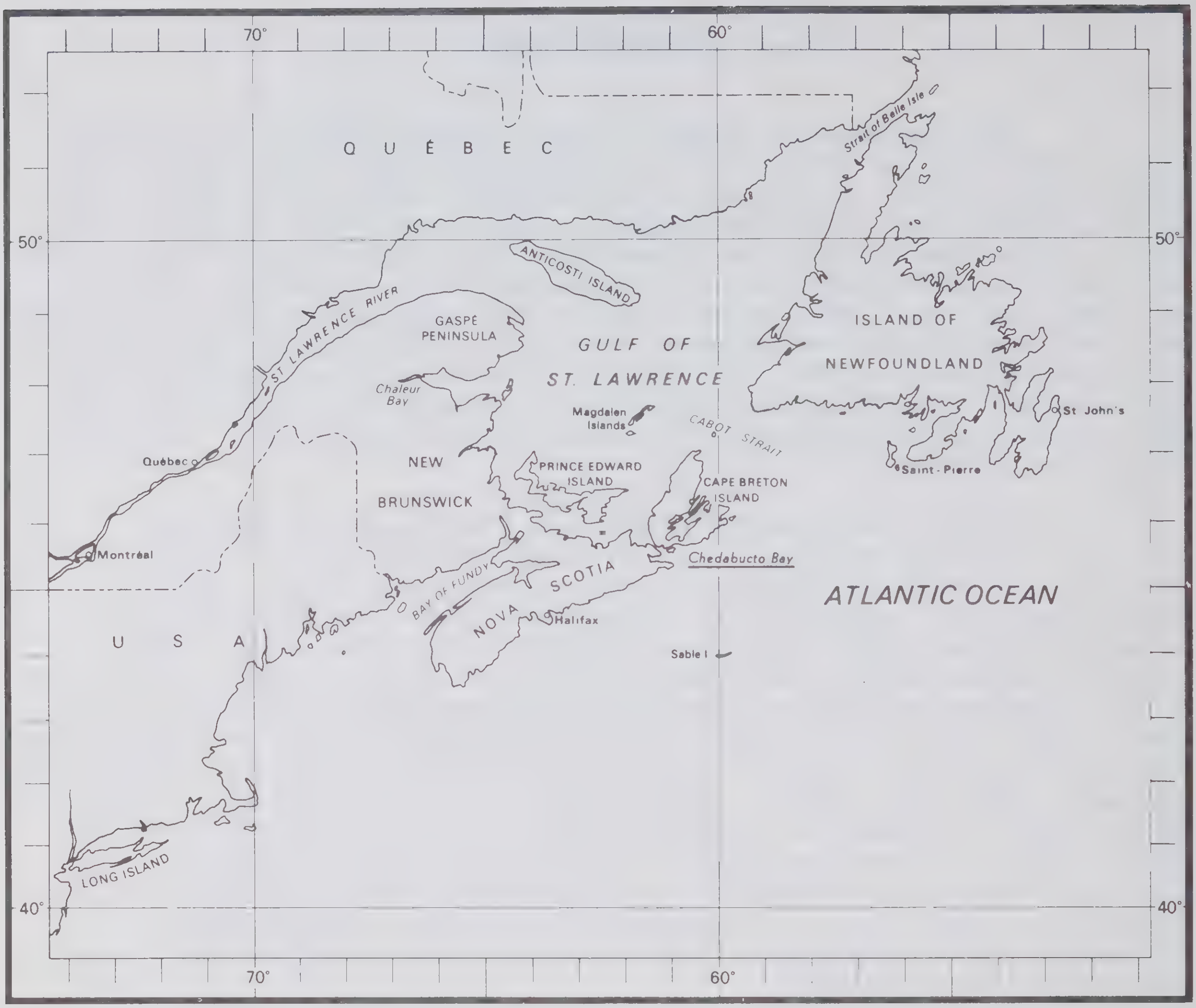
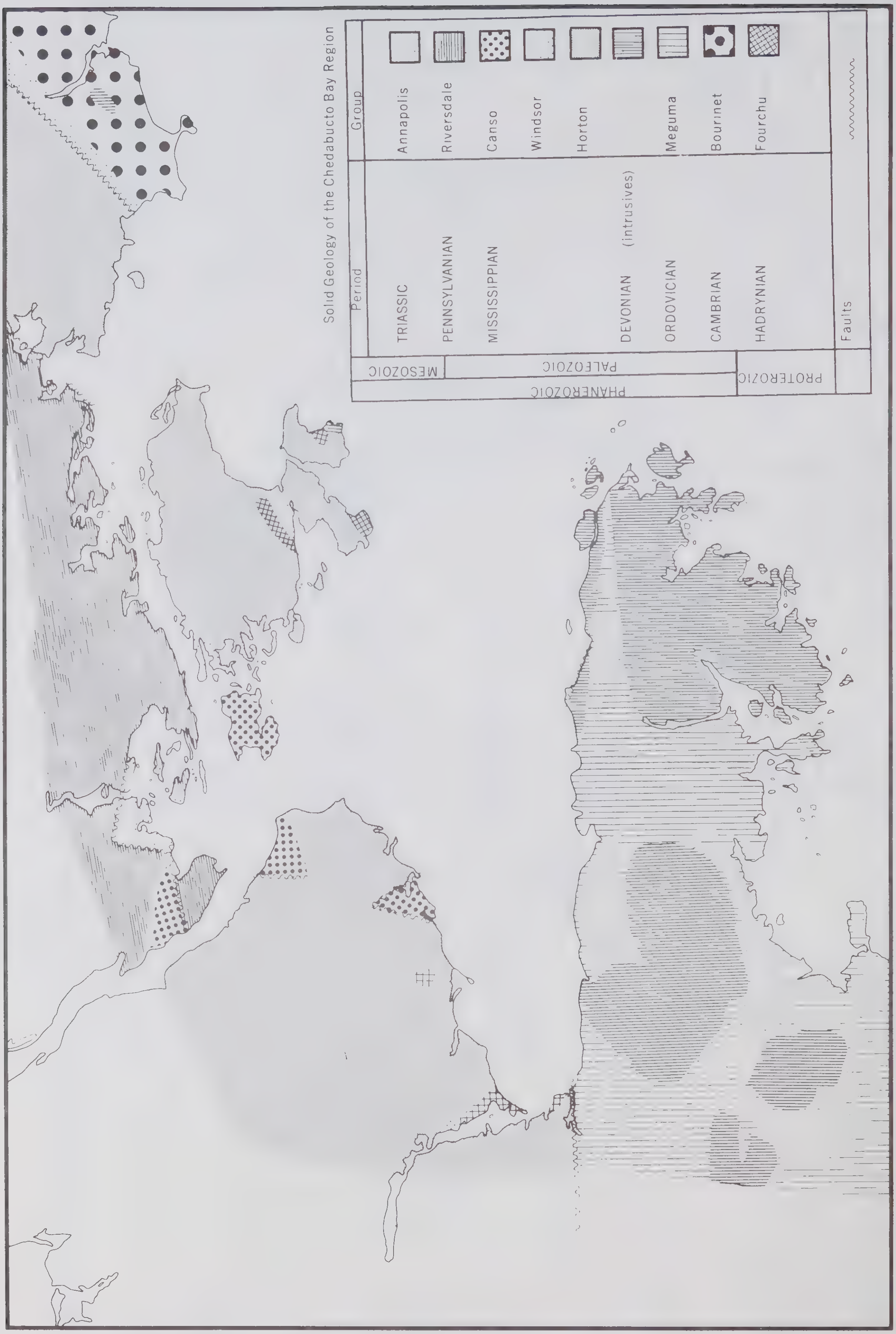


Figure 1. Chedabucto Bay location diagram.

Figure 2. Locations within the study area.


Solid Geology of the Chedabucto Bay Region
Period
Group
TRIASSIC
Annapolis
PENNSYLVANIAN
Riversdale
MISSISSIPPIAN
Canso
Windsor
Horton
DEVONIAN
(intrusives)
ORDOVICIAN
Meguma
CAMBRIAN
Bourinet
HADRYNIAN
Fourchu
Faults
MESOZOIC
PALEOZOIC
PROTEROZOIC
PHANEROZOIC

2. PHYSICAL BACKGROUND

2.1 Geology

The major structural feature of this region is the Chedabucto fault or fault complex of the Acadian orogeny which separates the resistant metamorphosed rocks of the Meguma group in the south from the relatively less resistant late Paleozoic sediments which lie to the north (Figure 3). The fault zone defines the south shore of the Bay and can be traced eastwards onto the Nova Scotian Shelf (King and Maclean, 1970).

South of the fault zone the stable Meguma platform is made up of tightly folded Ordovician quartzites and slates which have been intruded by Devonian granites (Webb, 1969). This block has been eroded to give an uplifted peneplain which slopes gently southwards and the rise of sea level has led to an indented shoreline which is in marked contrast to the long straight coast of the fault zone.

The Carboniferous beds which characterize the area north of the fault consist in part of a small basin within the uplifted basement and are relatively less resistant conglomerates, sandstones, shales, and limestones which have been folded and metamorphosed. Erosion along the fold axes provides a clear example of the general north-east/south-west trend in the Lennox Passage area. The erosion of these late Paleozoic rocks has led to the development of an undulating lowland area which has been drowned to produce an irregular coastline.

2.2 Surficial Sediments

The areas of bedrock exposure (Figure 4, unit 6) are confined largely to the more resistant uplands south of the fault zone. Where sediments are present in this area they are often very stoney and not deep. The remaining areas west of the Strait of Canso have a cover of locally derived glacial till (units 1, 2 and 3) with a few local deposits of outwash material (unit 4) or post-glacial alluvial sediments (unit 5). These latter units account for less than 3 percent of the land area (Hilchey et. al., 1964).

East of the Strait of Canso the till deposits are derived from the Carboniferous rocks and differentiation on a general basis produces a simple textural pattern related to the bedrock parent material.

The till deposits, which constitute the major unit of surface sediments in this region, were laid down directly by the ice as an unstratified and unconsolidated mixture of clay, silt, sand, gravel and boulders.

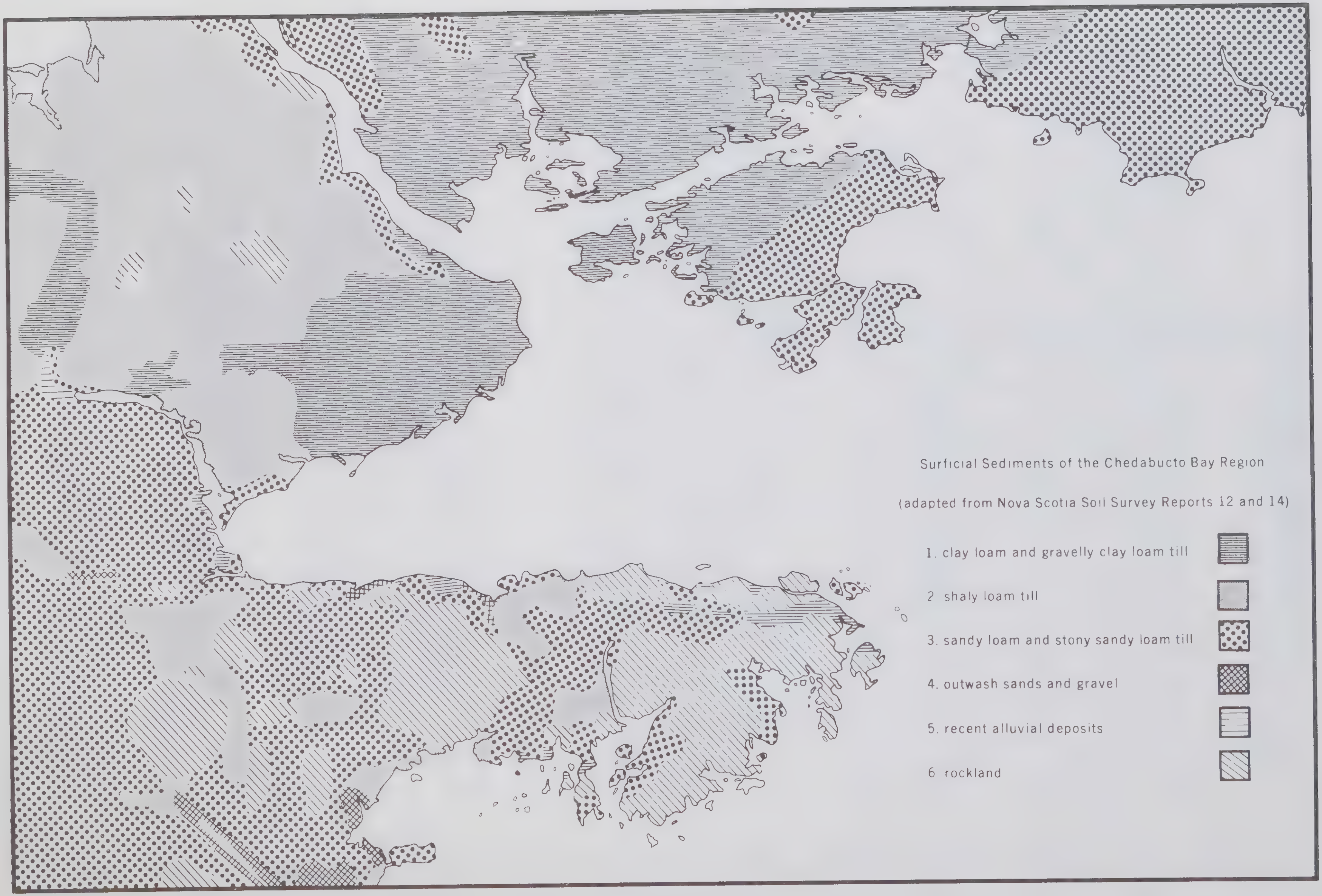


Figure 4. Surficial sediments.

The earliest ice advances which have been traced (Grant, 1971) indicate a west to east movement across Isle Madame. This was superceded by a flow to the north and north-west from a centre on the Scotian Shelf while the final phase of glaciation is related to the recession of an ice cover which had moved south over the area and which disturbed much of the existing till deposits.

2.3 Tides and Waves

The tides of this area are semi-diurnal, with mean and maximum ranges in the order of 4.3 and 6.6 feet respectively (Table 1). The tidal range is an important characteristic of the littoral zone as it affects the width of beach over which wave action can take place.

Table 1 Tidal ranges in Chedabucto Bay (Anon., 1970).

Location	Mean Tidal Range (ft)	Large Tidal Range (ft)
Canso	4.2	6.6
Guysborough	4.5	6.4
Port Hawkesbury	4.4	6.9
Arichat	4.2	6.3
Petit de Grat	4.4	6.6
St. Peters Bay	4.3	6.1

The east coast of Canada is a storm wave environment (Davies, 1964) where the most important waves in the littoral zone are those generated by local storms and a brief review of wind data and the occurence of storms provides an indication of the seasonal variations in wave intensity. This part of Nova Scotia is exposed to the full force of waves from the east and south and those coasts which are directly open to these waves have high energy littoral environments. The relatively shallow areas in the north and west of the Bay (Figure 5) refract the incoming swell waves so that beach orientation is often a reflection of the offshore topography.

The summary of wind data for the meteorological station at Canso (Table 2) shows that the highest wind speeds are mainly from the north-west and occur in the period December to February. The generalized data indicate that the extent of the winter

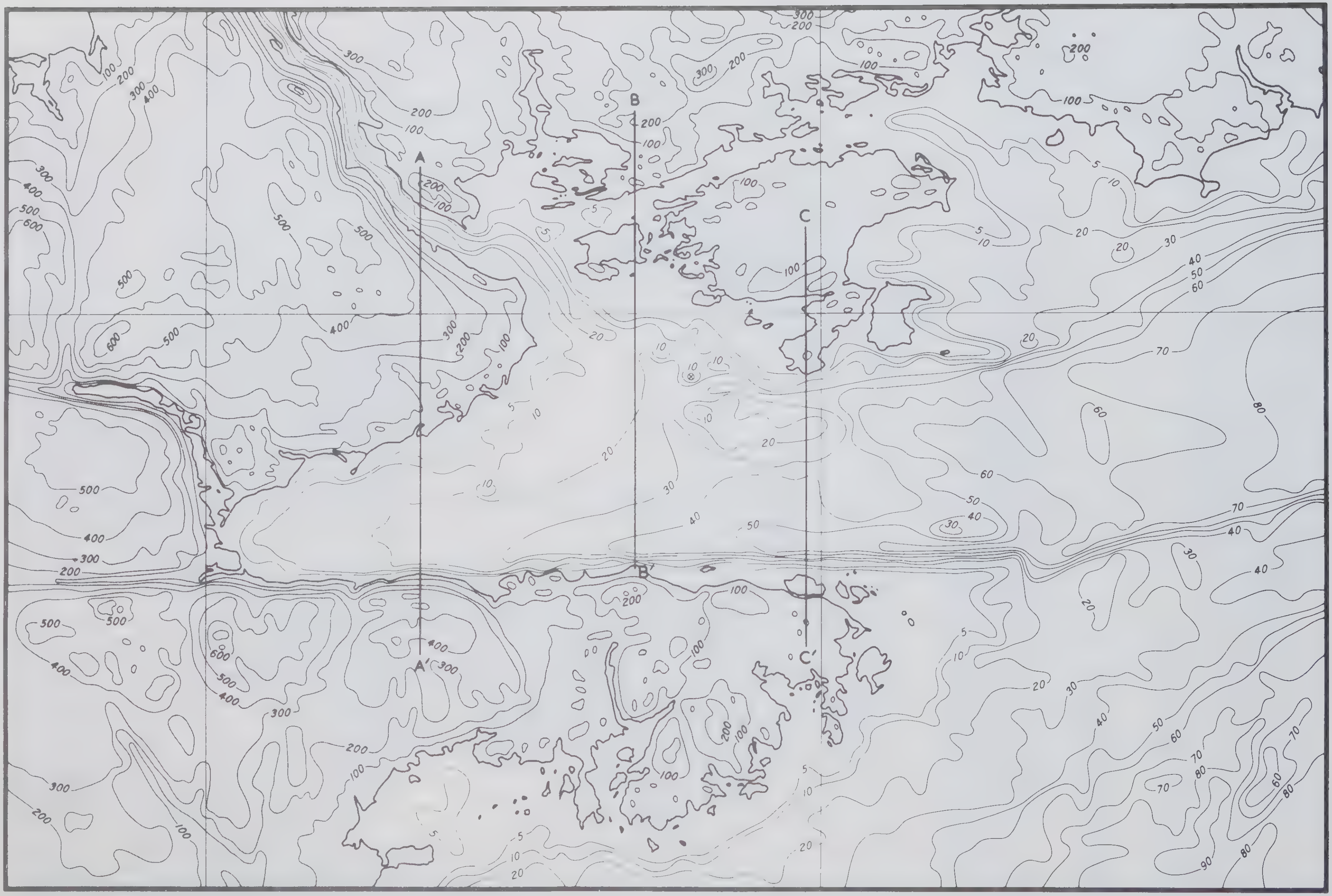


Figure 5. Topography and bathymetry. Heights in feet: depths in fathoms.

season, in terms of the wave climate and beach form, is probably from November to April, while the period of accretion and summer profiles is May to October when winds are from the south-west.

This agrees in general with Neu (1971) who states that from December to May "most of the wave energy occurs in the 7 to 12 seconds (wave) period band propagated from SSW to WNW". As part of the Project Oil investigation, Neu recorded wave data in Chedabucto Bay for 33 days in March and April, 1970. The results indicated that waves with a period less than 9 seconds were generated locally or over the adjacent shelf areas but that ocean waves with periods longer than 9 seconds prevailed 80 percent of the time.

Table 2 Summary of the wind data, 1964 to 1970, Canso, Nova Scotia (Anon., not dated)

	Average of the Monthly Mean Speeds (mph)	Prevailing Direction	Average of the Monthly Maximum Recorded Speed	Dominant Direction
Jan.	14.3	NW	42	NW/NW
Feb.	14.8	NW	40	NW
Mar.	13.8	NW	35	NE
Apr.	13.4	NW	33	NW/SW
May	12.7	SW	33	SW
June	11.6	SW	29	SW
July	10.6	SW	27	SW
Aug.	11.9	SW	29	SW
Sept.	11.4	SW	30	SW
Oct.	13.0	SW	34	SE
Nov.	13.7	NW/SW	36	NE
Dec.	14.8	NW	38	SE

3. THE COAST OF CHEDABUCTO BAY

The basic coastal trends are structurally controlled while the actual detail of the shoreline results from the erosion and submergence of the fault block. Grant (1970) estimated that submergence is taking place at a rate of about 0.5 feet per century through a combination of the eustatic rise in sea level and crustal subsidence. This has led to the drowning of lowlying coastal areas to produce a complex and irregular shoreline for much of the region (Owens, 1971a).

3.1 South Shore

The south shore of Chedabucto Bay between Canso and Guysborough is a straight, steep coast with a narrow offshore shelf (Figures 5 and 6). This is a resistant erosional shoreline composed largely of rock platforms and low cliffs with pocket beaches of shingle and coarse sand. The amounts of sediment in the littoral zone increase noticeably from east to west as indicated by the presence of spits and bars in the Salmon River - Guysborough area. South and west of Canso the irregular coast has resulted from the drowning of the southward sloping peneplain. Although this area was not examined in detail it was apparent that rock platforms with little beach material dominate the character of the shore. Occasional pocket beaches and spits interrupt the pattern but otherwise this area is an excellent example of a submerged resistant lowland. More sediment is available for reworking in the littoral zone of this region than along the south shore of the Bay, but in both cases there is a general scarcity of beach material.

3.2 West Shore

Between Guysborough and the Strait of Canso the relatively uniform south-east slope of the subaerial and submarine topography (Figure 5) gives rise to a generally straight shoreline. Few bedrock exposures occur and the coast is made up of actively eroding till cliffs (Photograph 1) alternating with wide accretional shingle beaches (Photograph 2). In an attempt to develop an equilibrium shoreline, beaches have built up across river exits and embayments while erosion is active where there are headlands and higher relief along the shore.

The till cliffs, which have a maximum height of 60 feet, are easily eroded by subaerial and marine processes but provide the beach zone with relatively little sediment. The till is composed largely of clay and silt sized material which is removed from the base of the cliff as a suspended sediment, leaving a few cobbles and boulders in the littoral zone. The beaches of the area are generally long and wide with

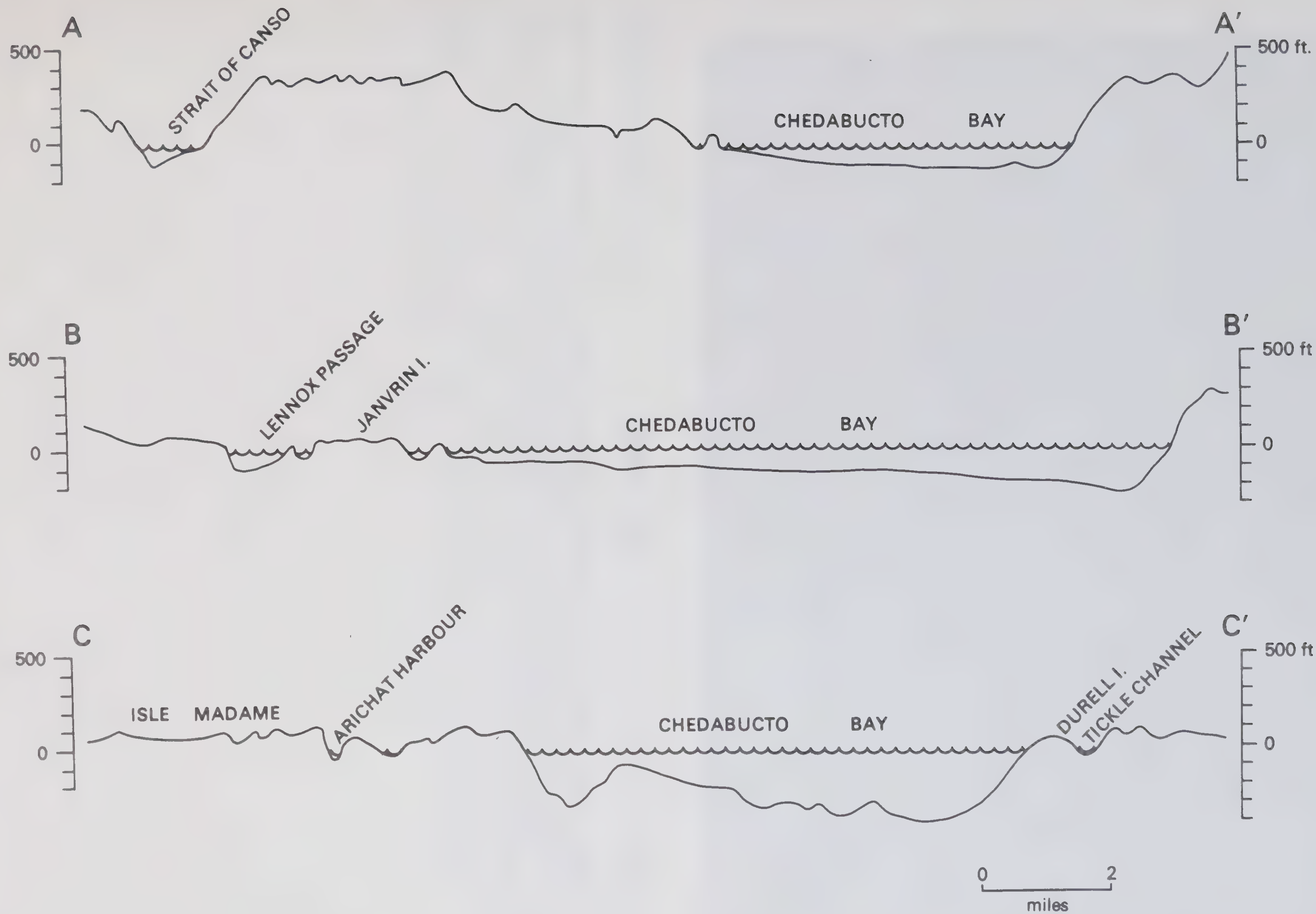


Figure 6. Profiles across Chedabucto Bay.

Photograph 1 CAPE ARGOS March 27, 1970

Air view of a partially eroded drumlin and an actively eroding till cliff with a narrow but heavily polluted beach at its base. The beach material here is almost entirely derived from the erosion of the cliff.

Photograph 2 MOOSE BAY June 9, 1970

Air view of the main section of the beach at mid tide before restoration. The main storm ridge is backed by a vegetated swale which in turn is replaced by a higher vegetation sequence. A squad of "slick pickers" is removing contaminated material manually (see also photograph 18), these are located by an arrow. No sections of this beach were paralyzed; although originally heavily polluted, most of the oil was buried or removed by normal wave action.

well-developed storm ridges and would appear to owe their growth to the landward movement of sediments from the offshore zone with the rise of sea level. It is doubtful that rivers and erosion of the till cliffs contribute significant amounts of coarse sediment under present conditions. Subsequent reworking and longshore sediment transport has produced some complex spit systems at several places along this coast. Where the till cliffs have narrow beaches at their base, waves attack the cliff directly and remove subaerially eroded material, but if the beaches are wide and there is a large sediment build-up, erosion is unlikely.

3.3 North Shore

East of the Strait of Canso, as far as Point Michaud, the drowning of an undulating lowland has given rise to a complex series of islands and inlets in an area of little local relief. Much of the coast is protected from direct wave action and beaches have not developed in these sheltered zones because of the low energy conditions.

The River Inhabitants is a particularly good example of a drowned valley. It flows into Inhabitants Bay which was the original flood-plain but is now a shallow bay with poorly developed beaches in a sheltered wave environment. South of Inhabitants Bay a series of well-developed bars and spits results from the greater availability of sediments and the higher energy level of the littoral zone, as this area is exposed to wave action from the south and east. The great variety of coastal types in this region results from the irregular nature of the drowned lowland area and the pattern of islands, bays and inlets. The beaches and spits of south Janvrin Island and Jerseyman Island are a direct contrast to the adjacent but sheltered areas of Port Royal or Arichat Harbour which do not receive the full force of waves from the Atlantic.

For most of the sheltered areas the coast is typified by a narrow beach, less than 100 feet wide, of coarse sediments resting on a till platform. This is usually backed by natural vegetation or a low till cliff which may not be subject to marine processes under normal conditions. There is rarely any evidence of storm ridge development, though bars and spits of limited size occur in most areas.

On the exposed shores of south Isle Madame and Petit de Grat the resistant Proterozoic and Devonian outcrops produce a rocky coast devoid of sediments which is very similar to the south shore of the Bay. To the north, in the Bay of Rocks, this gives way to a series of wide sand and cobble beaches oriented towards the refracted incoming waves from the east. Along the north shore, to the east of Lennox Passage, a large number of complex spits, bars and tombolos result from the

reworking of sediments in an energetic wave environment. As with the West Shore there is little indication that terrestrial sources are a major contributor of coarse material so that again the offshore zone appears to have been the major source for littoral sediments.

3.4 Summary

Apart from a few exceptions, in the north-east part of the Bay the beaches consist mainly of cobbles and boulders which are thrown up by wave action onto the higher parts of the beach to form a storm ridge. When these investigations commenced in April, steep winter profiles characterized most areas but by July these had been replaced by the more gently sloping accretional summer profiles, and these were still evident on all beaches in November. With this change in profiles came a marked increase in the amount of coarse sand and gravel in the intertidal zone.

Sand beaches are restricted to the Bay of Rocks, Point Michaud, and Blackduck Cove area, all at the eastern end of the Bay. These are wide, long beaches with shallow offshore zones and have well-established dune vegetation in the backshore areas.

The shoreline of Chedabucto Bay may be described as an erosional coast of rock and till exposures with pocket beaches. The growth of large accretional features is evident along the exposed north and west coasts but these beaches are supplied with little coarse material under present conditions. The sheltered coasts along the north shore have a limited sediment supply and are in a low energy wave environment with evident erosion in many areas as the region undergoes submergence.

4. SHORELINE STABILITY

"A beach is nothing more or less than a protective apron of rock waste fronting the land. Where beaches are wide and in equilibrium, erosion is unlikely. Where they are narrow and starved of material, wave attack is directed against the land with little to absorb its full force" (Kidson, 1966).

Coarse beach sediments, larger than 3 inches, were moved landward from the offshore zone with the rise in sea level and any sediment loss by erosion or excavation must be equated with the capability of natural replacement. Under present conditions, material of this size does not move into the littoral zone from offshore, thus the supply of coarse material is very limited and large-scale sediment losses cannot be replaced naturally.

4.1 Till Cliffs

At the base of till cliffs which are in active retreat there is usually a narrow cobble and boulder beach (Photograph 1). This material is derived largely from the erosion of the free face, which is a limited supply source as most of the till is clay and silt sized material. The retreat of the cliff leads to the development of a small platform at the cliff base on which rests the thin cover of beach material. In the sketch below it can be seen that on removal of the beach mantle the till base is exposed and becomes subject to erosion by marine agents. The protection that is afforded by the thin beach mantle is not great but with the loss of this mantle the level of the till platform is lowered by marine erosion and this in turn leads to a temporary acceleration of cliff retreat until an equilibrium condition is achieved by erosion of the free face.

Those till cliffs which have wide beaches at their base are not normally subject to direct wave action and erosion is a result of subaerial agencies, although the talus may be removed by littoral processes. These cliffs would be unaffected by beach loss unless large volumes of sediments are excavated to the extent that marine processes could begin to erode the base of the cliffs.

In sheltered areas, such as Inhabitants Bay, wave action is limited so that often, marine processes are not active in cliff erosion except under storm conditions. In these instances, subaerial rather than marine erosion may be consistently active though the infrequent storm would produce more dramatic results in the long term. The removal of the beach sediments which front these cliffs will have the same effect as on more exposed shores.

4.2 Beaches

Beaches are constantly changing in response to a variety of processes which may alter in intensity with the season, tidal cycle, or weather conditions. This is a complex environment which is still only partially understood.

The beaches of the region are made up largely of cobbles and pebbles and these sediments are being eroded, reworked and transported as the littoral zone strives towards the equilibrium which the processes demand. Although sand and fine sediments are fed into the coastal zone, particularly during the summer months, these are not a major contributor to the growth of beaches and large constructional features, except for areas at the east end of the Bay.

The loss of large volumes of cobble and larger size material would be harmful to the shore environment in this region. An instance where this has occured in a similar situation is reported by Robinson (1961) at Hallsands, on the south coast of England. As with most of the British coast, this beach owes its origin to the reworking of deposits during the post-glacial rise in sea level and is not now fed with coarse sediments from the offshore zone. At this locality there is no significant sediment supply from rivers or coastal erosion so that the beaches consist mainly of "fossil" material. Almost 500,000 cubic yards of shingle was removed in the period 1897 to 1902 along a half mile length of shore. This led to a recession of up to 20 feet between 1907 and 1957 in those sections of schist cliffs which had little protection at their base. The actual beach area to the north of the cliffs was lowered by as much as 12 feet and only the most southerly sections appear to have recovered in any way (Figure 7, profile 6).

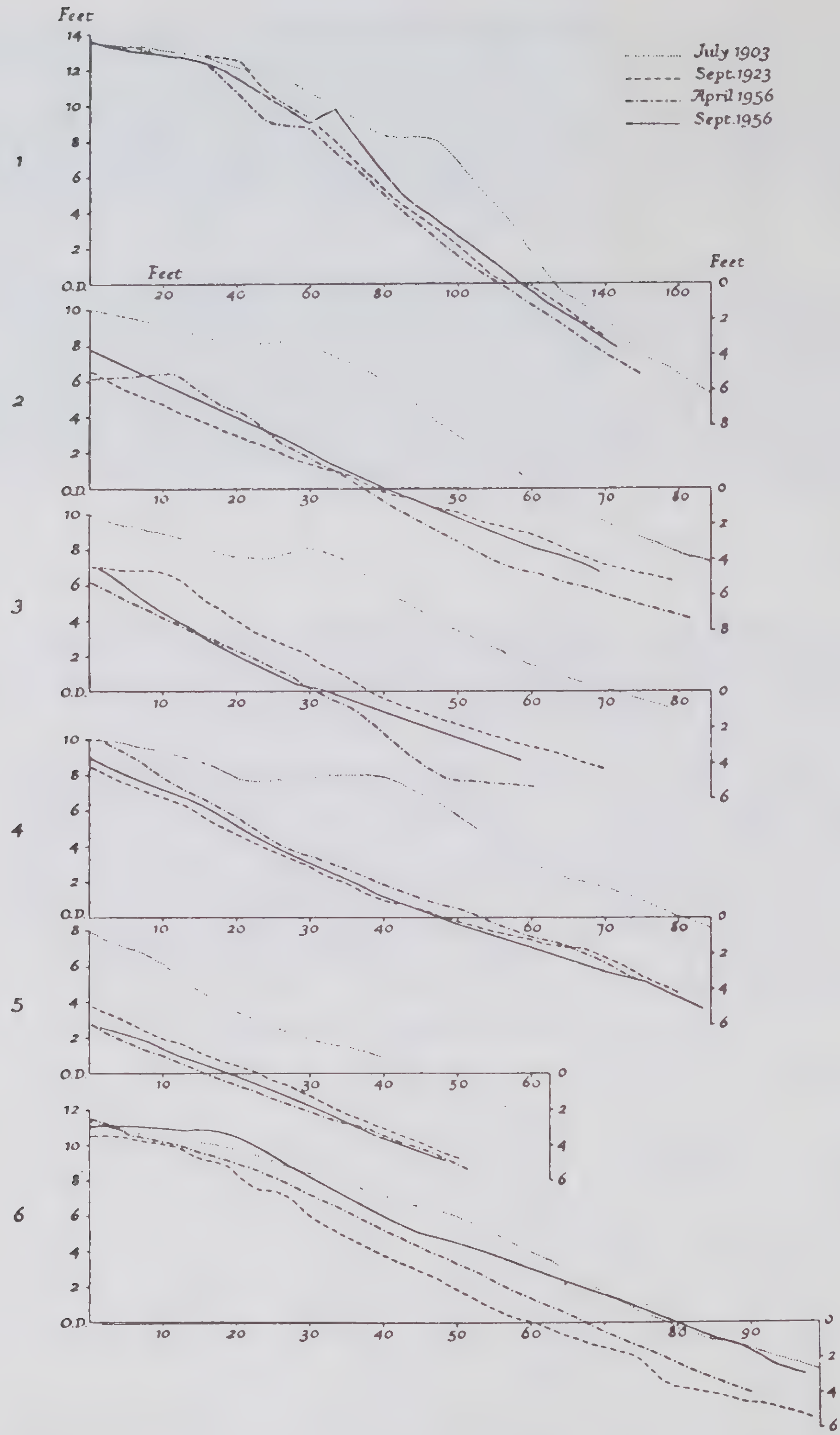


Figure 7. Beach Profiles from Hallsands, U.K. (Robinson, 1961) The changes in beach profiles along Sections 1-6 between 1903 and 1956. Vertical exaggeration for Section 1 is 1:5 and for remainder 1:2.5.

Although material removed from a beach may be replaced by longshore drift, providing the loss is not too great, there would be a net decrease for the shore as a whole in a region such as Chedabucto Bay, as this material cannot be replaced rapidly. On beaches which have a marked movement of sediment in one direction, should material be taken from the up-drift end, there is a danger that the beach may not fully recover as this is a section which would not normally receive much "new" material. In the same way, removal of sediments from a spit near the point of attachment may be harmful as most of the accumulation is concentrated at the distal end. This would apply to any constructional feature such as a bar, tombolo, or foreland which is formed by the longshore movement of material.

While it is important to consider the loss of material in terms of the plan form of the beach, more critical perhaps is the effect on the beach in profile. The storm ridge, or the sand berm, is built above the normal high water mark and material lost from this zone will only be replaced during the infrequent occasions when wave processes are active on these sections of the profile, providing that material is available for replacement.

5. OIL ON THE SHORE

5.1 Distribution

Those coastal areas which were contaminated by Bunker C from the "Arrow" are outlined in Figure 8. This diagram is only accurate for the Bay itself as many of the areas in the north-east and to the south and west of Cape Canso were only surveyed at the reconnaissance level and only those areas where oil was actually observed are indicated. The shore affected by oil was more extensive than is shown and it should be noted that contamination extended as far as St. Esprit, on the north shore some 5 miles east of Point Michaud, and Sable Island, which lies over 100 miles south-east of the Bay (Figure 1). The major spills took place in February and March but oil leaked from the wreck throughout the spring and summer.

The distribution diagram does not take into account the severity of the contamination or the frequency of reoiling. The shores of Janvrin Island, Crichton Island and Jerseyman Island and the south shores of Isle Madame and Petit de Grat were heavily polluted in February and subject to frequent reoiling in the following months. The area of Inhabitants Bay was not contaminated by the end of February but was subsequently heavily oiled. To the north and east of Petit de Grat contamination was relatively light and not continuous. The shore of the east section of Lennox Passage was not polluted due to the construction of a dam in the middle section.

Along the west coast of the Bay, from Cape Argos to Guysborough, most of the shore was contaminated but the amounts of oil on the beach decreased towards the south-west. Similarly on the south shore, the area around Canso was severly polluted while to the west of Half Island Cove the degree of oiling decreased and some sections were only lightly oiled.

South of Cape Canso several of the inlets and embayments were badly polluted though a full survey of this area was not carried out. The distribution diagram does not indicate the true extent of oil on this section of coast.

5.2 Behaviour of Oil on the Shore

Field observations by Asthana and Marlowe (1970) and by Drapeau (1970) during February and March were carried out, in part, to provide an understanding of the nature and behaviour of oil on the coast. Their conclusions are summarized below.

> Floating oil striking the shoreline behaves differently on different types of shore material.

Figure 8. Distribution of oil on the coast. x - Cerberus Rock

a) Bedrock is coated by a uniform layer of oil in the intertidal zone. The oil is not removed during tidal submergence but when exposed, flows into crevices and hollows.

b) Boulders and boulder beaches are affected in a similar way.

c) On sand and gravel beaches the oil remained on the surface and behaved as discrete sedimentary particles with unique hydrodynamic characteristics. The oil particles tended to float free and become concentrated along the high water line. Oil did not permeate sand but acquired a surface coating of sand particles (Photograph 3) while on gravel it was observed to have permeated as much as 18 inches. Where oil had been buried and later exposed by the normal accretion and erosion processes, the oil layers were less readily eroded and formed ephemeral ledges (Photograph 4) which were seen to crumble under the influence of gravity and sunlight.

Drapeau also noted that within one month of contamination, gravel bars directly exposed to wave processes cleaned themselves naturally and effectively, while in protected areas the beaches remained polluted. This is particularly evident in Photograph 5 of Blackduck Cove. The lagoon remained heavily polluted throughout the summer while the exposed side of the spit was "cleaned" by the end of March. In many of the exposed areas the intertidal zone was rapidly cleaned leaving a concentration of oil where it had been deposited above the high water mark beyond normal wave action (Photographs 6 and 7). Although oil polluted much of the upper part of the intertidal zone, wave action is continuously active in this zone and any oil laid down above mean high water mark would remain undisturbed by normal processes. During periods of storm waves or spring tides the remainder of the beach would be subject to wave action so that eventually all contaminated areas would be cleaned naturally. On cobble and shingle beaches, oiled material originally in the intertidal zone was often moved up the beach by storm or swell waves and deposited as part of the storm ridge. This may have occurred on several occasions so that with subsequent exposure the contaminated material would appear as layers interbedded with clean sediments (Photograph 4).

Exposed beaches are generally capable of self-cleaning (Johnston, 1970) but there is a threshold beyond which beaches become "paralyzed" (Drapeau, 1970). This situation may arise as a result of either heavy contamination or insufficient wave action. The beaches of Arichat Harbour and Inhabitants Bay, for

Photograph 3 HADLEYVILLE No. 2 July 10, 1970

The east end of the beach was made up of coarse sand and the oil remained as large pans which had a surface layer of fines or was buried to a depth of several feet. No oil was visible in the intertidal zone.

Photograph 4 HADLEYVILLE No. 2 July 10, 1970

On the west end of this beach at low tide. The layers of oil, when exposed by wave action, are more resistant to erosion but quickly collapse through sunlight and gravity.

Photograph 5 BLACKDUCK COVE April 25, 1970

Air view of spit, lagoon, and cove from the south at high tide. The lagoon is virtually cut off from the sea at low tide. This area was heavily polluted though the exposed section of the spit has been virtually cleaned by wave action whilst the beach in the lee of the spit remains paralyzed.

Photograph 6 HADLEYVILLE No. 2 July 10, 1970

The central section of the beach was characterized by a thick, continuous layer of oil ten to fifteen feet wide, above the high water line. This upper part of the beach, rarely affected by waves, remained paralyzed while the intertidal zone was cleaned by wave processes.

Photograph 7   JERSEYMAN ISLAND June 21, 1970

A badly contaminated cobble beach near the high water mark. Although this beach has been partially immobilized, wave action has begun to clean the area below the high water line by abrasion and burial. With storm waves the remainder of the beach will be combed down and the cleaning process will be extended to include all the contaminated material.

example, were paralyzed by a combination of both these factors while the heavily polluted but exposed beaches of Crichton and Jerseyman Islands remained mobile.

Four samples were collected for analysis of oil content by volume to determine the oil/sediment ratio on different beaches. Two were taken from the exposed, mobile beach at Moose Bay near the high water mark. The first was at a site which appeared clean and the second from material which appeared badly oiled. The results indicated 9 and 90 parts per million of oil, respectively. Two more samples were taken from the intertidal zone of a "paralyzed" beach at Arichat. These provided values of 4 and 5 per cent of oil. All the samples were from cobble beaches with a material size range of 1 to 6 inches.

5.3 Previous Beach Restoration Projects

The only restoration projects before the Chedabucto Bay operation resulted from the "Torrey Canyon" and "Santa Barbara" spills. Neither of these are directly comparable as the coasts of south-west England were cleaned with dispersents while the wide, firm, sandy California beaches were restored with relative ease.

The work of the Ministry of Technology, Warren Spring Laboratory, dates from 1960 and their involvement in the "Torrey Canyon" clean-up provided a practical demonstration of the various restoration techniques which are reported by Wardley Smith (1968a, 1968b, 1969, 1970). The fundamental conclusion from their experience is that no one method is adequate because of the variety of coasts and the amounts and types of oil involved. Despite this, it was stressed that the best defence is preparedness so that if oil does reach a shore the correct action may be taken immediately.

The various methods of restoration which were discussed are summarized below.

a) Burning. In most instances this is not satisfactory because of inefficiency and cost. Most oils, particularly Bunker C, do not burn readily and require a great deal of assistance; even then the oil burns very slowly. Heating often makes the oil more mobile so that it penetrates deeper into the beach and contaminates more material.

b) Absorbtion. The use of sawdust, peat moss, or similar material was found to be very useful but costly in terms of the labour required to spread and retrieve the absorbant.

c) Surface Coating. A crust could be formed on the surface of the oil with the application of large

quantities of powder or fine particles. This stabilized the surface but was not satisfactory for preventing the oil from moving and did not contribute to the removal of the oil.

d) Mechanical Removal. This method was found to be satisfactory for the removal of surface oil but not all the contaminated material was removed by the machinery. In combination with dispersants this method proved effective on cobble beaches. Machinery was developed to pick up contaminated material because of the unsuitability of available excavating or earth moving equipment, but this was elaborate, sometimes expensive, and not always successful.

e) Manual Removal. Effective on most beaches but expensive because of labour costs. Techniques include raking, shovelling and spreading absorbant materials.

f) Dispersants. Very effective in removing the oil, especially on cobble beaches, but toxic to marine life.

Each of these methods was considered in terms of applicability to different shoreline types.

a) Marshes. Manual removal would be best but burning may be effective during the non-growing season. However, unless the oil is thick the marsh is best left so that it could recover naturally.

b) Mud Flats. Although these areas would be left alone where possible, if restoration is necessary mechanical scraping or dispersants could be used for thick deposits.

c) Sand. Again it would be best to leave cleaning to natural processes if possible but if action is necessary, manual or mechanical raking would be effective. If the beach is heavily polluted, material could be pushed into the sea at low tide and the beach then sprayed with emulsifier. The sand would be reworked, cleaned, and returned to the beach without any sediment loss to the littoral zone.

d) Shingle. Apart from natural processes only dispersants could rid the beach of all contaminated material. No machinery was available which could clean the beach without the aid of dispersants.

e) Rock. These areas are best left to the action of waves and weather but burning or dispersants could be applied if necessary.

Following the Santa Barbara spill on the California coast the United States Federal Water Pollution Control Administration (FWPCA) initiated an evaluation of earth moving equipment in oil contaminated beach restoration operations (FWPCA, 1970). This project was carried out on wide, firm, flat sand beaches which had oil on the surface or in the surface layers. The results of this technique analysis are given below.

a) Grader/Scraper. This was found to be the best of the techniques evaluated. Contaminated sediments were pushed into wind-rows by the grader and then lifted by the scraper. This technique removed the least uncontaminated material but spillage from the scraper required a following pick-up crew. The grader became stuck on coarse sand unless expensive flotation tyres were in use and accuracy decreased if traction was low.

b) Scraper. Used on its own the scraper had a high spillage and like the grader, it required a flat beach and became stuck easily.

c) Grader/Front-end Loader. The loader was used to remove the wind-rows. Its performance in general is outlined below.

d) Front-end Loader. This machine was the least efficient of those tested as it removed too much uncontaminated material and had a high spillage. The same defficiencies apply to bulldozers and tracked loaders as bulldozers were found to "grind the oil several feet into the sand".

e) Ramp-Conveyor System. This method was developed to remove the material after the grader formed the wind-rows and was found to be valuable for very large operations.

Neither of these evaluations are directly applicable to the problems which were faced in Chedabucto Bay, as no wide, sand beaches required mechanical restoration and dispersants were ruled out of the programme. The conclusions from the work in Britain indicated that natural cleaning is best but even if restoration is necessary, shingle beaches were not cleaned properly by machines alone. The evaluation project by the FWPCA showed that the grader and bulldozer were the least efficient of the earth moving equipment tested. The performance was particularly bad if the vehicle had tracks, as clean and contaminated sediments were mixed by spillage and grinding.

5.4 Recontamination

Following the grounding and sinking of the "Arrow", large slicks contaminated over half of the shoreline in the Bay and leaks from the wreck continually spilt small amounts of oil throughout the spring and summer. These slicks were a great deal smaller and thinner than those of February and March but this oil did recontaminate many of the beaches along the north shore area.

Other sources of oil which led to shoreline recontamination were from the coastal areas themselves. The reworking of contaminated sediments by wave action led to the release of oil onto the water in small amounts. This was exemplified by the action of tracked vehicles working in the intertidal zone in the early phases of the project before better machine-operation techniques were developed. In particular at Arichat, which was the first area to be worked, a substantial amount of oil was released into the sea and this led to the reoiling of adjacent beaches (Section 6.4.1).

Oil was often contained in rock hollows and crevices above the limit of normal wave action. With spring tides or storm waves, some of these pools of oil were flushed, leading to recontamination of adjacent alongshore areas. This was judged to have been the reason for the reoiling of Indian Cove (Section 6.4.3).

6. BEACH RESTORATION

6.1 Deep Cove, February 1970

In mid-February, shortly after the sinking of the "Arrow", the Canadian Armed Forces bulldozed part of the bar at Deep Cove in an attempt to clean this beach (Photograph 8). It was estimated that some 3,000 cubic yards of sediment were excavated and removed during this operation (Asthana and Marlowe, 1970). This trial was not successful as oil and sediments were thoroughly mixed and not all the contaminated material was removed. This operation was also ineffective in terms of recontamination as this area was reoiled within a few tidal cycles and on several occasions subsequently.

The beach which was subjected to bulldozing is a narrow bar which joins two islands and the removal of large volumes of sediment was considered very damaging to the stability of the foreshore (Asthana and Marlowe, 1970). As a result of this experiment a recommendation was made to the Task Force that bulldozing should not be continued and that beach restoration should be restricted to manual methods (see also 6.4.6).

6.2 Selection of Beaches for Restoration

Before the assignment of the coastal geomorphologist to the programme, a series of beaches had been selected for restoration. These beaches included all accessible nationally rated shorelines (these are rated on a recreational basis), as well as community beaches in the area. The information regarding the location and extent of these beaches was supplied by the Emergency Measures Organization. Additional sections of coast were included in the programme as a result of public requests. In all, 30 miles of coast were restored by the Task Force.

The restoration programme was carried out by personnel of the Department of Public Works who determined whether beaches could be restored by manual or mechanical methods. Lightly oiled beaches were restored by squads of "slick pickers" (Section 6.3) while contracts were drawn up for the restoration of heavily contaminated sections. These were awarded to private companies after bids had been tendered.

No geological criteria were included in the decisions regarding which beaches were to be restored by the Task Force. Only after a section of coast had been designated were these criteria considered and the geological input was largely restricted to recommendations concerning those beaches actually under contract rather than advice related to the restoration programme as a whole in terms of the selection of beaches.

Photograph 8 DEEP COVE March 1, 1970

Air view from the north of armed services restoration work. This is a long, narrow bar which joins two islands and is open to wave action from the south and east. At the east end of the beach a spit has grown north to partially close off a small lagoon which is being infilled with mud and silt transported alongshore. The sea at the near side of the bar and the lagoon, are ice covered. Material has been piled up by the bulldozers for later removal.

6.3 Manual Restoration

Those sections which were designated for restoration but which were only lightly oiled were cleaned effectively by squads of "slick pickers". These units, local labour under the Department of Public Works supervision, removed oil and contaminated material with shovels and rakes (Photograph 9). The material was placed in plastic bags which were collected and removed to an approved dump site.

Although this method of restoration is comparatively expensive in terms of labour costs, it is most efficient and effective as only contaminated material is removed. The loss of sediment from a beach is low, so this has very little adverse effects on beach stability and except where the oil is deeply buried, it is possible to remove virtually all the contaminated material.

The sand beaches in the Point Michaud area and in the Bay of Rocks were successfully cleaned by this method. These shores were not in danger of recontamination and remained clean throughout the summer. Shingle and cobble beaches were harder to clean as the oil did not remain in cakes and pans on the surface but this method is still preferable to the use of heavy machinery on lightly oiled beaches.

Any material which was buried and not removed manually would probably be exposed in the winter or spring when the beach is combed down under vigourous wave action. This oil would be subject to wave action throughout the winter and by the following summer there should be little evidence of any contaminated material on the exposed beaches.

6.4 Contract Work

This section includes detailed accounts and analyses of the contract work on five beaches and a more general report on some of the other beaches where machinery was used. Not all of the sites which were contracted or where machinery rental was used are dealt with (MacKay, 1970) but the operations reported cover a representative cross-section of the shoreline areas in Chedabucto Bay where projects were carried out. The location of the sites reported here is given in Figure 2.

6.4.1 Arichat

The contract for a 3,700 foot section of beach was awarded to a low bid of $4,479 and the work was carried out over 9 days between April 30 and May 11, 1970 using a fixed-blade International TD15 bulldozer and an International TD9 skid shovel; both vehicles were tracked. A total of 422 cubic yards of material was removed and 40 cubic yards were brought in as clean replacement at Le Noir Forge.

Photograph 9 SLICK PICKERS May 29, 1970

Air view of the restoration of a sand beach manually. Oil and oiled seaweed is shovelled into plastic bags which are collected for dumping. This method is very effective as spillage is negligable and only contaminated material is removed.

Description of the contract area

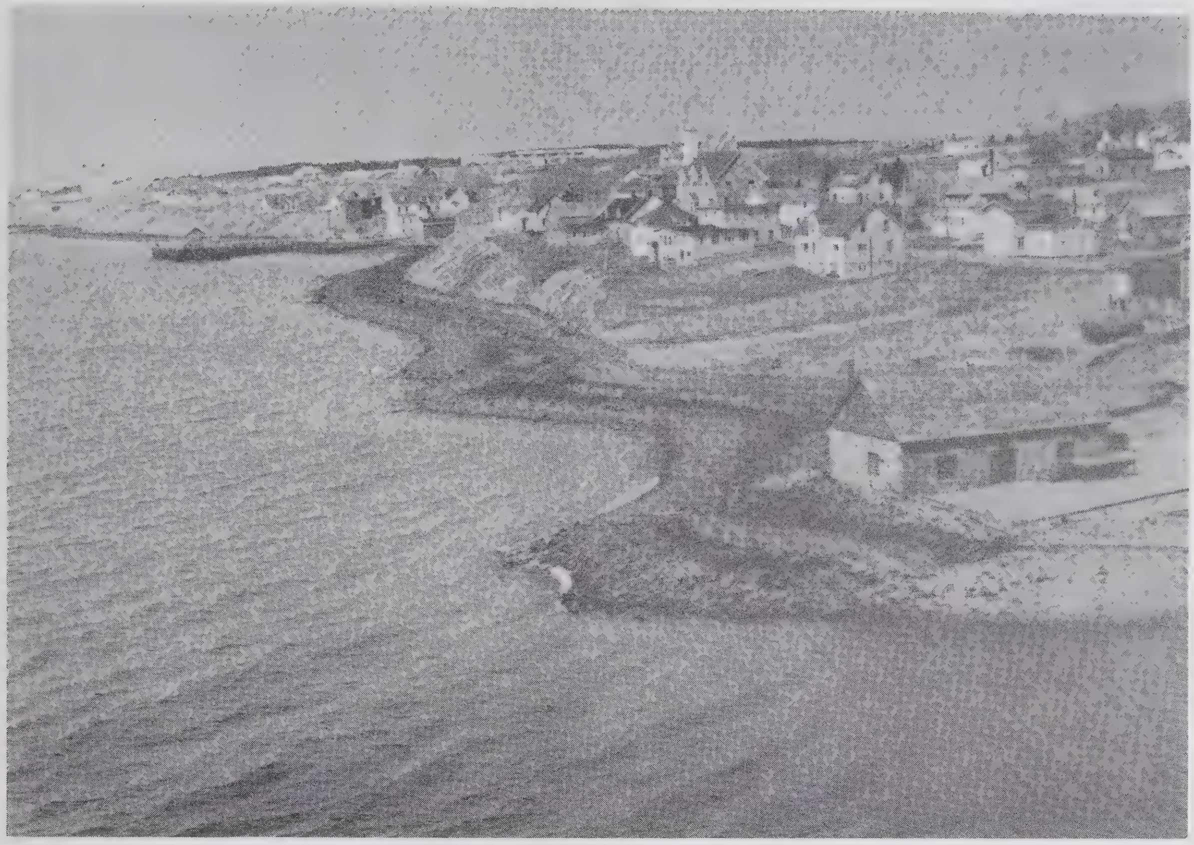
This is a sheltered location not exposed to the direct action of waves from the Atlantic with a tidal range in the order of 4 to 6 feet (Table 1). All of the contract area was badly oiled and the beach zone was effectively paralyzed; that is, oil prevented the normal movement of sediments by wave action. In detail the section may be subdivided into four units:

1) Between the Arichat wharf and Le Noir Forge the shore is characterized by a 20 to 25 foot till cliff (Photograph 10). The width of the beach zone averages 40 feet at low tide and is made up of sediments derived directly from the subaerial and marine erosion of the cliff. This beach mantle was rarely more than a foot thick, mostly gravel and cobble size with a few boulders and overlay the till platform which resulted from the retreat of the cliff. Sections of the backshore are undergoing active erosion but in general the supply of material into the littoral zone is minimal. As the stone content of the till is low, most of the eroded material is removed as suspended sediments.

2) A small prograding beach, about 100 feet long and 60 feet wide, fronts a low backshore in the area immediately west of Le Noir Forge. This beach appears to have resulted from accretion which occured as longshore sediment transport was interrupted by the small headland on which Le Noir Forge was constructed.

3) The Le Noir Forge headland is a till bedrock area which has a beach of till-derived material at its seaward end. The two small beaches on either side of the headland are of gravel and are areas of more rapid accretion.

4) The east half of the area is made up of a series of three mid-bay bars and low active till cliffs. The bars have built out on a shallow platform which has a maximum width of about 150 feet. Sediment accretion on these small bars has deprived the intervening areas of material so that the narrow beaches have not prevented wave erosion of the backshore cliffs. The till cliffs are generally lower than the west section, being between 5 and 15 feet high. Various sections have been riprapped or protected with wood structures to prevent the erosion of property.



Photograph 10 ARICHAT May 2, 1970

Air view of Le Noir Forge and west section at low tide. The bulldozer had made a "road" along the base of the active till cliff at the high water mark. This section was heavily contaminated and the beach zone was paralyzed. The contract area ended at the wharf.

Restoration

This contract was the first to be issued and work began on April 30. Contaminated material in the intertidal zone just to the west of the Le Noir Forge was heaped into piles and a "road" was bulldozed along the high water mark westwards toward the wharf. Some concern over the action of the contractor was expressed and it was decided not to remove any material beneath the cliff west of Le Noir Forge. As the "road" had already been bulldozed part of the way, the disturbed material on that section would be pushed against the base of the cliff but the contaminated material which had been piled up on the beach adjacent to Le Noir Forge would be removed.

On May 2, the "road" was bulldozed through to the west end of the contract section, near the public wharf and during May 2 and 3, the disturbed material in this section was pushed against the base of the cliff up to 4 feet above normal beach level. The piles of material west of Le Noir Forge were removed on May 5. The Department of Public Works site supervisor reported "fresh" oil on May 3 and 4 on those areas of the beach which had been subjected to machinery.

On May 6, a decision was made to remove and replace a section by Le Noir Forge which had been excavated to the till bedrock and areas were outlined in the east half of the contract where material should not be disturbed. During May 6 to 10, the areas designated for cleaning in the east section were dealt with by the contractor and on May 11 the contractor was released.

Effectiveness and Results of Operation

a) The use of a bulldozer to drive a road through the west section thoroughly mixed contaminated and uncontaminated sediments. As the beach was scarcely wider than the resulting road the contractor should have removed the layer of contaminated material while progressively moving westward.

   This action highlights an important point that the site supervisor should be aware of the implications of various actions so that he can make on the spot decisions which will be carried out by the contractor in order to prevent action which may be undesirable or endanger the stability of the beach.

b) The removal of material from the beach at the foot of the till cliffs may have accelerated basal erosion. The beach consisted of a thin cover of till-derived sediments overlying a till platform which was exposed as the "road" was bulldozed across the beach. In order to minimize the possible adverse effects of this action the disturbed sediments were pushed against the base of the cliff.

No material was removed from this area but the disturbance of the beach material led to the washing out of the fines and the release of oil into the littoral zone. The fines were transported along shore, in a band some 6 feet wide adjacent to the beach, while the oil was redeposited in thin layers near the high water line (Section 5.4). The section would have been better left, although at least this beach was no longer paralysed.

c) In the area of beach to the west of Le Noir Forge some 62 cubic yards of material was removed by the contractor, approximately a 12 to 18 inch layer of sediments. At the completion of the contract, this area was relatively clean, with only a small amount of reoiling near high water mark. On May 26 and 27, the beach was reoiled by a slick (not from the "Arrow") 30 feet by 5 feet which was 3 inches deep in parts. Peat moss was placed over the oil on the 28th and the contaminated material was removed manually to a depth of 12 inches on June 2. Some 2 feet of beach was removed in all and even those sections re-cleaned on June 2 had been slightly reoiled by June 3.

d) On the seaward edge of Le Noir Forge promontory, some 20 cubic yards of material were removed. This exposed the till base and the replacement of 40 cubic yards of larger, boulder sized material has increased the stability of this small section of beach. The clean material used as replacement has been contaminated.

e) In those areas east of Le Noir Forge which should have presented no difficulty for the contractor, i.e. only a shallow surface layer of material was contaminated, the use of a bulldozer with a straight blade was demonstrated to be particularly unsuitable. As material is pushed forward by the blade, clean and contaminated material is mixed. Spillage around the edge of the blade contributes to the inefficiency of the method as this material must be removed by a subsequent pass and is often ground in by the tracks of the vehicle. The only way in which the bulldozer could operate effectively was to pile up material for removal by other equipment and this tended to mix sediments rather than remove the contaminated layer.

f) The cleaning of selected sections of a shoreline is not effective because oil from adjacent contaminated areas as well as from offshore, can lead to reoiling. All of the beaches subjected to cleaning had been reoiled by June 4, either from alongshore or offshore.

g) The two basic criteria that all contaminated material should be removed and that there should be no danger of recontamination were not realized on this contract section.

h) The Arichat beach was badly contaminated and much of the littoral zone was "paralyzed". From this aspect, the restoration operation at least made the beaches mobile once more, though this could have been achieved by the use of a tractor-drawn rake or hoe. The contract was the first to be awarded and many of the lessons learnt here were applied to later projects.

6.4.2 Blackduck Cove

The contract for this 4,600 foot section of coast in the north and north-east of the Cove (Photograph 5) was awarded for $6,000 and the operation required 9 days between April 30 and May 12, 1970, using a fixed-blade Caterpiller D6C bulldozer and a Caterpiller 950 wheeled front-end loader. A total of 4,469 cubic yards of material was removed and 360 yards of clean boulders were brought in along the north shore.

Description of the contract area

The Cove is on the south coast of Nova Scotia but is not directly exposed to waves from the open sea. At the east end of the shallow cove a wide, medium to coarse grained sand beach is backed by a vegetated berm and brackish marsh. The beach is generally about 100 feet wide but at spring tides may be up to 300 feet wide. The abundance of sand in the offshore zone and the shallow nature of the cove indicate that there is an ample supply of sand-sized material for beach replenishment. The berm and backshore areas are stabilized by grass. Both ends of this section are areas of mud and silt accumulation presumably where the fines have been deposited by alongshore movement away from the centre of the beach.

The oil on the sand beach had "paralyzed" the sediments above high water and though some self-cleaning had taken place in the intertidal zone, oil had mixed with sand and seaweed to form large immobile cakes (Photograph 11).

The northern shore of the cove is a low silt/sand area which has a surface cover of large glacially derived boulders which protected a road and houses on the low till backshore from marine erosion. This area was heavily polluted and paralyzed.

Restoration

The contractor commenced work on April 30, 1970, in the sandy beach section. Contaminated material was bulldozed from the intertidal zone into large piles above high water for subsequent removal by the front-end loader (Photograph 12).

Concern was expressed over possible damage to the backshore vegetation by the machinery and the contractor thereafter maintained one track for the trucks.

Photograph 11 BLACKDUCK COVE April 25, 1970

Air view of sand beach at the east end of the cove near high tide. Oil is present as a thick, continuous layer along the high water mark and along the edge of the vegetation. Thick patches of oil and seaweed "cakes" cover much of the intertidal zone but the beach is still mobile. The cobbles and boulders along the north shore in front of the cottages are also heavily polluted but this section is paralyzed.

Photograph 12 BLACKDUCK COVE May 2, 1970

Air view of beach restoration work at low tide. A bulldozer has pushed the contaminated sediments above the high water mark for subsequent removal by a front-end loader. The contaminated material along the edge of the vegetation was removed by the bulldozer scraping backwards down the beach in order to minimize damage to the backshore. For the same reason, access to the beach from the backshore was carefully controlled to prevent the development of blow-outs.

The sand section was cleaned by May 2 and work then began on the north shore. The contractor agreed to remove material only from the upper part of the intertidal zone. An agreement was reached to replace one section where the road on the backshore was left open to wave action by the removal of the boulders.

Effectiveness and Results of Operation

a) The clean-up of the sand area was a successful operation but by piling the material above water level, mixing of oiled and clean material took place. Although spillage by the bulldozer was high it was of a much lower magnitude than on a cobble or mixed beach. Ideally the material should have been lifted off the beach rather than pushed around for removal by other equipment.

b) Controlled access to the beach by the trucks using only one track resulted in damage to only one area of vegetation rather than wide-spread disturbance. Where contaminated material was found near the edge of the berm vegetation, the bulldozer was used to scrape down, rather than dig or push up the material. This again helped to minimize damage to the vegetated zone, (the destruction of the vegetation could have led to blow-outs).

c) At the extremities of the sand beach the bulldozer often sank above its tracks in the silt and mud. This led to a thorough mixing of contaminated and clean material. In "soft" areas like this, very little can be done with heavy machinery.

d) The sand areas were easily cleaned as the oil was only on the surface and this type of material is more easily handled than gravel or cobbles. An angle-blade on the bulldozer may have been an improvement, as this would have reduced spillage.

e) Although the sand beach was clean after the contractor had completed his work, it was reoiled on various occasions subsequently from alongshore areas. By the end of July the intertidal zone was again covered with cakes of oil, sand and seaweed over large areas and these were still evident in November. The heavily polluted lagoon at the west end of the Cove released some of its oil throughout the summer and this was transported alongshore towards the beach by incoming waves (Section 5.4).

6.4.3 Indian Cove

(Fox Island Main)

This 850 foot beach was contracted for $2,000 and involved four days work between May 15 and May 20, 1970. A Caterpiller 950, wheeled, front-end loader removed 1,368 cubic

yards of beach material during this period. The volume of sediment excavated on subsequent occasions is not available.

Description of contract area

This is a small concave pocket beach set back between two rock headlands. The beach has a maximum width of 130 feet and the material varies from coarse sand in the lower parts of the intertidal zone to cobbles at the crest of the storm ridge, the typical mixed beach of this area, with the dominant size in the cobble range.

The well developed storm ridge in the eastern half is backed by a low vegetated swale and a brackish pond. In the west half there is a continuous level area of old beach material fronted by a poorly defined beach crest ridge. The oil was confined to an area above normal high water level as a 6 to 12 inch thick caked layer approximately 10 feet wide which extended for almost the entire length of the beach (cf Photograph 6).

Restoration

A small experiment using a front-end loader with a 3 cubic yard capacity bucket was carried out on May 13 to test the effectiveness of this equipment for lifting off the oiled carpet. The encouraging results led to the use of the front-end loader for this operation and the beach was cleaned between May 15 and 20.

By May 25, the eastern 300 feet had been badly reoiled in the high water mark zone from the adjacent oiled rock areas (Photograph 13). On June 11 to 13, a bulldozer with a front bucket was rented to re-clean the beach. Oil was removed from the rock areas manually and limestone was spread over the rocks to stabilize the remaining oil, but recontamination occurred on several occasions throughout June and July (Section 5.4).

Beach profiles

To provide some measure of the effects of sediment removal, the beach was surveyed by 17 levelled profiles on May 14, 21, 29, June 14 and November 11. Six of these profiles, all from the east end of the beach, are presented in Figure 9.

In order to ascertain the distribution of sediment removal across the beach, profiles were taken immediately before (May 14) and after (May 21) the contract work. The limits of the thick, continuous layer of oil on the beach are shown (Figure 9b) and it is evident that the beach had been able to clean itself except for those areas above the reach of normal wave action. Most of the sediment removal was from this zone and as was noted earlier (Section 4.2), this part of the beach is not an area of accretion except during periods of storm waves or spring tides. Sediment replacement is, therefore, not rapid in this zone.

Photograph 13 INDIAN COVE May 25, 1970

The east section of this pocket beach at low tide, four days after completion of the contract work. Before restoration, a thick continuous layer of oil, six to ten feet wide, covered the length of the beach above the high water line. (see photograph 11). The figure is standing near the high water line. Recontamination was from alongshore and the oil concentrated in the upper part of the intertidal zone.

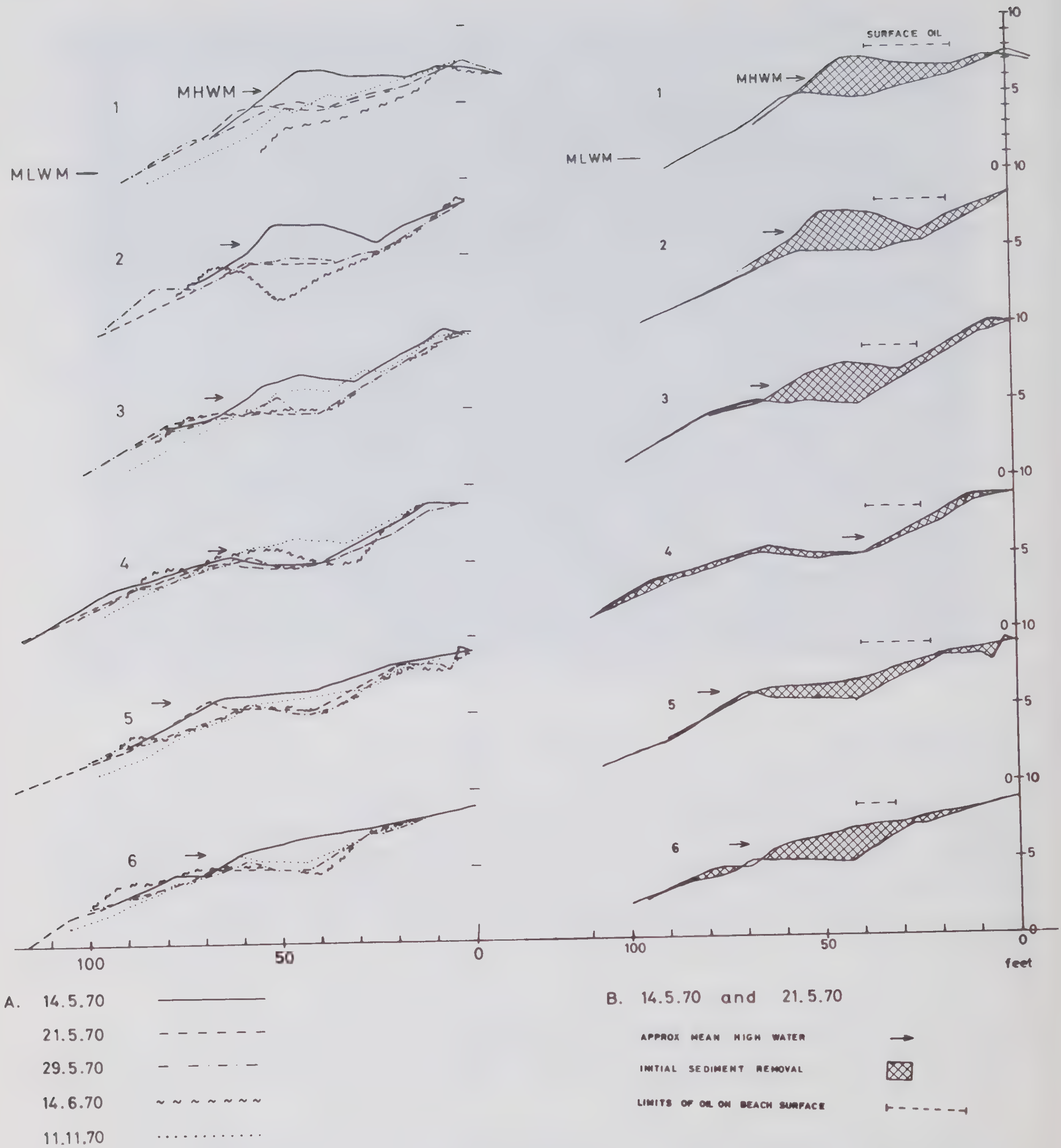
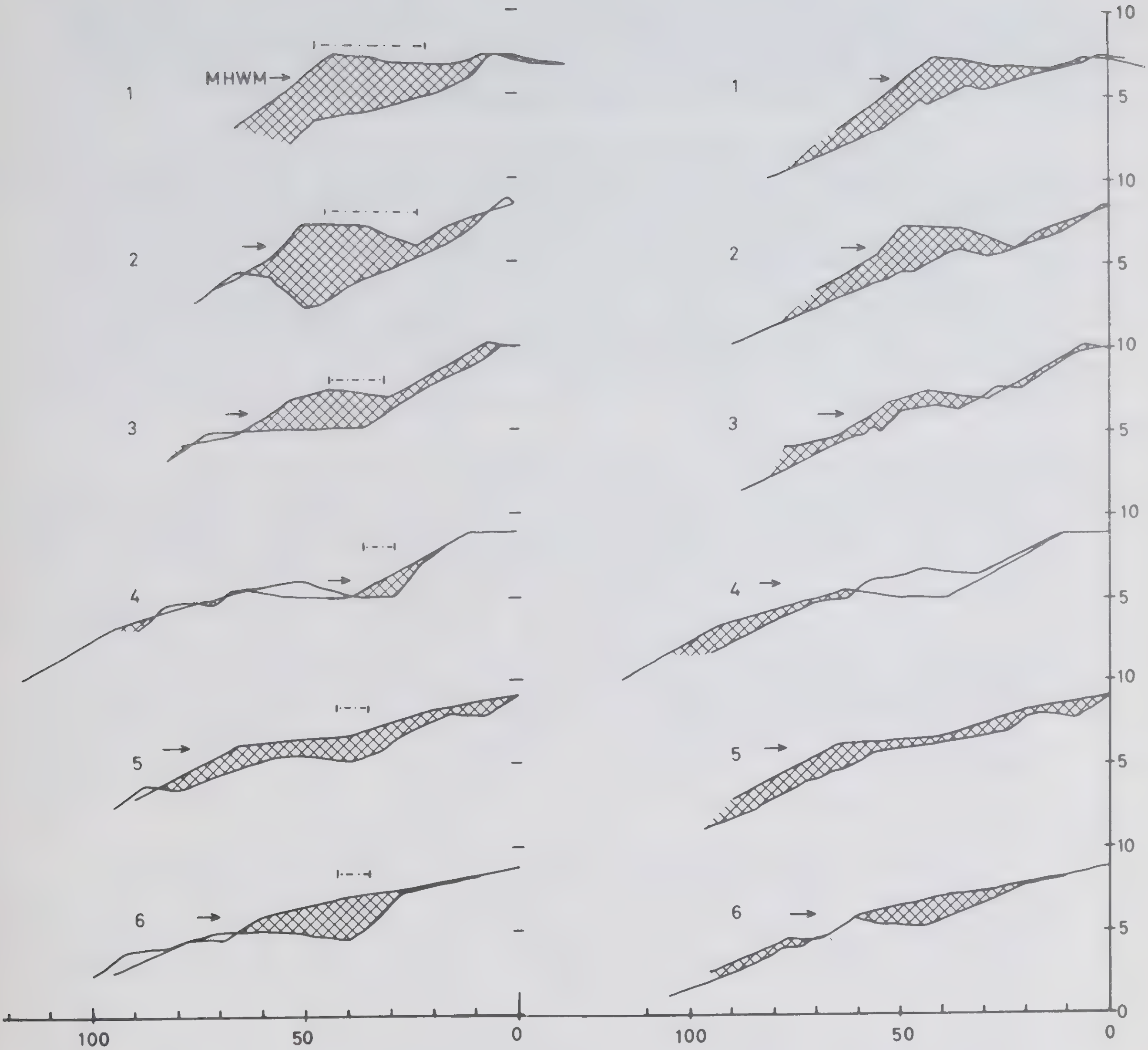


Figure 9. Beach profiles at Indian Cove.



C. 14.5.70 and 14.6.70

D. 14.5.70 and 11.11.70

TOTAL SEDIMENT REMOVAL

LIMITS OF REOILING

NET SEDIMENT LOSS

As a result of a reoiling at the east end of the beach, further excavation took place and the effects of this are shown in Figure 9c, which compares the original situation with that following the second removal operation (June 14). Parts of this beach lost as much as five feet of sediments.

The fact that some replacement of material occurred during the summer months is indicated in Figure 9d. However, the profiles show that in the intertidal zone material had been removed by wave action. The beach had been unaffected by storm waves before November 11 and this should have been a period of net accretion in the intertidal zone. It is likely that the material was pushed up into the area around high water mark but the effect of winter storms combing down the beach may lead to a further lowering of the foreshore. Reprofiling during the spring of 1971 will give a better indication of any long term effects.

Effectiveness and results of operation

a) A carefully handled front-end loader was able to remove the one foot deep oiled layer efficiently. The bucket was used to cut under the oil and lift it, disturbing little of the uncontaminated sediments. This worked well as long as the operator did not attempt to fill the bucket, in which case the effect was to push rather than lift and spillage was very high. From the point of view of the contractor, this method is wasteful as the loader bucket is filled to only about one quarter of its capacity. Thus the efficiency of his operation suffers.

b) Beaches where the oil is only on the surface are relatively easy to clean mechanically, as it is necessary to remove little sediment. However, with the equipment available, spillage is high unless the operator is particularly careful. A machine which could get under the oil carpet, in the manner of a fork lift truck, would be ideally suited to this type of situation.

c) This beach was generally well cleaned except for occasional patches of oil contaminated material which were spilled by the loader; these were removed manually.

d) The mixing of oil covered pebbles or cobbles with fine material leads to a surface cover of sand and silt sticking to the stone, which disguises the oil beneath. Subsequent washing and reworking by wave action tends to remove the surface material and may give the impression that the beach has been recontaminated although in fact this is a result of the mixing of material during cleaning.

e) Reoiling of the eastern section took place (within 4 days) during a period of spring tides as oil was released from pools and cracks in the adjacent rock areas (Photograph 13). High temperatures during the preceding week had encouraged

the oil to collect in hollows and this was then easily washed out by the waves. The actual timing of the reoiling coincided with the spring tides so that the pools of oil had not been within the range of normal wave action for several weeks (Section 5.4).

f) The oil which recontaminated the beach was generally less than half an inch thick but would cause as much inconvenience as the thick layer removed earlier. Unless all sources of recontamination are dealt with, the chance of reoiling will remain. It is not practical to clean only one section of a coast, as oil from alongshore may spread to cleaned areas, which was demonstrated by the fact that this beach was reoiled several times.

6.4.4 Half Island Cove

This 1,500 foot section of beach was restored during a six day period between May 26 and June 15 at a cost of $3,000. A Caterpiller 950 wheeled, front-end loader was used and 1,761 cubic yards of material were removed.

Description of the contract area

This is a wide, shingle beach partially set back from the general trend of the straight coast on the south shore of the Bay. The beach has a maximum width of about 80 feet and the lower intertidal zone is composed of fines and gravel while the storm ridge and upper beach zone are shingle. The alongshore movement of material appears to be from east to west.

The most easterly 600 feet of the contract area, commencing from a low rock platform, is backed by a 20 foot active till cliff. The next 500 feet form a ridge some 120 feet wide which has a lagoon in the rear. The central 200 feet were excluded from the contract. This area consists of a bedrock, boulder and shingle zone backed by a 5 foot active till cliff. The westerly 400 feet form a steep narrow beach with a shingle storm ridge, and a low wooded backshore. At one point, a rib of rock extends from the mid-tide zone seawards for almost 100 feet.

No oil patches were visible on the surface as all the contaminated material had been reworked by wave action and in parts had been buried to a depth of three feet. The rock platform just to the east of the contract section was badly oiled and much of this oil had collected in pools and cracks.

Restoration

The beach was surveyed with 16 levelled profiles on May 25, prior to cleaning which commenced on the 26th. A short experiment with a road grader was conducted on May 26 to the west of the contract section on a wide, low sloping part of the beach.

The contractor left the area on the 28th but as the work was not completed satisfactorily he returned on June 13 and 14.

On June 14 and 15 material was removed from one section of the beach below high water mark. This was used as clean replacement for Phillips Harbour which has been subjected to machinery. The profiles were resurveyed on May 30 and June 15.

Results and effectiveness of operation

a) The tests with the road grader were not productive as this machine requires a firm, flat or low angle surface. While it was able to work with some efficiency along the flat crest of the ridge, difficulties were encountered on the low beach face slope. As has been reported elsewhere (Section 5.3), a grader is useful and effective on firm, low sand beaches where the oil is on the surface. On a gravel or shingle beach it is of no practical value.

b) The basic problem in this contract area was to remove buried, contaminated material. At the first attempt the contractor merely scraped over the surface and as this was not acceptable, he returned and removed more beach material. There still remained a great deal of contaminated sediments after the contractor was released (Photograph 14). The only way in which the method employed could have succeeded would have been to remove most of the upper sections of the beach without spillage. This is both undesirable and impractical and may have led to adverse effects, such as increased erosion of the till cliffs or breaching of the lagoonal ridge.

c) Even if the beach were cleaned adequately, there is a danger in this area of recontamination by oil from the adjacent rock platform and other areas alongshore.

d) This beach was not paralyzed by the oil and wave processes had been active and effective in cleaning and burying much of the contaminated material. Following the restoration programme the beach was no cleaner in terms of recreational purposes, so that there was little net gain from the operation. There was no large-scale removal of sediment and no damage to the beach itself.

6.4.5 Hadleyville No. 1

This 4,500 foot beach to the west of Oyster Point was contracted out for $9,450 and work took 8 days between June 2 and 11. A Caterpiller D6C bulldozer and a Caterpillar 950, wheeled, front-end loader removed 3,980 cubic yards of material.

Photograph 14 HALF ISLAND COVE June 3, 1970

Close-up of contaminated cobbles after restoration. This may have resulted from spillage, mixing, or the exposure of buried oiled material. Scale is graduated in decimeters.

Description of contract area

This well developed, prograding, steep shingle beach is one of several similar formations on the north-west coast of the Bay. The longshore movement of material is to the east and large depositional ridges are evident at Oyster Point, just east of the contract area. Parts of the backshore have large well-preserved former storm ridges which attest to the gradual seaward progradation of the shoreline.

The intertidal zone has a maximum width in the order of 100 feet in the central parts of this section and the material distribution is fines in the lower intertidal zone with a shingle storm ridge above normal high water mark.

The shallow nature of the offshore area has allowed the beach to become oriented towards the dominant direction of refracted wave approach, that is, the south-east.

There was very little contaminated material visible on the surface of this beach. The oiled sediments had been reworked by wave action and were buried to a maximum depth of 4 feet. In some areas a layer of oiled material on the upper parts of the beach had been buried and the seaward edge eroded, so that a band of contaminated sediments was evident in the beach face slope (Photograph 15). The only method of cleaning with machinery involved the large-scale removal of mixed clean and oiled material.

Restoration

The contractor began work on June 2 and in many sections of the beach the bulldozer was used to pile up, above high water mark, material which was subsequently removed by the front-end loader. Where the oil was deeply buried, the loader made a cut down the beach and then excavated along the beach parallel to the ridge. The work was completed on June 11.

Beach profiles

Ten profiles were surveyed across the beach on May 21, June 13, July 12 and November 11, 1970. Five of these profiles, from the central and east sections, are presented in Figure 10.

The "before and after" situation is given in Figure 10b and it is evident that material was removed largely from those areas above mean high water mark. In some places over three feet of sediments were removed in order to excavate the buried oil. Profile 6 is an example of the removal of the storm ridge crest which is well above the limits of normal wave action.

One month after completion of the contract (Figure 10c) very little sediment replacement had taken place even though the rate of accretion should be at its maximum at this time of year. This can be attributed to the fact that the intertidal beach was relatively undisturbed by the machinery and that there was no wave action on the higher parts of the beach during this period.

Photograph 15 HADLEYVILLE No. 1 May 30, 1970

This was one of the worst parts of this beach, in the west section at low tide. Contaminated material has been buried and subsequently exposed as layers in the beach face slope. This well developed beach is backed by a series of vegetated swales and former ridges.

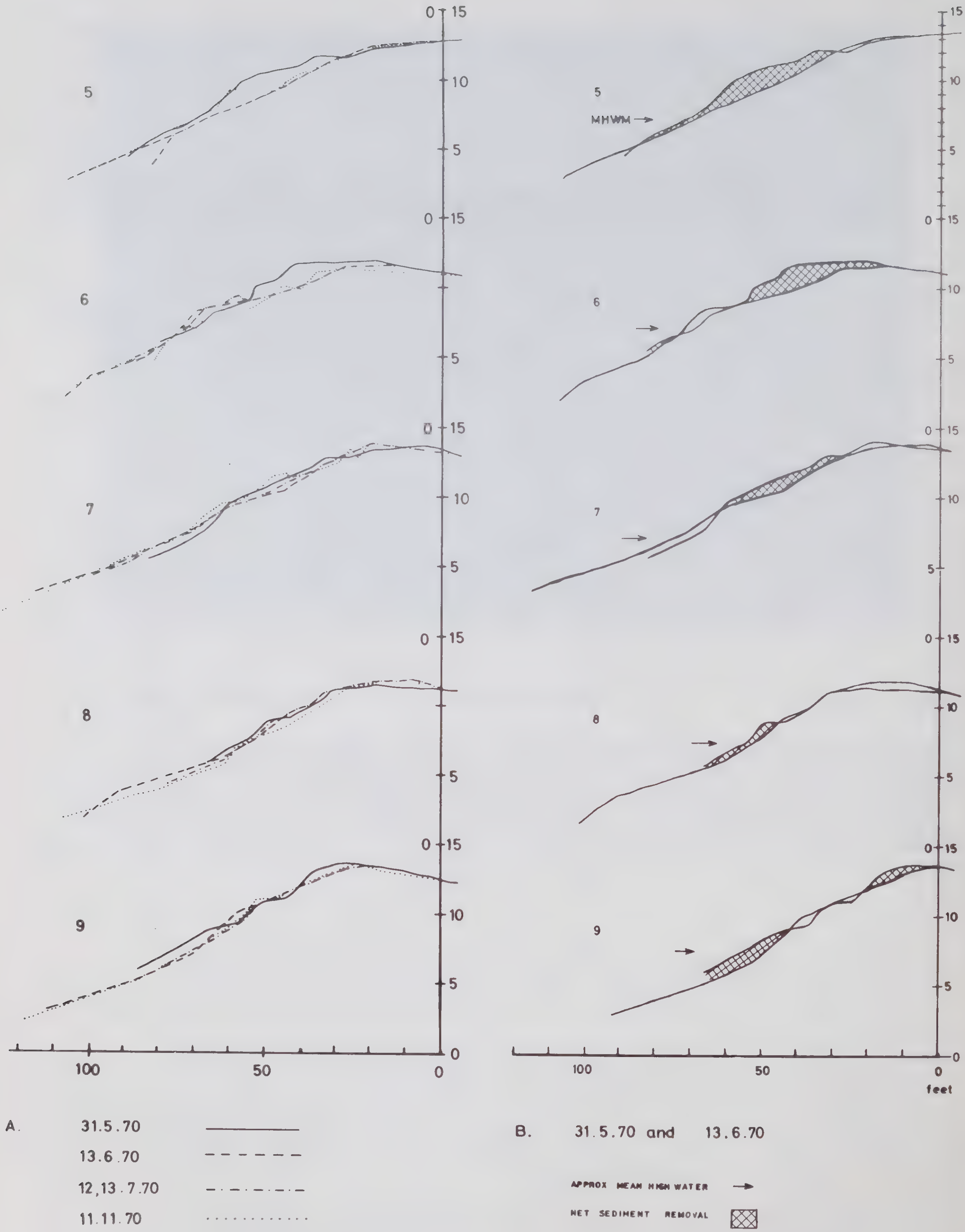
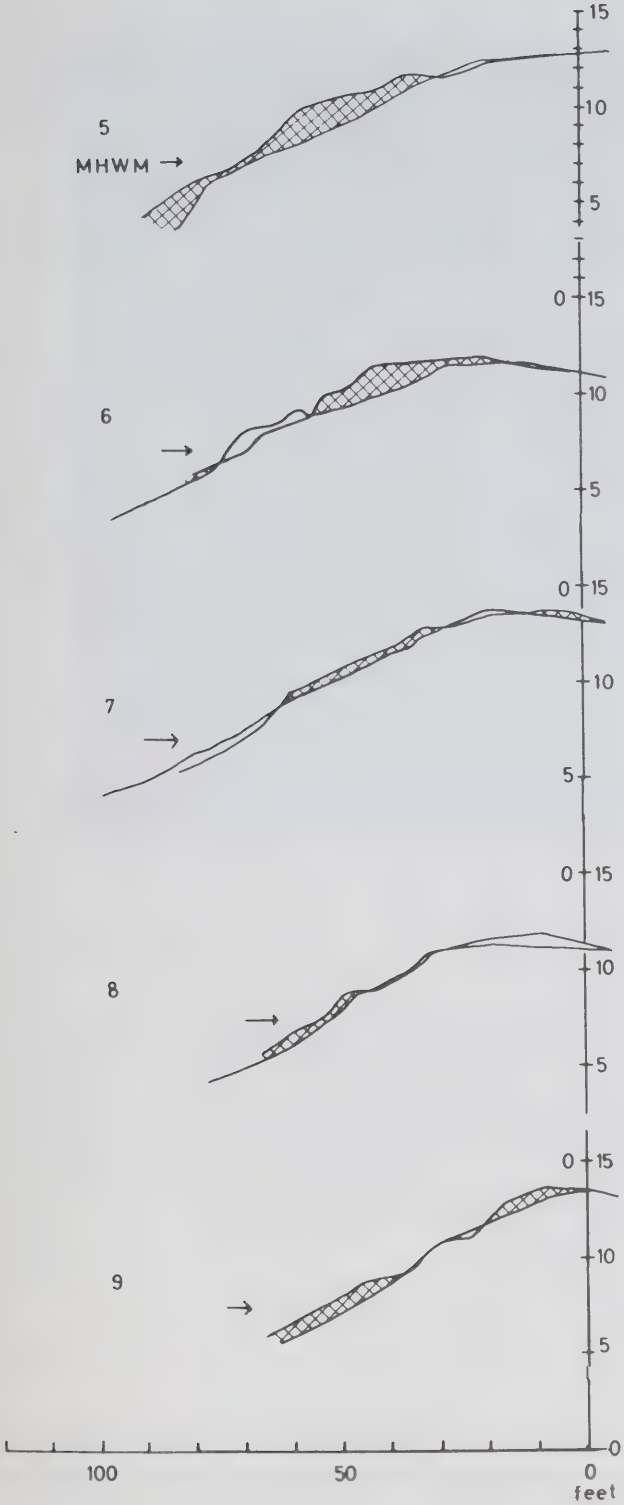
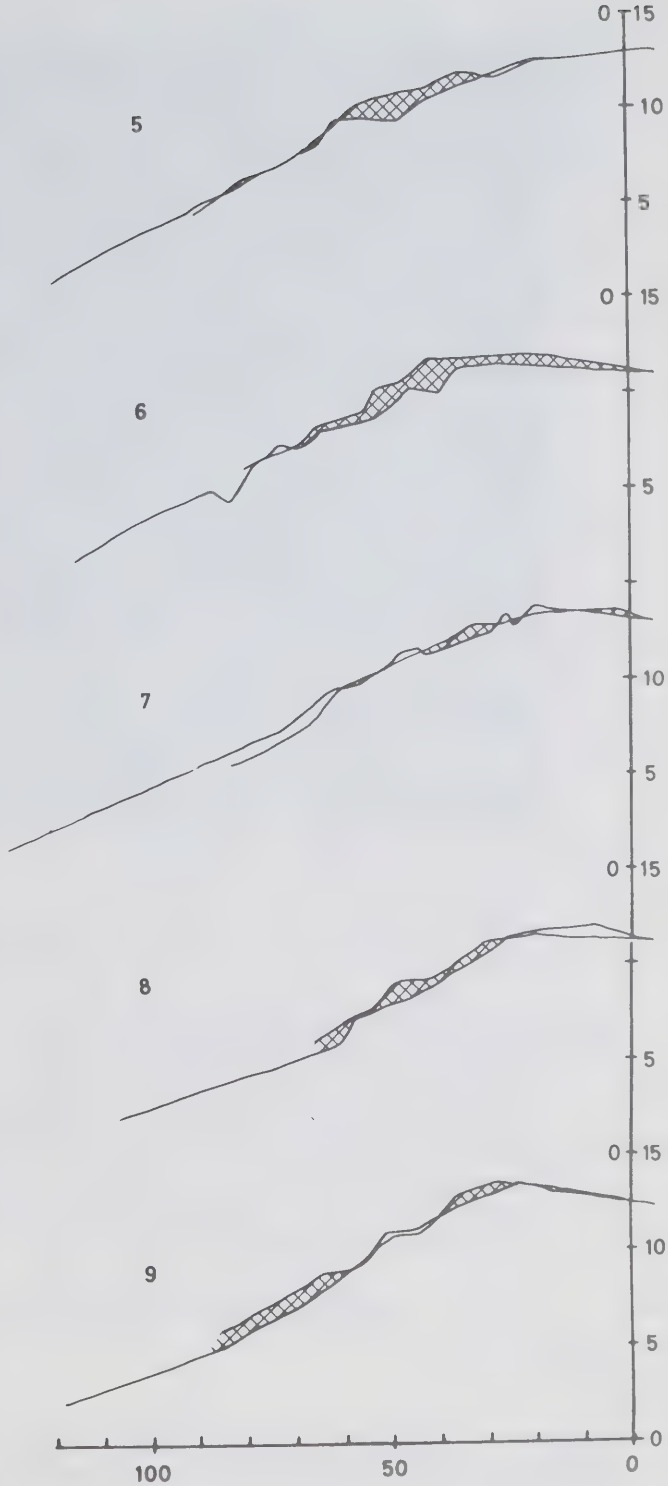


Figure 10. Beach profiles at Hadleyville No. 1.



C. 31.5.70 and 12.7.70

NET SEDIMENT LOSS



D. 31.5.70 and 11.11.70

NET SEDIMENT LOSS

Few changes took place in the fall and the November profiles (Figure 10d) record little significant erosion or accretion except for the presence of some small ridges near the high water mark. As most of the excavation took place above the intertidal zone, the profile changes on the lower part of the beach are probably due to normal variations in response to the littoral processes.

Effectiveness and results of operation

a) In certain sections where the contaminated material was buried, removal of large volumes of sediments is very inefficient. Of the total amounts excavated, only a small percentage of the material was contaminated (Section 5.2). There is no adequate method for mechanical cleaning of beaches where oil is buried.

b) Contaminated material was still evident after the contract had been completed. The oil on the cobbles acquired a surface coating of fines which were easily knocked off or washed off by wave action.

c) Unlike Arichat, Blackduck Cove or Indian Cove this beach was not paralyzed and normal beach processes were active. Wave action will succeed in cleaning the beach where machinery can only be used to remove large portions of the beach. (This situation is similar to Half Island Cove.)

d) The use of a bulldozer to pile up material for removal mixes clean and oiled material. This does not contribute positively to the clean-up operation.

e) It is unlikely that removal of sediments would have any long-term adverse effects on this beach provided large volumes are not removed from any one location. This should not detract from a critical analysis of the operation which was basically an inefficient way of dealing with the problem. Mechanical methods are not suited to this type of cleaning. Natural processes are more effective and less costly.

6.4.6 Other Contract Beaches

Moose Bay

This large cobble beach and spit complex on the west coast of the Bay is one of the most impressive coastal features of the area (Photograph 16) (Johnson, 1925). The west half of this beach was restored manually (Photograph 2) while the eastern 5,000 feet were put out to contract for machine work.

The problems which were faced in this area were similar to those of Half Island Cove and Hadleyville No. 1 as the oiled material was deeply buried and in order to remove it, several feet of sediments were excavated.

Photograph 16 MOOSE BAY May 21, 1970

Air view of the east end of Moose Bay at mid tide. The bedrock island of Ragged Head, lower right, is connected to the mainland by a double tombolo. This is the western of the two arms and it gives way to a wide, long beach.
The eastern limit of the area which was later restored is indicated by an arrow.

The initial attempt at manual cleaning was not continued to the end of the designated section because of the expense, which was double that for machinery on a cost per foot basis. The manual method is less harmful to the beach as little uncontaminated sediment is excavated. Although this approach does not lead to the removal of all the oiled material, mechanical methods are little better as there is no suitable equipment available.

Hadleyville No. 2

This beach is distinguished from Hadleyville No. 1 as it is a separate coastal unit to the east of the area discussed above (Section 6.4.5). This beach area is little different from Moose Bay or Hadleyville No. 1 but is included as an example of the types of contamination which were experienced in the operation.

At the west end of this cobble beach oil had been reworked, buried, and subsequently exposed (Photograph 4). In the central area, a carpet of thick oil up to 15 feet wide was left above the limits of normal wave action (Photograph 6). At the east end of the beach cobbles gave way to coarse sand and here the oil remained as large discrete pans above the high water line (Photograph 3).

This area is a valuable example of how to deal with different situations using the methods available.

a) The type of beach contamination in the west section represents a situation which is best left alone unless urgent requirements necessitate action. The waves had removed most of the surface oil by reworking and burial and the beach would continue to clean itself as this is an exposed beach which receives the full force of storm and swell waves from the east. To remove the contaminated material completely would require deep excavation which is harmful to the beach equilibrium and at present would be an inefficient method as no equipment is available which will remove all oiled sediments from cobble beaches.

b) The thick cover of oil above the high water mark could be removed by a carefully operated front-end loader which has been shown to be effective in this type of situation (Section 6.4.3). No attempt should be made to remove the other sediments which have been reworked or buried by the waves for the reasons outlined above. The action taken would merely be to remove the surface layer of oil which has effectively paralyzed that section of the beach.

c) The large pans of oil on the surface of the sand can be removed manually, as these are easy to pick up with a shovel. The use of men rather than machinery is recommended, as the spillage from the equipment could be ground and mixed with clean sand. Any oil which is buried will be reworked by waves as it is exposed during the winter months. Should some of this be left on the beach the following summer, pickers could again restore the contaminated areas.

Eddy Point

This foreland is made up of two cobble beaches which enclose a fresh-water lagoon on a shallow offshore platform (Photograph 17). According to local inhabitants the north-west shore is retreating at a rate of nearly one foot per year and as the upper part of this beach was covered with a continuous layer of oil, it was decided that this would be removed and then replaced with clean rip rap to serve as protection against any possible erosion resulting from sediment removal.

Profiles 1, 3, and 5 in Figure 11b were surveyed across this beach and show the effect of the restoration programme. Along the south-east limb of the foreland, the profiles 7, 9, 11, and 13 in Figure 11b show how the removal of buried oil led to excavation in the zone above high water mark, the area where beach replenishment is least active.

Subsequent surveys in July and November (Figures 11c and 11d) indicate that no significant replacement had taken place on the north-facing beach and by November the beach had lost what little sediment had built up immediately following restoration. Profile 7 was levelled across the point of the foreland, which is the most mobile section of this feature, and the loss of material is probably the result of normal processes rather than the contract work. On the steep south-facing beach some build up had taken place along the high water line but this has not been sufficient to return the beach to its previous level.

Deep Cove

Following the initial work on this beach (Section 6.1) further removal of contaminated sediments was carried out in June on the seaward side of the bar and in the lagoon. An angle-blade bulldozer was used in the lagoon area to remove the surface oil but the vehicle tracks acted to grind in the oil which had been buried. Although the machine was relatively efficient in scraping the surface, it did not affect the oil which was interbedded with the silt and mud except to mix it to a depth of several feet.

Photograph 17 EDDY POINT June 12, 1970

Air view looking north. The lagoon is fresh water and rarely breached by the sea. The contract limits are given by the arrows.

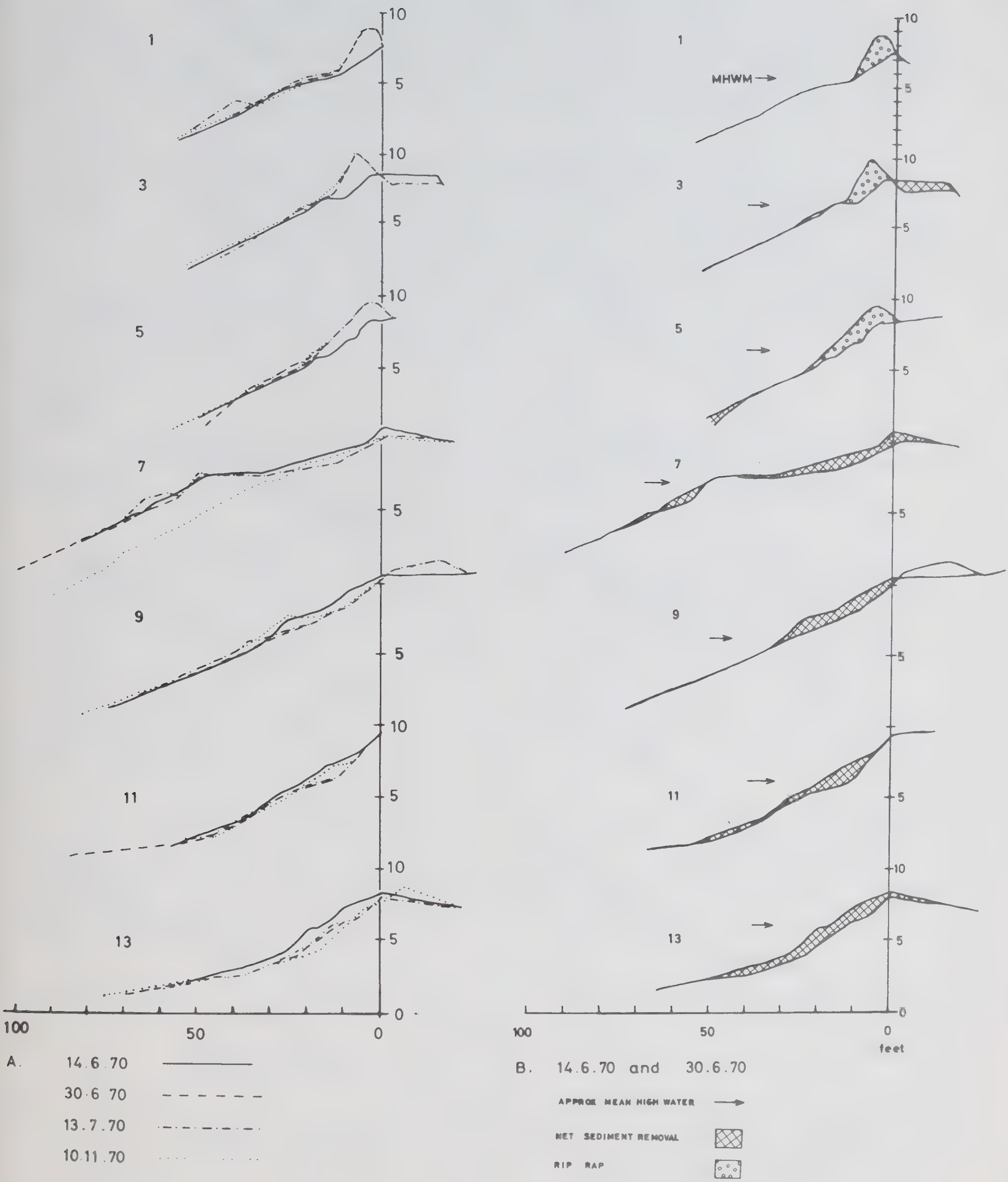
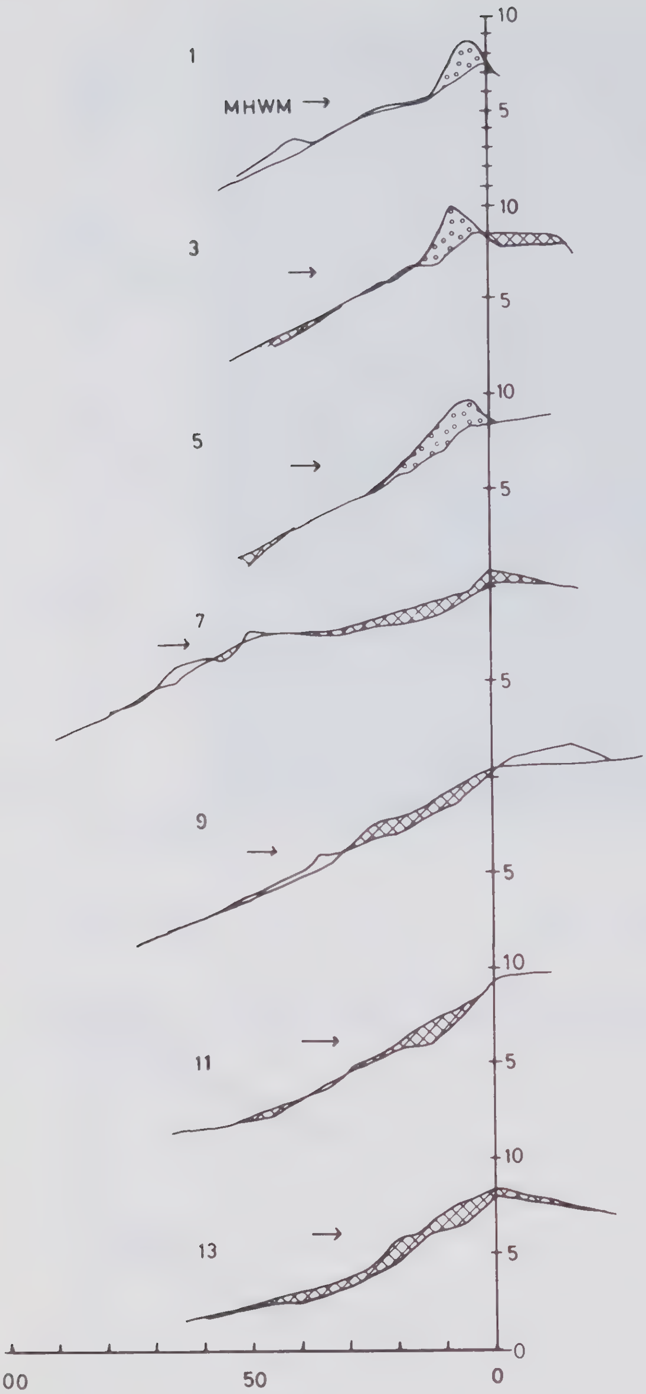


Figure 11. Beach profiles at Eddy Point.



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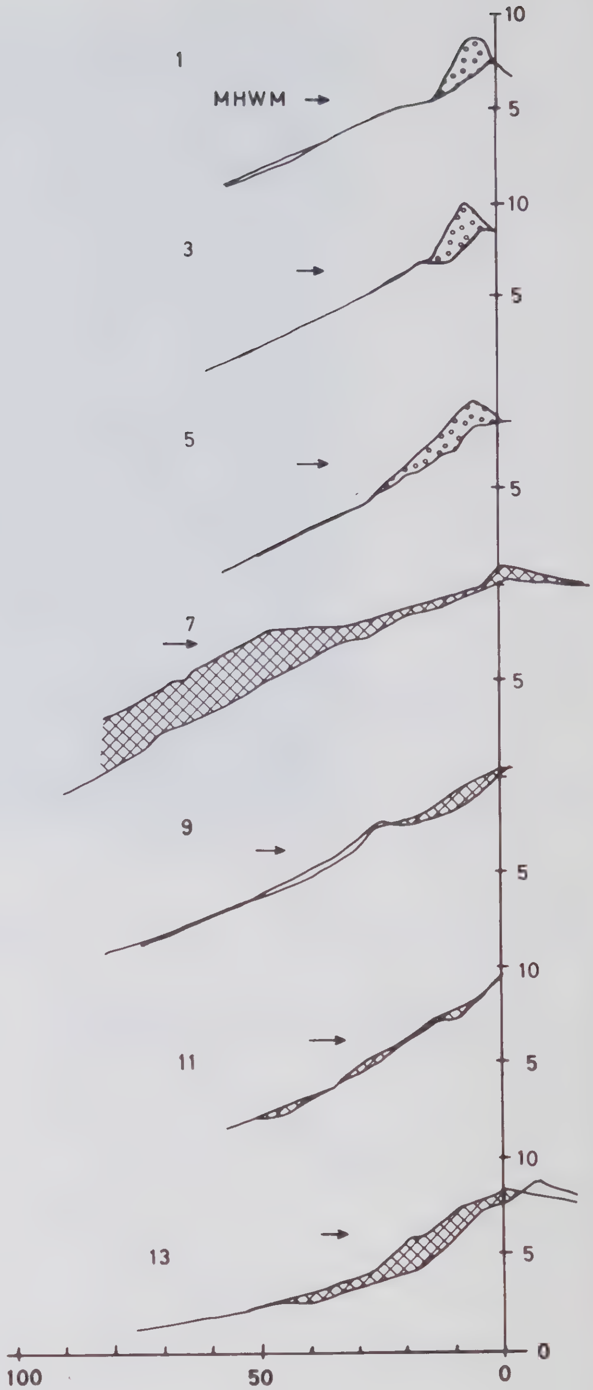

D. 14.6.70 and 10.11.70

NET SEDIMENT LOSS

NET SEDIMENT LOSS

The bar beach was restored again and material was replaced to prevent any major damage to this sensitive feature. Unfortunately the replacement sediments were taken from the spit which enclosed the lagoon and the cut into the spit which resulted will not be filled by natural processes as it is not in the active beach zone.

When this location was revisited in November it was evident that waves had washed over the road on the crest of the bar. Whether this would normally occur is not known but even if this is the case, there is clearly a serious danger that the bar could be breached. On such an unstable beach any interference with natural processes may easily accelerate erosion. Some rip rap was replaced in one section of the bar and this may help protect this part of the beach.

Walkerville

Several contracts were given to restore the paralyzed beaches in the sheltered areas of the north shore. One of the problems in this area was that oil was continuously moving alongshore from other contaminated areas.

This beach section near Walkerville on the north coast of Inhabitants Bay is in this sheltered area and has a very thin sediment cover which overlies a till platform and is backed by low till cliffs. The surface layer of cobbles was paralyzed by the oil and was removed by a bulldozer and a tracked front-end loader. This action exposed the fine, till-derived, sediment which overlay the bedrock and clean and contaminated material were mixed by the churning of the tracked vehicles (Photograph 18). With the removal of the surface layer of large material, the fines were removed by wave action.

In a situation like this it would have been preferable to rake over the paralyzed sections so that the beach material would be made mobile. Removal of sediment is harmful as the supply of cobble size material is very limited and erosion of the till platform and backshore may result from the loss of the protecting beach apron.

6.5 Summary

6.5.1 Sand Beaches

a) On lightly oiled sand beaches self-cleaning will take place. If restoration is necessary the manual removal of contaminated material with shovels and rakes is very efficient and effective, as this method does not involve the removal of large volumes of uncontaminated sediments (Section 6.3).

Photograph 18 WALKERVILLE June 27, 1970

This beach on the sheltered north shore of Inhabitants Bay has had its thin surface layer of cobbles removed. This has exposed a wet layer of fines about three feet deep. These clean sediments have been thoroughly mixed with contaminated material by the tracked bulldozer and front-end loader.

b) If the beach is heavily oiled and restoration is essential, a well-operated, wheeled, front-end loader which can cut under the oil is an efficient method of removal. The aim of the machine operator should always be to lift material in small quantities rather than push it forward and fill the bucket. There will, however, be spillage which can be removed manually. No beaches in this area were suited to the use of graders.

c) Sand beaches can be cleaned for recreational purposes as long as there is no danger of recontamination (Section 6.4.2).

d) When machinery is working in the beach zone it is important to prevent damage to the backshore vegetation as this could easily lead to blow-outs.

6.5.2 Cobble and Shingle Beaches

a) Natural processes are effective in cleaning this type of beach unless wave action is limited and/or pollution is very heavy. Paralyzed beaches are by definition inactive and there is little self-cleaning at these locations (Section 5.2).

b) It was found that cobble beaches could not be cleaned for recreational purposes with the methods available. Machinery alone could not remove all of the contaminated material.

c) Where oil lies on the surface of a beach it can be removed efficiently by a well-handled, wheeled, front-end loader (Section 6.4.3). Even in these situations there is spillage from the equipment but this could be picked up manually.

d) A lightly oiled or reworked beach with buried oil can only be restored by excavation methods. This often involves the removal of 3 to 5 feet of sediments and these are usually taken from those areas above high water mark where natural replacement is slowest (Section 6.4.5). If the storm ridge or beach crest is in any way lowered, attempts should be made to replace all the excavated material (Section 6.4.6).

e) Local contractors have been removing beach material for construction purposes on several of the beaches in this area for a number of years. The beaches do not seem to have been damaged, as far as is known, but the long term effects are not understood, so that this activity should not be used as an argument for large-scale beach excavation. The lesson from Hallsands is an important one (Section 4.2).

f) Where oil is buried in the beach it should only be removed if it is absolutely necessary. The low amounts of oil in these reworked sediments, often as low as 10 parts per million, mean that large volumes of material are removed for very little oil.

g) Mechanical restoration is best applied to the situations where oil is on the surface or where the beach zone is paralyzed. In the case of the latter, the object of restoration should be to remobilize the beach so that normal self-cleaning can be effective, rather than attempt to clean it by excavation (Section 6.4.1).

6.5.3 Recontamination

a) No beaches should be cleaned if there is a danger of recontamination. This was evident in the instances where re-excavation was necessary (Section 6.4.3).

b) It is not recommended that a contaminated shoreline be cleaned in sections as reoiling is likely unless the beaches can be protected or the adjacent oiled areas stabilized. Recontamination is usually light but is nevertheless undesirable.

6.5.4 Machinery

a) Earth moving equipment is designed to excavate or remove large volumes of material and operators are trained to carry out this type of work. In order to use machinery for restoration purposes, time must be taken to train and supervise the operators so that they meet the requirements of this more delicate type of operation.

b) Mechanical methods were found to be useful to remove surface oil from beaches and to remobilize paralyzed beaches. In the latter case, reworking of sediments rather than removal is more applicable. In particular, a well-handled, wheeled, front-end loader was found to be efficient to remove surface oil from cobble beaches (Section 6.4.3).

c) Bulldozers were not satisfactory for restoration work, particularly the large 100 to 120 fly-wheel horse-power machines which were awkward to handle in confined areas. Spillage from the blade and grinding by the tracks were the major defects of this equipment, the same being true for all tracked vehicles.

d) Material which is to be removed should be lifted directly from the beach. If it is piled up and later removed this doubles the chances of spillage and further mixing with clean sediments takes place.

e) None of the machines were able to operate adequately on soft mud or silt areas. Even if traction was maintained this led to grinding and mixing to depths as great as four feet (Section 6.4.2).

6.5.5 Beach Profiles

a) Accurately surveyed profiles across beaches are a valuable tool in establishing short and long term changes in the character of the beach as well as showing the immediate effects of excavation.

b) The profile of a beach varies with each tidal cycle so that care must be exercised in isolating long term changes. Meaningful analysis requires a set of observations over a period of not less than a year. The results of the continuing survey programme on the Chedabucto Bay beaches will be reported at a later date (Owens, 1971b).

7. DISCUSSION

7.1 Selection of Beaches for Restoration After a Spill

In coastal areas affected by an oil spill, the decisions related to beach restoration should be based upon geological, wildlife and socio-economic factors. In the first instance, those shores which would require cleaning for tourist or other economic reasons should be established. The decision as to whether these beaches should be cleaned and the methods to be used would then be related to geological criteria. In certain instances, natural self-cleaning may be the acceptable solution; elsewhere the use of manual or mechanical methods may be required. Where cleaning is to be carried out, it is necessary to assess any possible adverse effects which this action could precipitate. For example:

1) removal of large volumes of material in an area of limited sediment supply could seriously affect the stability of the beach and backshore zones.

2) bars or lagoons could be breached by the disruption of the local shore environment as a result of sediment removal in particular localities.

3) destruction of backshore vegetation, particularly in sand areas, could lead to blow-outs and aeolian erosion.

In all instances, the possibility of upsetting the delicate balance of the shoreline must be considered when assessing the socio-economic requirements, for in most instances, natural self-cleaning is more effective and is more desirable from the geological viewpoint. Certain shores such as rocky cliffs or shingle ridges are virtually uncleanable unless dispersents are employed, and in these areas piecemeal restoration of some beaches should be considered in the light of recontamination from the alongshore areas which would not be cleaned.

The system of rating which was used for the selection of beaches in this operation is one based solely on recreational capability (Anon., 1967). This was the only available information on the beaches of this area but was not a rating based on applicable criteria. Moose Bay has been given, for example, a "2" rating but is in fact used only by members of the small local community. The same rating was given to Point Michaud which is completely different geologically and very popular as a picnic and bathing beach. Distinction between different types of beaches for restoration is best defined by the actual recreational use, wildlife considerations, socio-economic factors, and upon

geological characteristics such as material size, sediment supply, sediment movement, and wave energy conditions. With this in mind the selection of beaches for restoration should be based on:

a) Does the beach need to be restored?
b) Can the work be done effectively without damaging shoreline stability?

These aspects of the restoration project could be discussed in a regional study (Section 7.2) and once the priorities have been outlined they would only require minor modification according to the actual seriousness of the contamination on given shores.

7.2 Regional Coastal Studies

The correct assessment of areas to be restored and the methods to be used requires an understanding of the coasts involved. To date, only a few small sections of Canada's coast, which is the longest in the world, have been investigated geologically and geomorphologically. Should future restoration projects be necessary, this means that at least reconnaissance surveys would be required before planning the operation. Such a survey should be able to provide a detailed outline of those sections of coast which would require restoration, on a priority basis, according to the character of the shore and economic, social, or wildlife requirements.

The shoreline of Chedabucto Bay had been reviewed very briefly by Johnson (1925) but until field investigations were carried out during this operation, this was the total available information on the coastal environment. Some basic research on the nature of Canada's coasts must be regarded as a necessity for future operations. This information would also greatly benefit tourist and conservation programmes.

The coasts of western Europe and the United States have been studied at least at the reconnaissance level so that restoration projects in these areas have benefitted from the existing storehouse of information. Studies of this nature could be carried out readily in Canada, at least for the areas where spills are likely, by the use of aerial photography, topographic and bathymetric maps, with additional field work for areas requiring more detailed study (Owens, 1971a). The investigation of Canada's shoreline has been neglected and this operation has brought to light the lack of even the most basic general studies in this field.

7.3 Oil on Arctic Beaches

The arctic beaches are in a low energy environment where the intensity of wave and ice action varies with location and exposure (McCann and Owens, 1970). If oil were to pollute these coastal areas, the natural self-cleaning processes would act on a greatly reduced scale when compared to the exposed beaches of Chedabucto Bay. A more valid comparison would be the sheltered environments of the lagoon at Blackduck Cove or the beaches of Inhabitants Bay where wave action is very limited. In these areas it is expected that the oil will persist for several years before it is degraded by natural processes.

The lower energy conditions in the arctic mean that waves and wave-induced currents are relatively less effective in moving pebbles and cobbles when compared to lower latitude shorelines and this results in less transport and abrasion within the beach zone. Contaminated beaches would be cleaned only very slowly by this action and as there is less movement and redistribution it would be very easy for the littoral zone to become paralyzed.

If beach restoration is required in an arctic area it is important to consider that replenishment of a beach by natural processes takes place at a slower rate than in lower latitudes. Therefore, even if a beach appears to be building up, it may take a long time for any losses to be replaced. A documented example of the removal of beach sediments in an arctic location is given for the Point Barrow, Alaska area by Hume and Schalk (1964). It is reported that, as well as producing a shoreline retreat of 10 feet in 12 months, the lowering of the beach allowed ice floes to advance further inland and nearer shoreline installations. This mechanism of ice pushing up the beach is common in polar regions and such action would likely damage or destroy any fixed object in its path.

Although wave action is restricted in the arctic environment, ice may play an important role in reworking the contaminated sediments as broken ice is moved within the littoral zone (Owens and McCann, 1970). This would act to reorganize the oil and beach material but would do little to clean the beach. Biodegradation is particularly slow in this environment, so that the oil on a beach would be expected to remain much longer than in more temperate areas.

7.4 Concluding Remarks

The coastal zone is an extremely complex environment, being at the interface of land, sea and air. The processes which operate within this zone are numerous and not fully understood. Any one segment of coast is the result of a series of interactions which involve a great number of variables and should any one of these variables be altered significantly, this may lead to a trend towards a different equilibrium situation which will satisfy the new process demands.

Although the coast is largely a response to the active processes of winds and waves, its form and nature closely affect these processes. Beach slope, material size, and sediment supply, for example, are critical factors in assessing longshore movement and beach nourishment. The construction of groynes, jetties and other harbour facilities has led, in the past to unexpected alterations of the adjacent coast due to the generation of new and the adjustment of existing processes, resulting in a different process-response environment. A great deal of research has now been carried out in this area and coastal engineering is regarded as a field of study in its own right. In general, work has been concerned with the construction of artifical shorelines, but outside of a few isolated examples little is known of the effects of sediment removal. In one documented case, referred to earlier (page 17) beach material was used for construction purposes and this was followed by a shoreline recession of 20 feet in 50 years. This example is in an area where there is no present-day supply of sediment. The beach, once deprived of sediment, could not be replaced and this "demonstrates most effectively how dangerous it is to tamper with a beach, how wrong it is to make assumptions about the drift of beach material without a full investigation and how important it is to study each part of the coast intensively and not apply general ideas too readily." (Steers, 1964).

Regional studies of Canada's coastline should be undertaken to determine the features and character of different shoreline units. A knowledge of our coasts is essential if adequate contingency plans are to be formulated. During restoration projects, geologists should be consulted to assist in the planning and operation of programmes in order to take into account local shoreline and beach conditions. In particular, the selection of beaches to be restored and the application of the various restoration methods should be carefully considered.

The role of the coastal geomorphologist or geologist is to make the planners aware of the possible effects of a beach restoration operation. Detailed analysis of a shore area requires time, particularly to determine seasonal as well as long term changes. For this reason, the preparatory work necessary for the provision of adequate information for decision making, should be carried out before a spill occurs. If this type of research is not undertaken, it will be possible to provide only a cursory assessment of the coastal environment based on a qualitative judgement. The function of the geologist in this type of operation is a very important one, but one which should be ahead of, as well as working within, the operation.

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Many members of the Project Oil team contributed directly towards the field work, but in particular Dr. C.S. Mason, Dr. D.H. Loring (of the Bedford Institute), and J. MacKay (of the Department of Public Works) provided a great deal of support and advice throughout the operation. The Dominion Hydrographer, Dr. A.E. Collin, is thanked for allowing this work to be undertaken in place of normal duties and for providing valuable technical assistance in the production of this report. Thanks go to Dr. S.B. McCann (McMaster University) for reviewing the manuscript, Miss M. Rossiter for assistance in the preparation of diagrams and tables, and to the Chart Drafting section of the Canadian Hydrographic Service for final drawing. Mr. F.G. Barber encouraged the preparation of this report and editorial assistance was provided by Miss T. Millington.

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See also

Report of the Task Force - OPERATION OIL to the Minister of Transport:
Volume I - Report of the Task Force
Volume II - Report of the Scientific Coordination Team to the Task Force
Volume III - Reports of Other Units to the Task Force.

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No. 20

Zero padding as a means of improving definition of computed spectra

R.F. Henry and P.W.U. Graefe

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ZERO PADDING AS A MEANS OF IMPROVING DEFINITION OF COMPUTED SPECTRA

R.F. Henry and P.W.U. Graefe

1971

Forward

The concept of zero-padding was first introduced in computation of correlation functions via the Fast Fourier Transform. However, its use in the contexts described in this report has not been discussed in any publication known to the authors.

Abstract

A simple method of data modification is described which in conjunction with the Fast Fourier Transform provides economically a detailed picture of the frequency content of a sampled time signal and thus improves the accuracy with which the positions of spectral peaks can be determined. Also discussed is the use of zero-padding in the accurate computation of Fourier transforms of functions which are non-zero over a finite interval.

Improved Spectral Peak Determination

This note describes a method for facilitating the accurate determination of the frequencies of periodic components in oceanographic records. In theory, such frequencies can be found to any specified degree of accuracy by identification of peaks in spectra computed from sufficiently long records, but in practice, the available records may be relatively short. Record length may be limited by simple logistic considerations, such as availability of ship time. Other constraints on record length which the writers have encountered are

a) in computer simulations of certain problems in fluids (for example, resonance studies of coastal bays and inlets), accumulation of round-off error acts as 'noise' which swamps the computed record if the computation is continued for too long;

b) in determining components which have periods of the order of 10 years or more, practically all the reliable quantitative records available are relatively short;

c) in calculating local values of period for phenomena with slowly-varying frequency, records are divided into short lengths in each of which the period is assumed to be approximately constant.

When discrete spectra are calculated from relatively short time records, accurate determination of peaks presents two difficulties. Firstly, the frequency resolution may not be sufficient to separate components whose frequencies differ

only slightly; nothing but use of longer records can rectify this problem. The second difficulty, the solution of which is discussed here, is that points in the computed spectrum are normally too widely spaced to outline peaks adequately.

As an illustration, consider the time record shown in Fig. 1(a), which consists of four periodic components of equal amplitudes and of frequencies 3.0, 8.5, 15.0 and 15.5 Hz: the spectrum of this signal is shown in Fig. 2(a). Suppose that in practice only a portion of this record, such as that of length T in Fig. 1(b), is available for analysis. The best approximation to the true spectrum (Fig. 2(a)) which can be obtained from this truncated record is shown in Fig. 2(b). This represents the convolution of the true spectrum with the Fourier transform of a rectangular function of length T through which, in effect, we view the original record (Jenkins and Watts, 1969). It will be noted that the frequencies at which peaks occur in the 'smeared' spectrum, Fig. 2(b), agree well with those of the periodic components shown in Fig. 2(a) except for the 15.0 and 15.5 Hz components which lie too close together to be separated with the resolution ($\Delta f = 1/T = 1$ Hz.) attainable with record Fig. 1(b).

Since $\Delta f = 1/T$ is the finest spacing at which independent spectral estimates can be formed from a record of length T, practically all conventional algorithms available for computing spectra from sampled data are so arranged as to give values of the spectrum at points Δf apart. Consequently, when the record Fig. 1(b) is sampled, the discrete spectrum normally computed is that shown in Fig. 2(c).

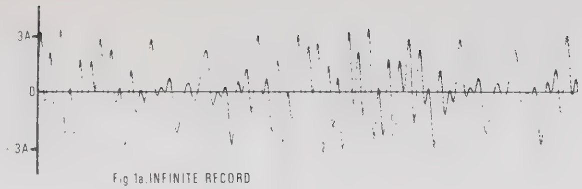


Fig 1a.INFINITE RECORD

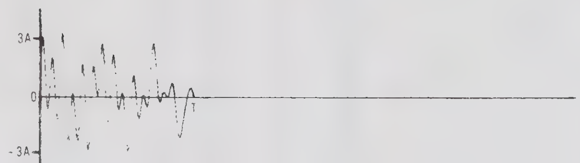


Fig 1b RECORD OF LENGTH T=1sec

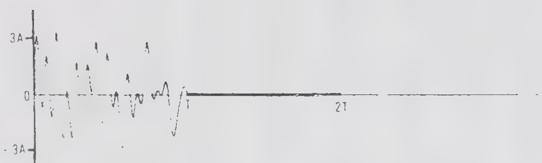


Fig 1c RECORD OF LENGTH T, AUGMENTED BY 1 SEC OF ZEROS

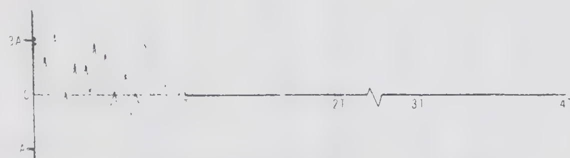


Fig 1d RECORD OF LENGTH T AUGMENTED BY 3T SEC OF ZEROS

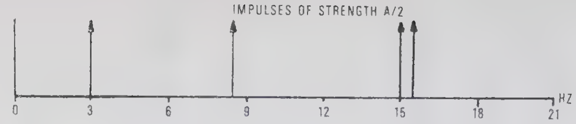


Fig 2a FOURIER TRANSFORM OF INFINITE RECORD

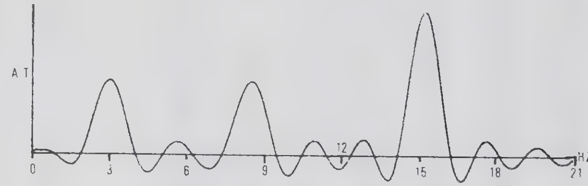


Fig 2b CONTINUOUS FOURIER TRANSFORM OF T SEC. OF RECORD

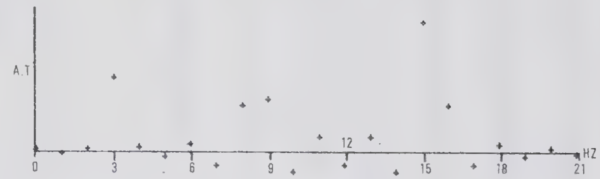


Fig 2c. DISCRETE FOURIER TRANSFORM OF T SEC OF RECORD

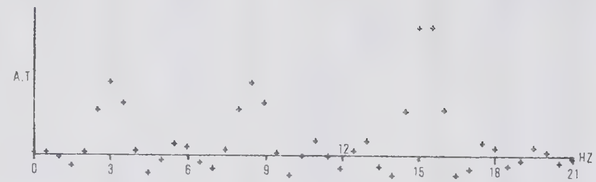


Fig 2d DISCRETE FOURIER TRANSFORM OF 2T SEC AUGMENTED RECORD

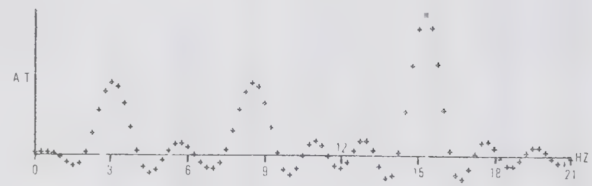


Fig 2e DISCRETE FOURIER TRANSFORM OF 4T SEC AUGMENTED RECORD

Since the component periods are unknown, the coincidence of a sample point in Fig. 2(c) with a peak in Fig. 2(b), as happens at 3 Hz, is accidental and in general, peaks in Fig. 2(b) are often poorly defined by the widely spaced sample points in Fig. 2(c). For instance, it can be seen from the neighbouring sample points that the second peak in Fig. 2(b) lies in the interval 8.25 to 8.75 Hz but its precise location is hard to judge. However, the situation would obviously be improved if the number of samples could be increased, say for instance doubled as in Fig. 2(d), where the points are $\Delta f/2 = 1/2T$ apart. Here the position of the second peak is far better defined.

The values needed for the improved plot, Fig. 2(d), can be found in two ways. Firstly, instead of using a standard Fourier transform algorithm to compute the N values in Fig. 2(c) according to the formula

$$(1) \qquad S(n.\Delta f) = \Delta t. \sum_{j=0}^{N-1} x(j.\Delta t)\ e^{-2\pi i.jn/N}, \quad n=0,\ldots,N-1$$

where $i=\sqrt{-1}$ and $x(j.\Delta t)$, $j=0,\ldots,N-1$ are the sampled values taken from the record, Fig. 1(b), it is possible to write a special-purpose algorithm to calculate the 2N points in Fig. 2(d), which are given by

$$(2) \qquad S(m.\frac{\Delta f}{2}) = \Delta t. \sum_{j=0}^{N-1} x(j.\Delta t).e^{-2\pi i.jm/2N}, \quad m=0,\ldots,2N-1.$$

A simpler method of achieving the same result without special programming is to augment the record in Fig. 1(b) to that shown in Fig. 1(c) by adding an equal length of null

record, then sampling at intervals Δt as before and performing the standard discrete Fourier transformation on the 2N data points thus obtained. The spacing of points in the computed spectrum is governed by the augmented record length 2T and so, in this case, is equal to $1/2T = \Delta f/2$ as required for Fig. 2(d). Of course, the samples $x(j.\Delta t)$, $j=N,\ldots,2N-1$ are all zero and consequently the standard Fourier algorithm in this case gives

$$S(m.\frac{\Delta f}{2}) = \Delta t. \sum_{j=0}^{2N-1} x(j.\Delta t)e^{-2\pi ijm/2N} \quad \text{(by analogy with equation (1))}$$

$$= \Delta t. \sum_{j=0}^{N-1} x(j.\Delta t)e^{-2\pi ijm/2N} + \Delta t. \sum_{j=N}^{2N-1} x(j.\Delta t)e^{-2\pi ijm/2N} \tag{3}$$

$$= \Delta t. \sum_{j=0}^{N-1} x(j.\Delta t)e^{-2\pi ijm/2N} + 0, \quad m=0,\ldots,2N-1$$

Thus, the values computed in this manner are identical with those given by (2). Essentially this second method of obtaining the points in Fig. 2(d) involves data modification, whereas the first method involves modifying the Fourier transform algorithm. Without question, modifying the data by simply adding zeroes is the easier technique in practice.

The process of augmentation can be continued indefinitely. For instance if the augmented record in Fig. 1(b) is augmented to double its length by the addition of a further length 2T of null record as shown in Fig. 1(d) and then sampled, the corresponding computed spectrum is as shown in Fig. 2(e). This has double the number of points in Fig. 2(c) and gives an even better representation of Fig. 2(b). Doubling the record

length at each stage is the most natural choice, if, as is normally the case, a Fast Fourier Transform with radix 2 is used to compute the spectrum.

When records are augmented by zeroes, the use of weighting, for example by means of a Hamming data window, is to be recommended. This does have the disadvantage of increasing the effective filter bandwidth, that is of broadening each peak in Fig. 2(b), but its main effect, reduction of the noticeable side-bands associated with the peaks, is particularly desirable with augmented records, since otherwise the increased number of spectral points shows up every side-band to full effect. The data window should be based on the original record length rather than on the length of the augmented record.

Application to Finite-Length Functions

It is sometimes necessary to compute accurately the Fourier transforms of functions which are known to be zero outside a certain specified range. If the Fourier transform is computed in the most obvious manner by using samples of the function taken only in that part of the range where it is non-zero, the result may be seriously misleading. A good illustration of this difficulty occurs when it is necessary to obtain the Fourier transforms of so-called 'data-windows' (Jenkins and Watts, 1969), for instance, the rectangular or boxcar data window:

$$f(t) = 1, \qquad |t| \leq \frac{T}{2}$$

$$= 0 \qquad |t| > \frac{T}{2}$$

shown in Fig. 3(a). The continuous Fourier transform of this function is

$$F(\omega) = 2 \sin\left(\frac{1}{2}\,\omega T\right)/\omega$$

which is shown in Fig. 4(a).

If the Fourier transform of this window or some more complicated finite-range function has to be computed from sampled values rather than analytically, the simplistic approach would be to use only samples from within the range where the function is non-zero. Suppose for instance that the Fourier transform of the rectangular data window is computed with a conventional algorithm using the equi-distant samples shown in Fig. 3(b). The resulting discrete Fourier transform is shown in Fig. 4(b). There are two different but equivalent explanations of why Fig. 4(b) is such a poor representation of the true Fourier transform, Fig. 4(a). These are

a) the interval (1/T) between the computed points shown in Fig. 4(b) is determined by the length of record in the time domain, in this case T, on which the computed transform is based. Unfortunately, apart from the point at $\omega = 0$, the computed spectral points coincide with zeroes

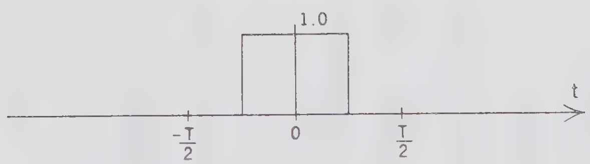
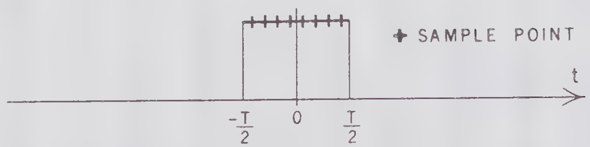


Fig. 3a : RECTANGULAR DATA WINDOW OF LENGTH T



3b : RECTANGULAR WINDOW 3a WITH EQUI-SPACED SAMPLES

Fig. 3c : IMPLIED CYCLIC FUNCTION CORRESPONDING TO 3b

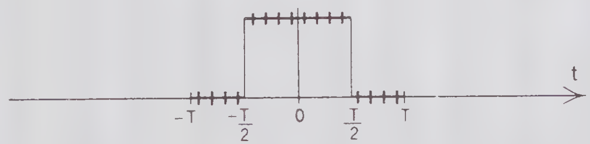


Fig. 3d : RECORD 3b AUGMENTED WITH EQUAL NUMBER OF ZEROES

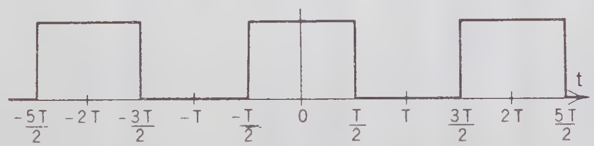


Fig. 3e : IMPLIED CYCLIC FUNCTION CORREPONDING TO 3d

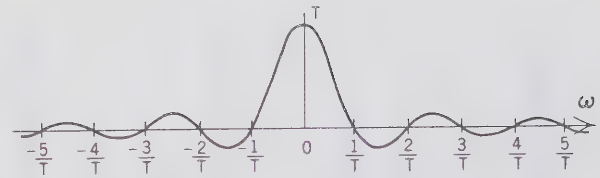


Fig. 4a : CONTINUOUS FOURIER TRANSFORM OF RECORD 3a

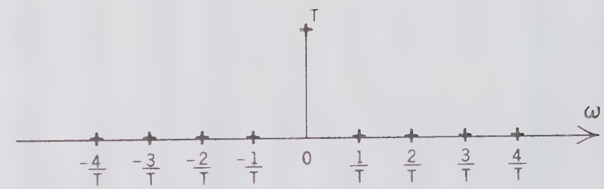


Fig. 4b : DISCRETE TRANSFORM OF UNAUGMENTED RECORD 3b

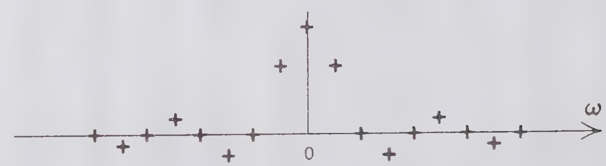


Fig. 4c : DISCRETE TRANSFORM OF AUGMENTED RECORD 3d

Fig. 4d : DISCRETE TRANSFORM OF RECORD AUGMENTED TO LENGTH 4T

of the true spectrum $F(\omega) = 2 \sin(\frac{1}{2}\omega T)/\omega$.

b) The fact that only a zero-frequency component is found in the computed transform (Fig. 4(b)) implies that the corresponding sampled function in the time domain is constant over $-\infty < t > \infty$. Thus it appears as if the original rectangular function of Fig. 3(a) is in effect repeated at intervals T all along the time axis as shown in Fig. 3(c). In fact this cyclic repetition of the original sampled function is an inherent property of discrete Fourier transforms. It may be noted that the repetition interval is equal to T, the length of record on which the computed transform is based.

A better representation of the true Fourier transform, Fig. 4(a), can be obtained by zero-padding. For instance, in Fig. 3(d), where the original samples are augmented by an equal number of zeroes, i.e. increasing the effective record length from T to 2T, the resulting computed spectrum is that shown in Fig. 4(c). Since the computed spectral values are now spaced 1/2T apart, a much better representation of the true spectrum is achieved. Fig. 4(d) shows additional improvement obtainable by further zero-padding, the case illustrated corresponding to an effective record length of 4T. This shows, in terms of interpretation a), how zero-padding can be used to improve the poor definition in Fig. 4(b).

In terms of interpretation b), the effect of zero-padding is to increase the interval at which the original

function is repeated along the time axis. For instance, when the sampled rectangular window of Fig. 3(b) is padded with an equal number of zeroes as shown in Fig. 3(d), the corresponding cyclic time function implied under interpretation b) is that shown in Fig. 3(e). Since in the limit, when the number of zeroes added is increased indefinitely, the spacing between repetitions of the original function becomes infinite and hence the implied cyclic function is identical to the original function. It is reasonable to expect that even a lesser degree of zero-padding will result in some improvement over the unpadded case.

In conclusion, it can be seen that zero-padding essentially results in a legitimate form of interpolation between the coarsely-spaced spectral points obtained by straightforward computation of the discrete Fourier transform of the original sample points.

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On the water of the Canadian Arctic archipelago, an atlas presentation of 1961 and 1962 data

F.G. Barber and A. Huyer

Marine Sciences Branch
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ON THE WATER OF THE CANADIAN ARCTIC ARCHIPELAGO, AN ATLAS PRESENTATION OF 1961 AND 1962 DATA

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1971

CONTENTS

1. Introduction

This work represents a step in the development of those oceanographic data observed in and near the Canadian Arctic archipelago (Fig. 1) during the years 1960, 1961 and 1962 (Table 1); a presentation of part of the 1960 observations is in preparation. In the latter work it is emphasized that the data exhibit considerable variability, most of which is believed to be real. No change in this consideration has occurred and for this reason a separation by year for the presentation here has been maintained. It may be that with increased understanding a significant integration of such data will become possible.

In 1961 the distribution of stations occupied north of Foxe Basin and Hudson Strait* was uneven (Fig. 2), but was much better than usual from Barrow Strait and Smith Sound to Davis Strait. On this account it was decided to limit the presentation here for the 1961 data to the latter area. Similar consideration of the 1962 station distribution (Fig. 3) led to a more widespread presentation which includes most of the data (Table 1) north of Foxe Basin and Hudson Strait. Other presentations of portions of the data of Table 1 include a heat budget for Barrow Strait (Huyer and Barber, 1970) and a fresh water budget for the Gulf of Boothia (Huyer and Barber, 1971). Ford and Hattersley-Smith (1965) utilized the data observed in Nansen Sound in 1962 in CCGS "John A. MacDonald", while Milne, Herlinveaux and Langleben (1962) and Milne (1963) reported on the 1961 ICE PACK observations and on that portion of the 1961 CCGS "Labrador" data in M'Clure Strait respectively.

Data from three of the 1962 "Labrador" stations in Smith Sound (consec numbers 1 to 3) were utilized by Collin (1965 p8) in a determination of the volume transport through Smith Sound and data at "John A. MacDonald" 1962 station (consec number 57) were presented by Palfrey (1968 p57).

More recently Muench (1971) in a detailed examination of northern Baffin Bay utilized most available data including those for 1960, 1961 and 1962.

2. Method

The plotting sheet used (CHS 750) was derived from chart 7000 titled "Arctic Islands" and was provided in quantity by the Canadian Hydrographic Service. In the preparation of

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*Data obtained in 1961 in the Hudson Bay system have been reported (Anon., 1964a; b).

Table 1 A listing of the relevant data for 1960, 1961 and 1962 indicating the year obtained, name of ship, Cruise Reference Number and a literature reference to the data if available:

Year	Ship	CRN	Reference
1960	"Westwind"	00670	Anon., 1964c
1960	"Theta"	329	Anon., 1964d
1960	"Labrador"	340	do
1961	"Edisto"	00688	-
1961	(ICE PACK)	763	Herlinveaux, 1961
1961	(PCSP)*	339	-
1961	"John A. MacDonald"	344	Anon., 1966b
1961	"Labrador"	341	do
1962	"Atka"	31966	-
1962	"John A. MacDonald"	359	Anon., 1966a
1962	"Labrador"	362	Anon., 1967a
1962	(ICE CAMP)	784	Herlinveaux, 1963
1962	"Salvelinus"	377	Anon., 1963a
1962	(land-based)	363,4,5,6,7	Anon., 1963b

*Polar Continental Shelf Project

the sheets the data were placed beside the appropriate station in blue ink* and the sheet subsequently became the final line drawing. The values were derived from individual station graphs of each variable against depth using available bathythermograph data and T-S diagrams where appropriate; interpolated data of the data records were not used. Occasionally, it occurred that some of the tabulated values of the data records did not fit the overall interpretation. When these appeared anomalous, an attempt was made to determine whether they were real or due to blunders. In only the two instances below was it possible to find definite errors:

CRN	Consec	Depth	Sal Correction	Temp Correction	Justification
362	4	20		0.36	bathythermogram shows positive temperatures above 50 m.
359	7	10	31.376	1.89	inadvertantly omitted from data record.

In some instances it seemed almost certain that a blunder had occurred and here the values were "corrected" or deleted. These are shown in Table 2, which includes a statement concerning the basis for the suspicion. It is realized that these alterations are largely a matter of our interpretation.

The contour interval is indicated on each distribution. A dashed contour represents one additional to the interval indicated there, while a dotted contour indicates that the configuration is in doubt, because either the interpretation or the observation is in doubt. Many of the 1962 stations were located within the archipelago and in contouring in this area it was assumed that there was continuity in the particular distribution in spite of the barriers posed by the islands. For this reason, many of the contours initially extended across these land areas and subsequently had to be removed.

3. Data

It appears that the 1961 salinity determinations for each of the three ships were carried out with conductivity bridges after completion of the field work. The "Edisto"

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*In many of the figures the values so placed are still visible.

Table 2. Suspected errors in the tabulated values of salinity and temperature in two of the data records for 1962.

CRN	Consec	Depth (m)	Correction Sal (⁰/oo)	Correction Temp (°C)	Justification
359	4	200	33.308		strong density inversion.
		250	33.690		
	13	0	29.690		strong density inversion.
		10	30.130		
	18	0	delete		strong density inversion; correction not obvious.
	33	350	delete		strong density inversion; correction not obvious.
	37	500	delete	delete	density inversion and somewhat low temperatures.
		600	delete	delete	
	39	200	delete	delete	slight inversion where strong gradient expected.
	41	10	29424		a number of density inversions.
		20	31020		
		30	31170		
		50	31411		
	58	0	26260		strong density inversion.
		10	26700		
362	6	600		delete	
	13	450		delete	
	14	378		delete	
	17	10	31146		
		30	31176		
	18	223	delete		
	46	145	33480		density inversion.
		194	33553		
	52	10	32600		density inversion.
		20	32839		
	53	300		0.54	bathythermogram shows temp. at 275 m positive and increasing; "Atka" station shows 0.54 at almost the same depth and position.

samples were analysed "approximately two months after they had been collected", while those of "Labrador" and "John A. MacDonald" were not analysed completely until February and May of 1962 respectively. It appears likely that the determinations for the 1962 samples would have been carried out over a similar period of time.

The vertical distribution of salinity at about 1150 m and deeper (Fig. 4) observed in "Labrador" in 1961 and 1962 suggests the existence of a small systematic error of about 0.01 $^{o}/oo$ between the data of the two years. Consideration of the temperature data (Fig. 4) does not suggest that systematic differences occur in the data of the two surveys.

4. Remarks

A feature of most of the shallower presentations is the strong gradient in each distribution across Baffin Bay. The feature appears to be associated with a movement of a relatively low-salinity cold water out of the archipelago and along the east coast of Baffin Island. This is a view of long-standing of course, which recently received some additional support in a note (Huyer and Barber, 1971) about the distribution of freshwater within the archipelago (after Tully, 1958). The interpretation was based on the oceanographic data for 1961 and 1962 and on the distribution of freshwater in relation to precipitation (Fig. 7) and runoff (Sanderson and Philips, 1967), particularly around the Gulf of Boothia.

Data on observed ice conditions during 1961 are available (Archibald et al., 1962) and Dunbar (1962 p5) remarked that, "In general the 1961 season was unfavourable for navigation in the Queen Elizabeth Islands". The 1962 season within the archipelago was relatively light (Anon., 1963c; Markham and Hill, 1963; Black, 1965; Lindsay, 1968) and may have contributed to higher than usual near-surface temperatures and greater storage of seasonal heat (Huyer and Barber, 1970). This situation appears to be reflected in those near-surface temperature data obtained in the "John A. MacDonald" (Fig. 13) from the Gulf of Boothia north to western Lancaster Sound, Barrow Strait, western Jones Sound and north to the vicinity of Tanquary Fiord.

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6. List of Figures

Figure 10. The distribution of depth and temperature on a σ_t surface during 1961 from data observed in "Labrador". (a) 27.50. (b) 27.60. (c) 27.65. (d) 27.70. (e) 27.72. (f) 27.74.

Figure 11. The distribution of depth, temperature and dissolved oxygen on the 34.4°/oo surface in 1961. (a) Depth. (b) Temperature. (c) Dissolved oxygen (includes 1962 data).

Part B

Figure 12. The distribution of salinity and temperature near the surface during 1962 from data observed from "John A. MacDonald", "Labrador", by the land-based parties and from the Ice Camps. (a) 0 m. (b) 10 m. (c) 20 m.

Figure 13. The distribution of salinity and temperature during 1962 from data observed on "John A. MacDonald", "Labrador", "Atka", and from the Ice Camps and by the land-based parties. On the distributions at 200 m and greater the hatched lines show the approximate limits of areas where the greatest depth in any section of the channel is less than the depth of the distribution. (a) 30 m. (b) 50 m. (c) 75 m. (d) 100 m. (e) 150 m. (f) 200 m. (g) 250 m. (h) 300 m. (i) 400 m. (j) 500 m. (k) 600 m. (l) 800 m. (m) 1000 m.

Figure 14. The distribution of depth and temperature in 1962 on the 34.4°/oo surface. (a) Depth. (b) Temperature.

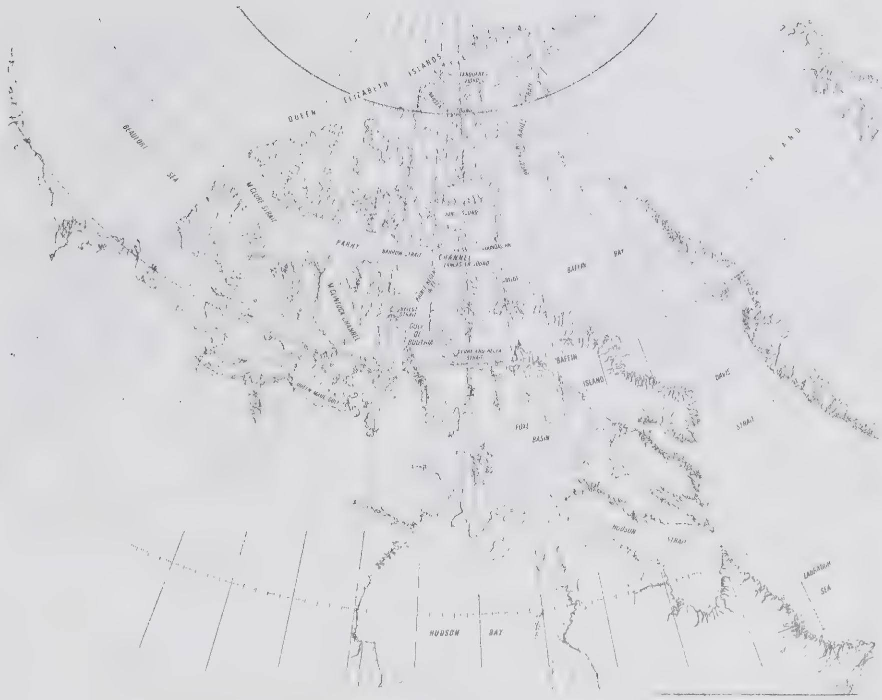


Figure 1. Some place names in the Arctic archipelago.

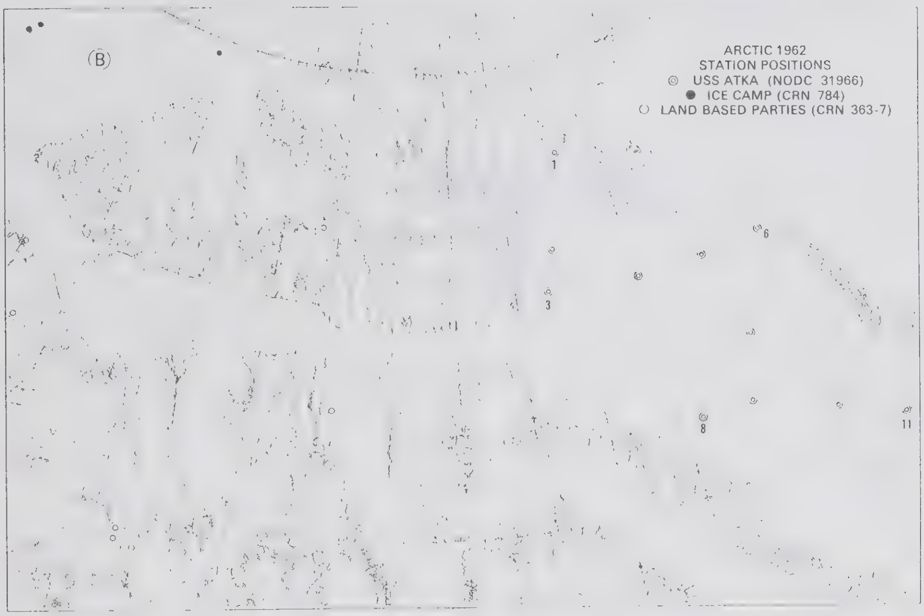
Figure 2. The approximate positions of stations occupied in 1961 north of Foxe Basin and Hudson Strait.

Figure 3

The approximate positions of stations occupied in 1962 north of the latitude of Hudson Strait. (a) Stations occupied on surveys by CCGS "Labrador" (Anon., 1967a) and CCGS "John A. MacDonald" (Anon., 1966a). (b) Stations occupied by five land-based parties (Anon., 1963b), from the pack ice (Herlinveaux, 1963) and on a survey by USS "Atka".


ARCTIC, 1962
STATION POSITIONS
• Labrador
John A. MacDonald
Bathythermograms

3(a)



(B)
ARCTIC 1962
STATION POSITIONS
USS ATKA (NODC 31966)
ICE CAMP (CRN 784)
LAND BASED PARTIES (CRN 363-7)
1
6
3
8
11

3(b)

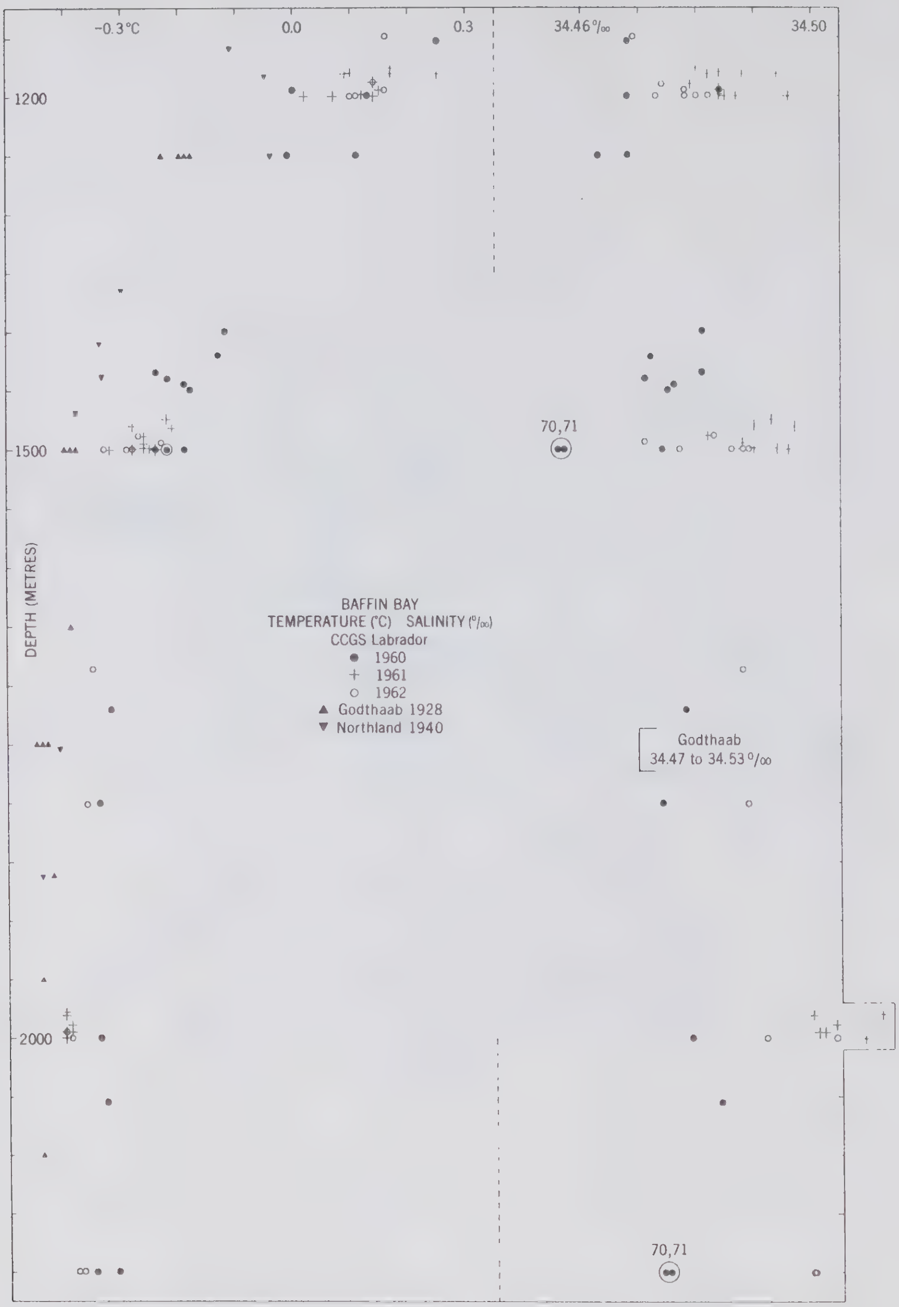


Figure 4. Temperature and salinity values at depth below about 1150 metres in Baffin Bay from a number of sources. The circled salinity values at "Labrador" stations 70 and 71 in 1960 are believed to reflect sampling error.

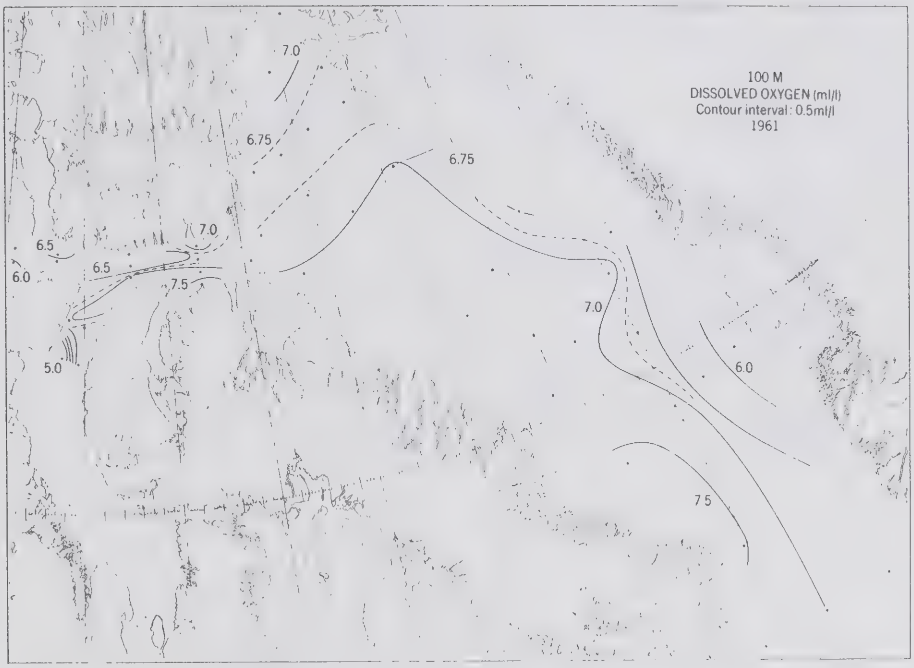
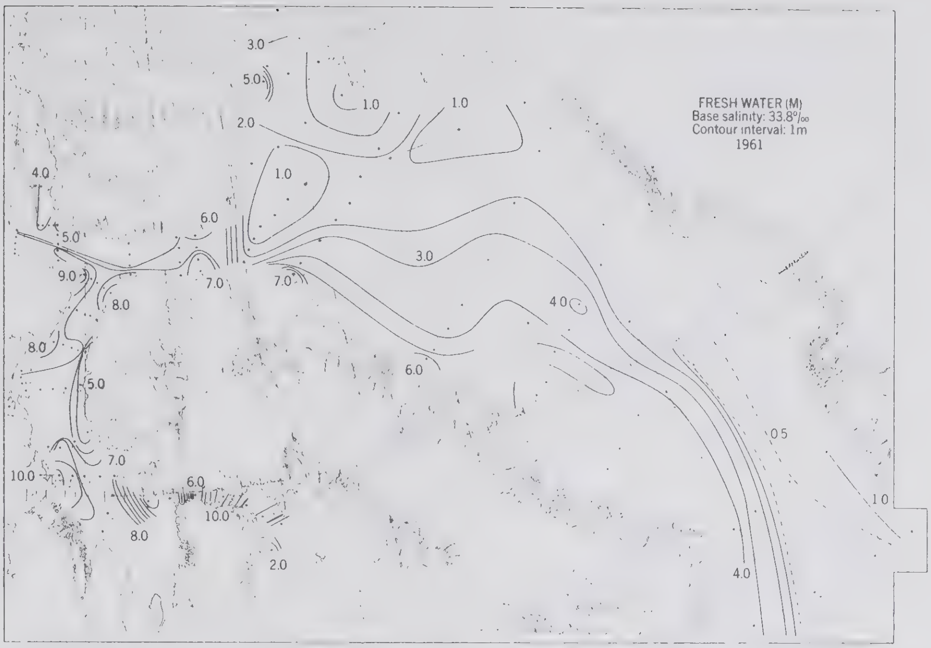


Figure 5. The distribution of dissolved oxygen at 100 m observed in "Labrador" in 1961. The relatively low value in the Gulf of Boothia (consec number 39) could be due to error.

Figure 6

The distribution of the amount of fresh water. (a) 1961.
(b) 1962.


FRESH WATER (M)
Base salinity: 33.8‰
Contour interval: 1m
1961

6(a)


FRESH WATER (M)
Base salinity: 33.8‰
Contour interval: 1m
1962

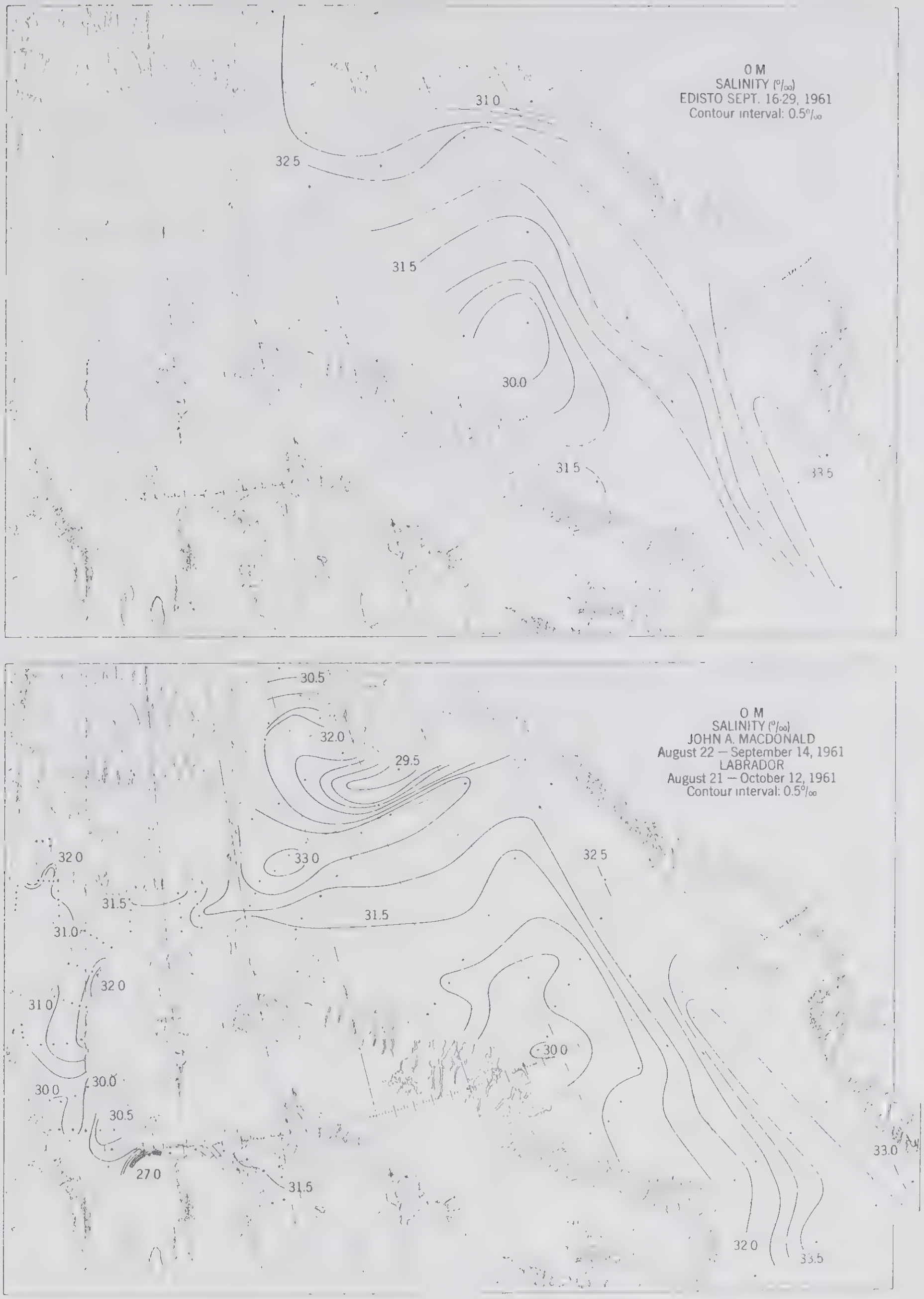
6(b)

Figure 7. Precipitation in the Northwest Territories (Anon., 1967b).

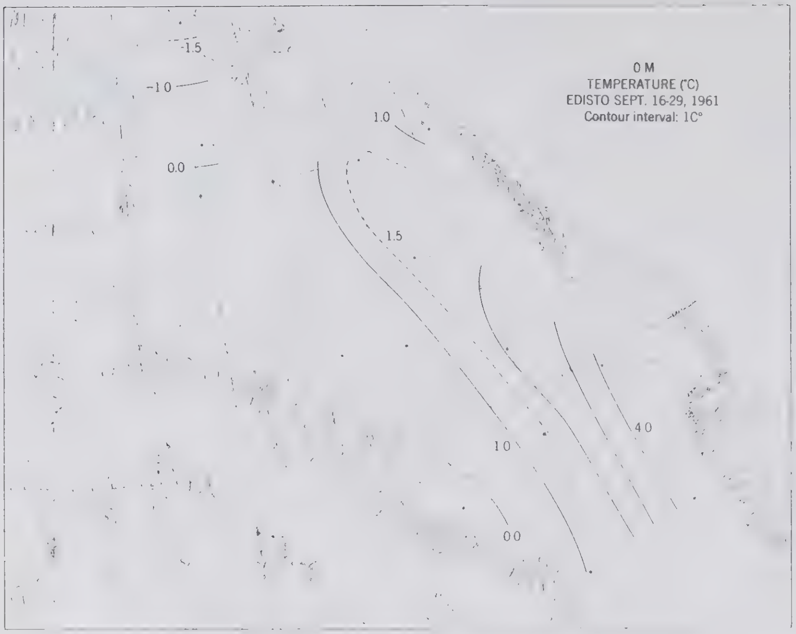
PART A

Figure 8

The distribution of salinity and temperature at the surface during 1961 from data observed in the vessels "Edisto", "John A. MacDonald" and "Labrador". (a) Salinity. (b) Temperature.


O M
SALINITY (°/oo)
EDISTO SEPT. 16-29, 1961
Contour interval: 0.5°/oo
31.0
32.5
31.5
30.0
31.5
33.5
O M
SALINITY (°/oo)
JOHN A. MACDONALD
August 22 — September 14, 1961
LABRADOR
August 21 — October 12, 1961
Contour interval: 0.5°/oo
30.5
32.0
29.5
33.0
32.5
32.0
31.5
31.5
31.0
32.0
31.0
30.0
30.0
30.0
30.5
27.0
31.5
33.0
32.0
33.5

8 (a)


0 M
TEMPERATURE (°C)
EDISTO SEPT. 16-29, 1961
Contour interval: 1C°


0 M
TEMPERATURE (°C)
JOHN A. MACDONALD
August 22 – September 14, 1961
LABRADOR
August 21 – October 22, 1961
Contour interval: 1C°

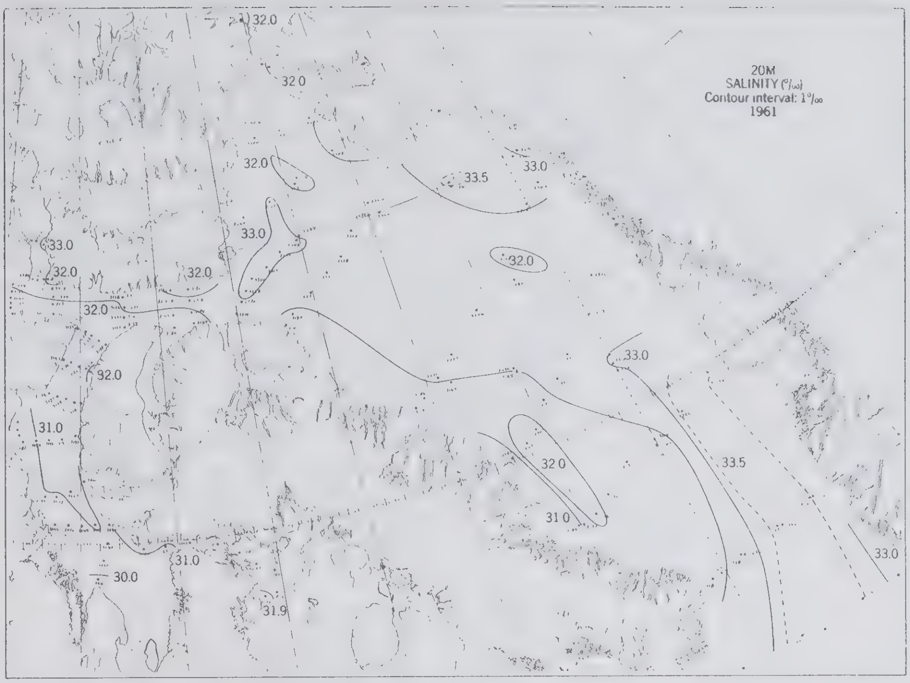
8 (b)

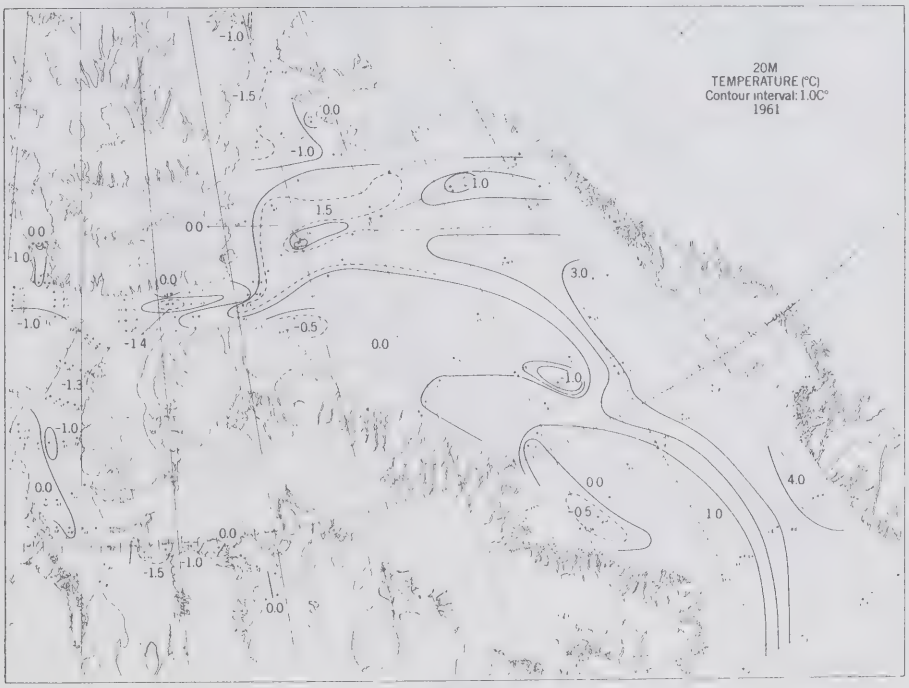
Figure 9

The distribution of salinity and temperature at standard depths during 1961 from data observed in the "Edisto", "John A. MacDonald" and the "Labrador". (a) 10 m. (b) 20 m. (c) 30 m. (d) 50 m. (e) 75 m. (f) 100 m. (g) 150 m. (h) 200 m. (i) 250 m. (j) 300 m. (k) 400 m. (l) 500 m. (m) 600 m. (n) 800 m. (o) 1000 m.

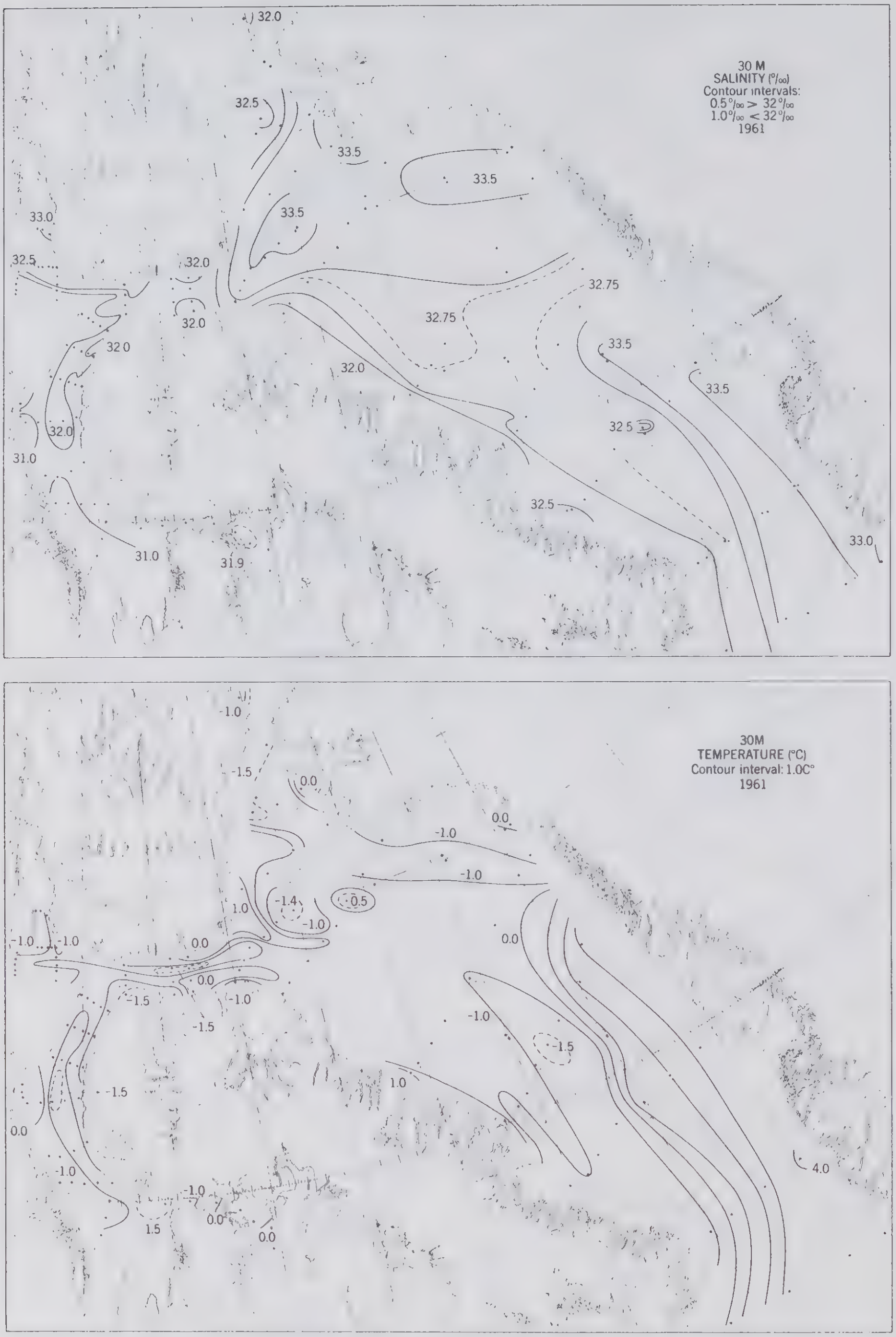

10M
SALINITY (°/₀₀)
Contour interval: 1°/₀₀
1961
10M
TEMPERATURE (°C)
Contour interval: 1.0C°
1961

9 (a)

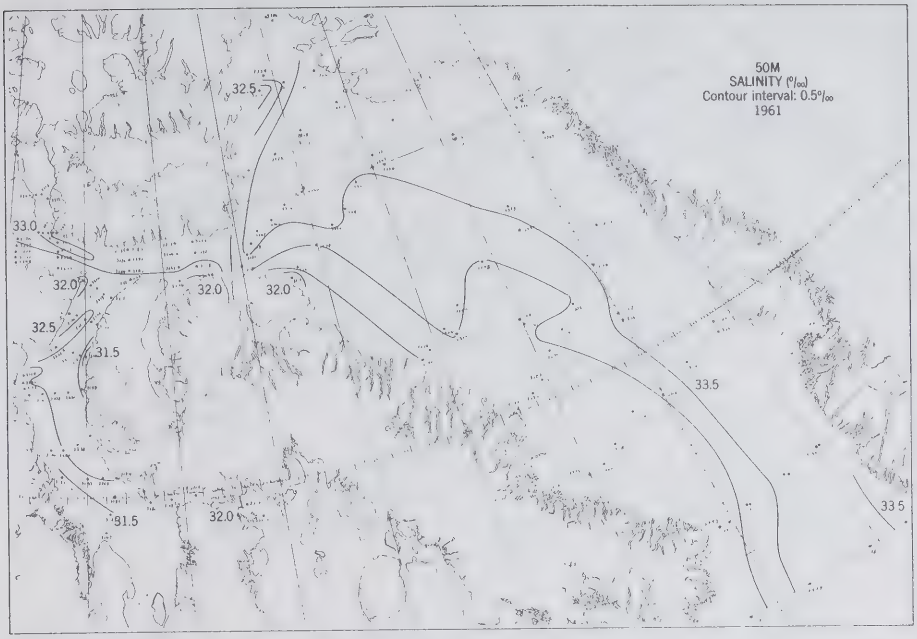

20M
SALINITY (‰)
Contour interval: 1‰
1961

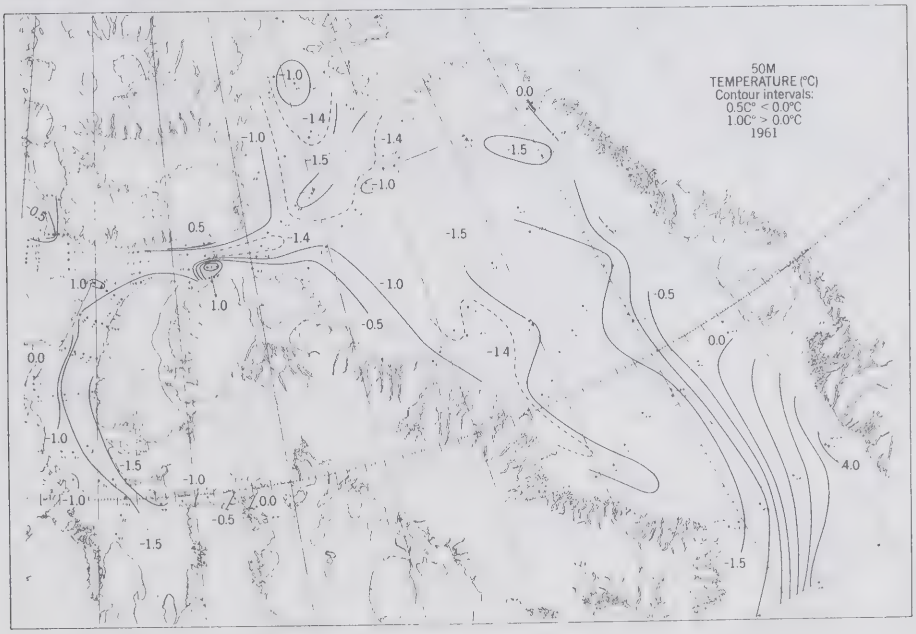

20M
TEMPERATURE (°C)
Contour interval: 1.0C°
1961

9 (b)

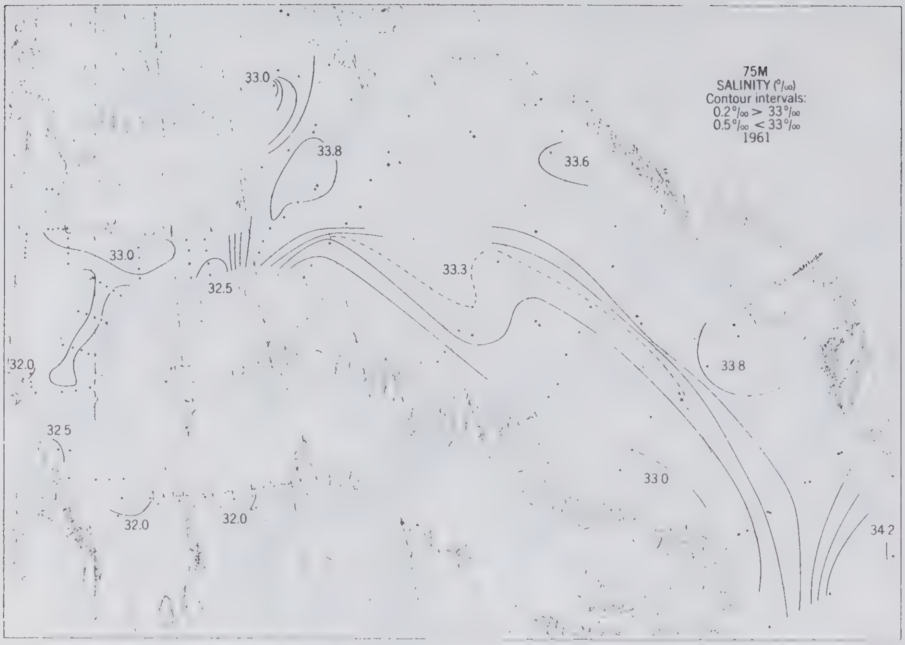

30 M
SALINITY (°/oo)
Contour intervals:
0.5°/oo > 32°/oo
1.0°/oo < 32°/oo
1961
30M
TEMPERATURE (°C)
Contour interval: 1.0C°
1961

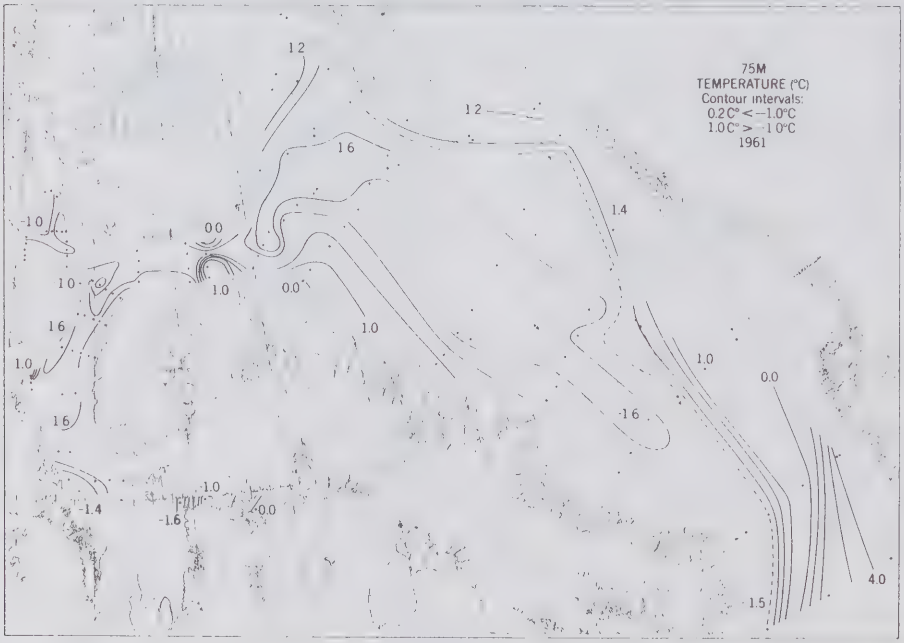
9(c)


50M
SALINITY (°/₀₀)
Contour interval: 0.5°/₀₀
1961
32.5
33.0
32.0
32.0
32.0
32.5
31.5
33.5
31.5
32.0
33.5

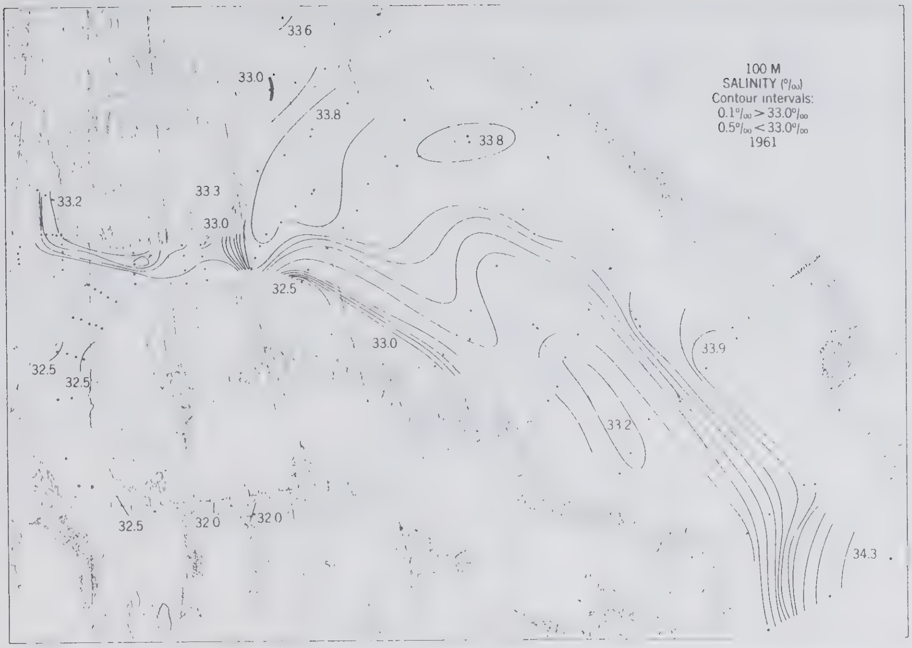

50M
TEMPERATURE (°C)
Contour intervals:
0.5C° < 0.0°C
1.0C° > 0.0°C
1961

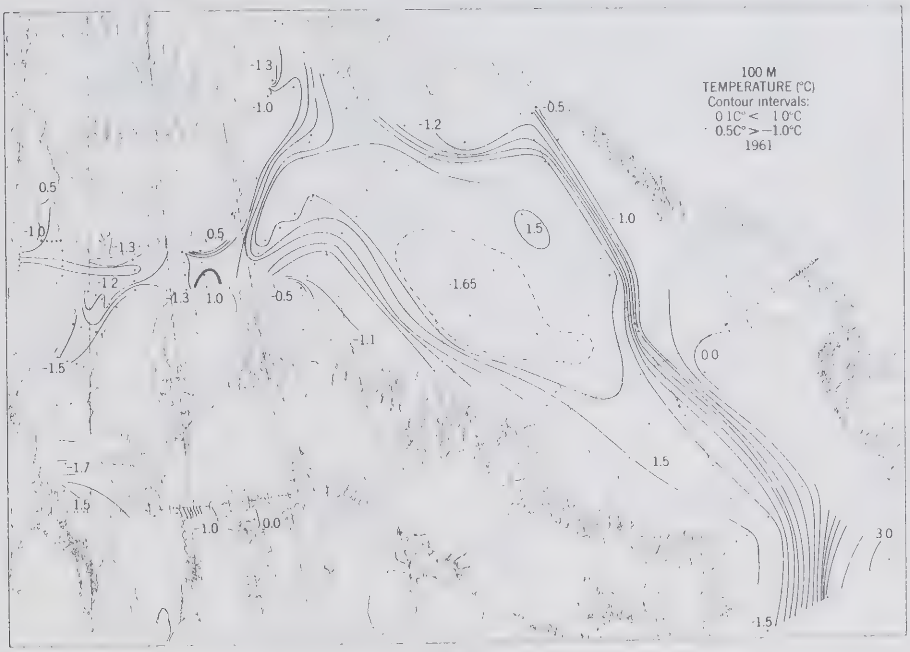
9(d)


75M
SALINITY (°/oo)
Contour intervals:
0.2°/oo > 33°/oo
0.5°/oo < 33°/oo
1961
33.0
33.8
33.6
33.0
32.5
33.3
32.0
33.8
32.5
33.0
32.0
32.0
34.2


75M
TEMPERATURE (°C)
Contour intervals:
0.2C° < -1.0°C
1.0C° > -1.0°C
1961
1.2
1.2
1.6
1.4
-1.0
0.0
1.0
1.0
0.0
1.6
1.0
1.0
1.0
0.0
-1.6
1.6
-1.0
-1.4
-1.6
0.0
4.0
1.5

9 (e)

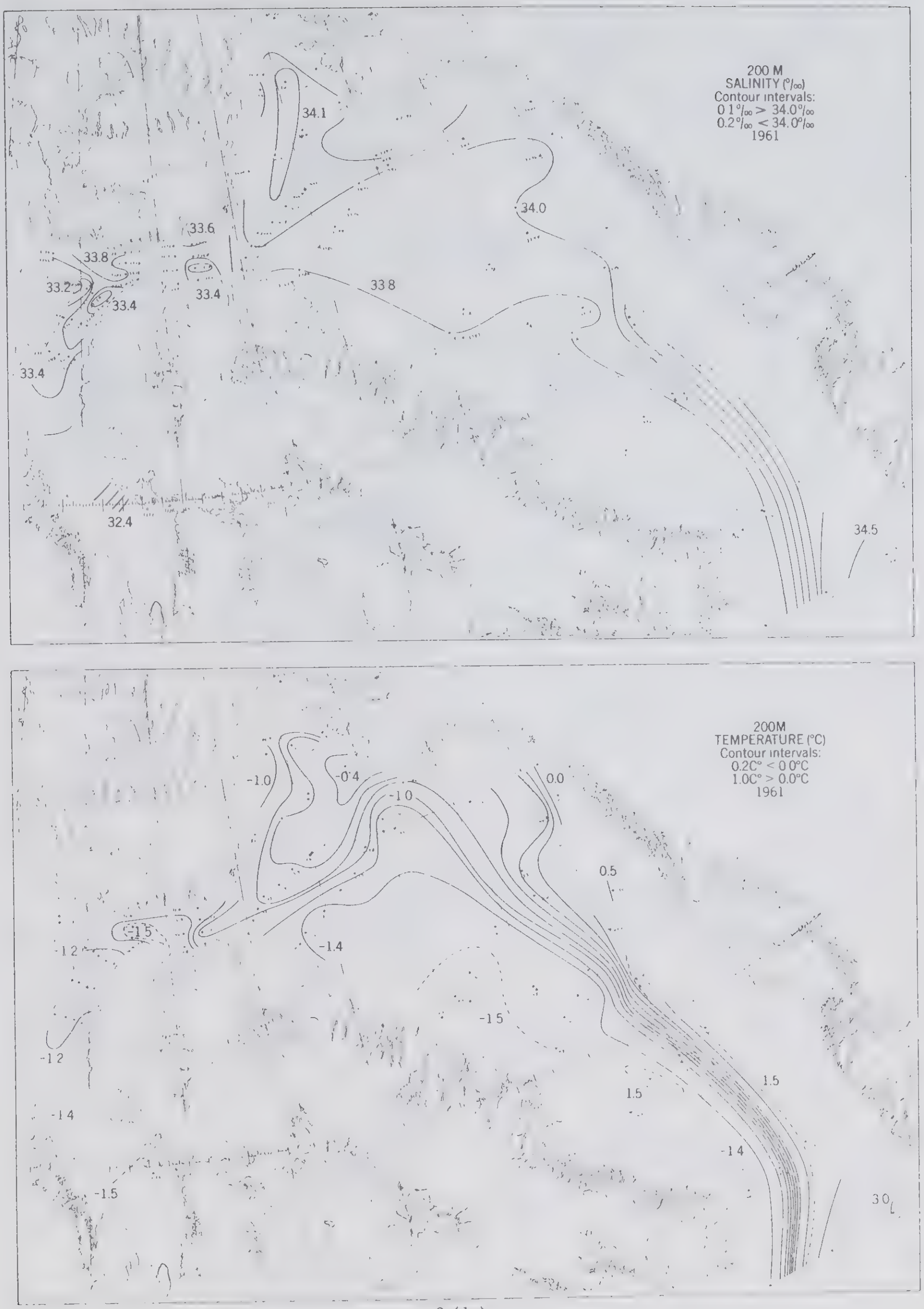

100 M
SALINITY (°/oo)
Contour intervals:
0.1°/oo > 33.0°/oo
0.5°/oo < 33.0°/oo
1961
33.6
33.0
33.8
33.8
33.3
33.2
33.0
32.5
33.0
33.9
32.5
32.5
33.2
32.5
32.0
32.0
34.3


100 M
TEMPERATURE (°C)
Contour intervals:
0.1C° < 1.0°C
0.5C° > -1.0°C
1961
-1.3
-1.0
-1.2
-0.5
0.5
-1.0
-1.3
0.5
1.5
1.0
-1.2
-1.3
1.0
-0.5
-1.65
-1.1
0.0
-1.5
1.5
-1.7
1.5
-1.0
0.0
3.0
-1.5

9 (f)

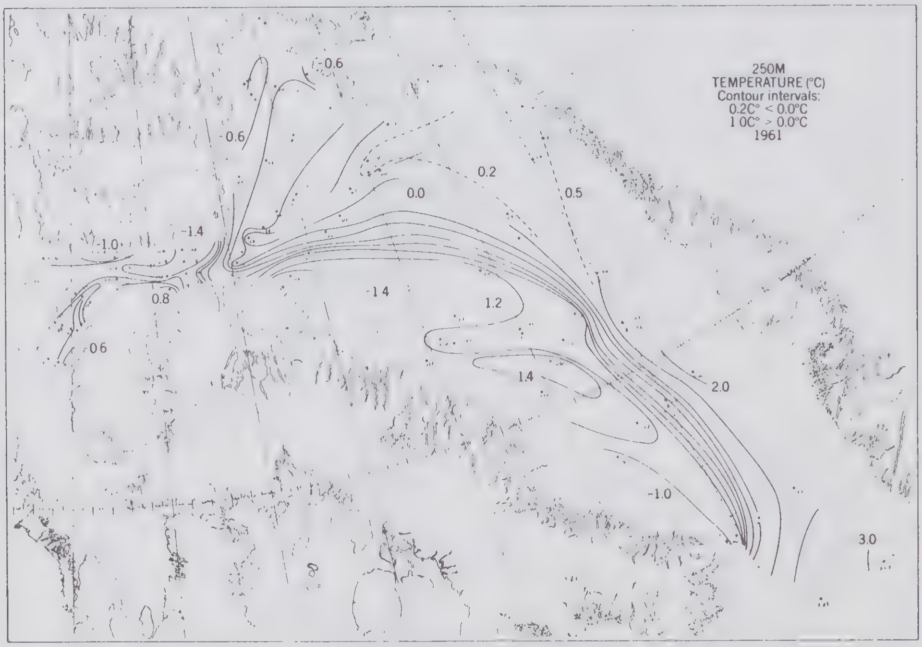

150 M
SALINITY (°/oo)
Contour intervals:
0.2°/oo > 33°/oo
0.5°/oo < 33°/oo
1961
150M
TEMPERATURE (°C)
Contour intervals:
0.2C° < 0.0°C
1.0C° > 0.0°C
1961

9(g)


200 M
SALINITY (°/∞)
Contour intervals:
0.1°/∞ > 34.0°/∞
0.2°/∞ < 34.0°/∞
1961
34.1
34.0
33.6
33.8
33.2
33.4
33.4
33.8
33.4
32.4
34.5
200M
TEMPERATURE (°C)
Contour intervals:
0.2C° < 0.0°C
1.0C° > 0.0°C
1961
-1.0
-0.4
-1.0
0.0
0.5
-1.5
-1.2
-1.4
-1.5
-1.2
1.5
1.5
-1.4
-1.5
-1.4
3.0

9 (h)

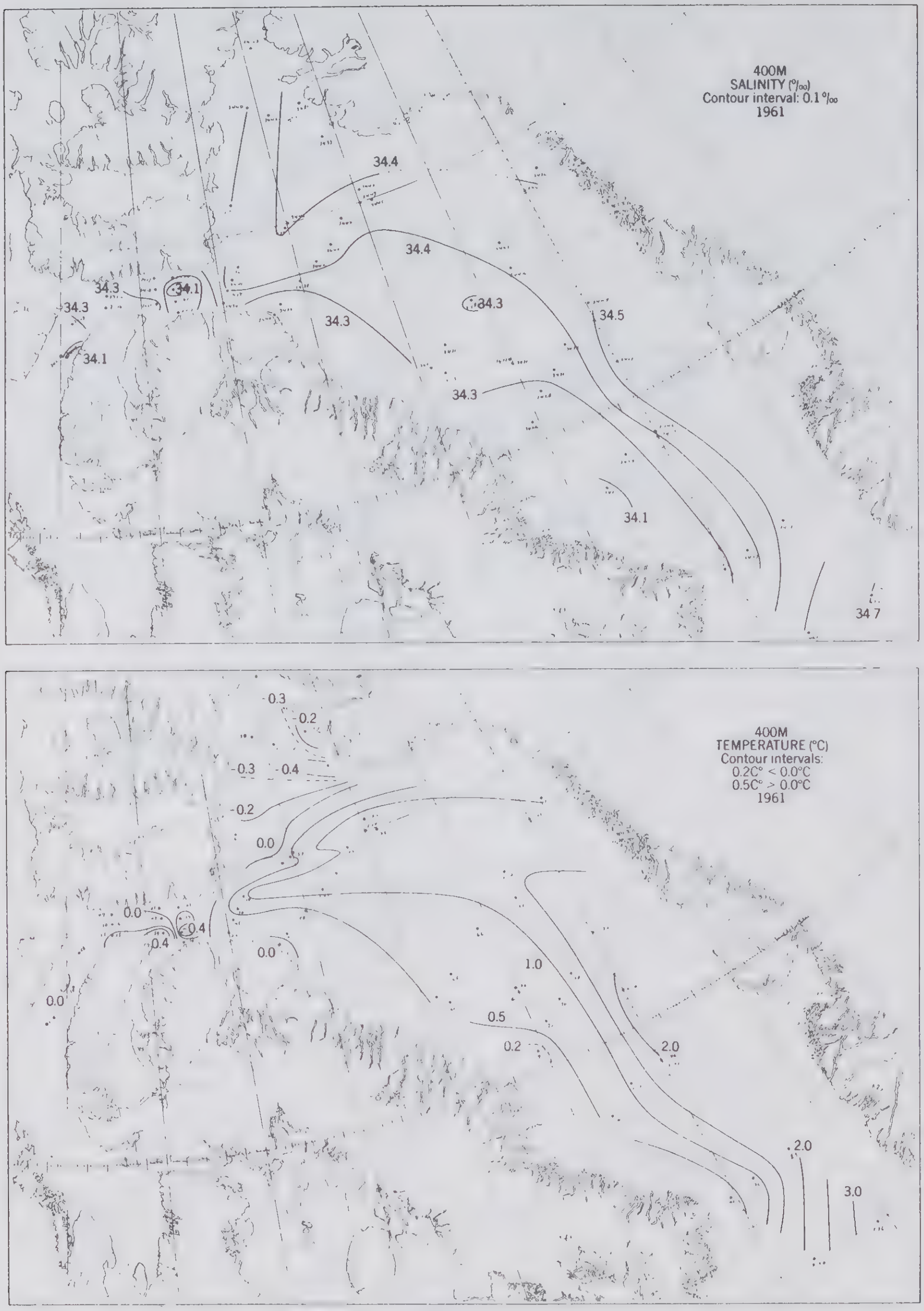

250 M
SALINITY (°/oo)
Contour interval: 0.1 °/oo
1961
34.1
34.2
34.1
34.0
33.7
34.0
33.7
33.9
34.0
33.8
33.9
34.0
34.6


250M
TEMPERATURE (°C)
Contour intervals:
0.2C° < 0.0°C
1.0C° > 0.0°C
1961
-0.6
-0.6
0.2
0.0
0.5
-1.4
-1.0
0.8
-1.4
1.2
0.6
1.4
2.0
-1.0
3.0

9(i)

300M
SALINITY (‰)
Contour interval: 0.1 ‰
1961
34.2
34.3
34.2
34.0
34.0
34.0
34.0
34.1
34.2
34.0
34 4
34.6
300M
TEMPERATURE (°C)
Contour intervals:
0.2C° < 1 0°C
0.5C° > 1 0°C
1961
-0.5
-0.5
0.0
0.5
0.5
-1.4
1.0
-0.5
-1.2
-0.5
1.0
0.0
2.0
0.5
1.5
0.2
3.0

9(j)

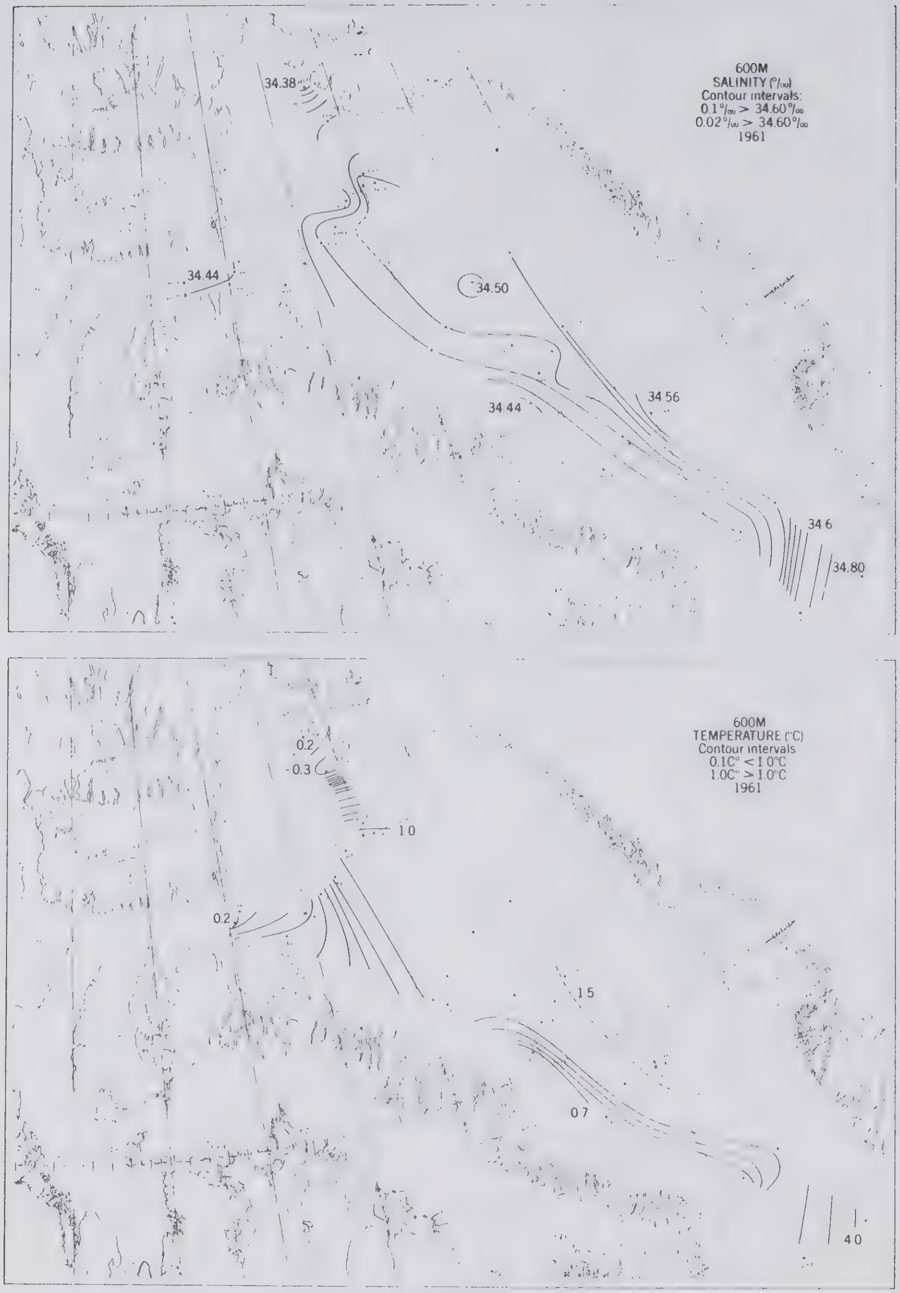

400M
SALINITY (‰)
Contour interval: 0.1 ‰
1961
34.4
34.4
34.3
34.1
34.3
34.1
34.3
34.3
34.5
34.3
34.1
34 7
400M
TEMPERATURE (°C)
Contour intervals:
0.2C° < 0.0°C
0.5C° > 0.0°C
1961
-0.3
-0.2
-0.3
-0.4
-0.2
0.0
0.0
0.4
0.4
0.0
0.0
1.0
0.5
0.2
2.0
2.0
3.0

9(k)

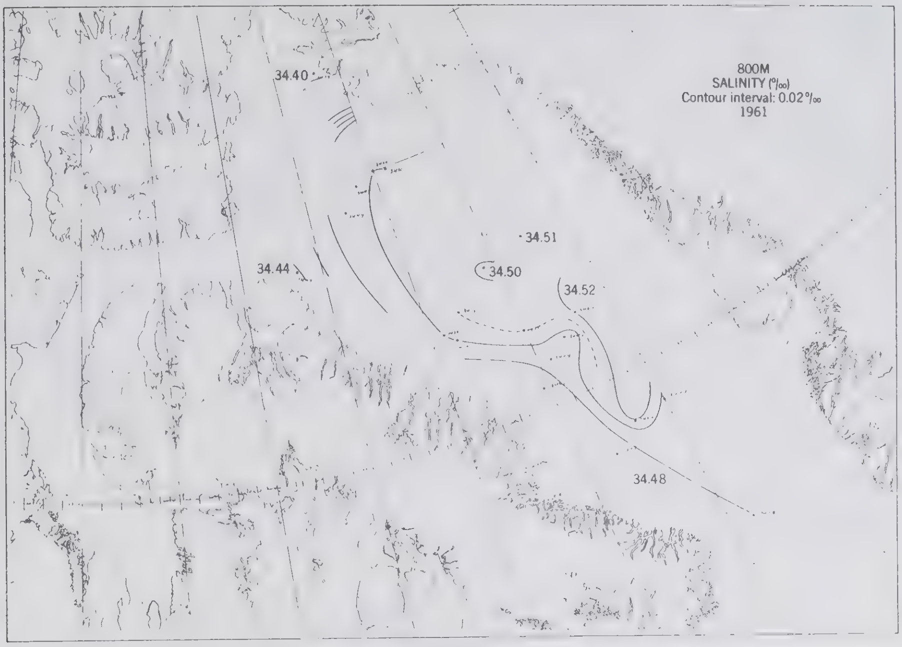
500M
SALINITY (‰)
Contour interval: 0.1‰
1961
34.4
34.2
34.4
34.4
34 5
34 55
34 4
34 8

500M
TEMPERATURE (°C)
Contour intervals:
0.2C° < 1.0°C
0.5C° > 1.0°C
1961
0.2
0.2
1 2
0.4
0.8
0.4
0.6
1 5
0.6
2 0
4 0

9(1)


600M
SALINITY (°/oo)
Contour intervals:
0.1°/oo > 34.60°/oo
0.02°/oo > 34.60°/oo
1961
34.38
34.44
34.50
34.44
34.56
34.6
34.80
600M
TEMPERATURE (°C)
Contour intervals
0.1C° < 1.0°C
1.0C° > 1.0°C
1961
0.2
-0.3
1.0
0.2
1.5
0.7
4.0

9 (m)


800M
SALINITY (‰)
Contour interval: 0.02 ‰
1961
34.40
34.44
34.51
34.50
34.52
34.48


800M
TEMPERATURE (°C)
Contour interval : 0.1C°
1961
-0.1
0.2
0 8
0 9
0 3

9 (n)

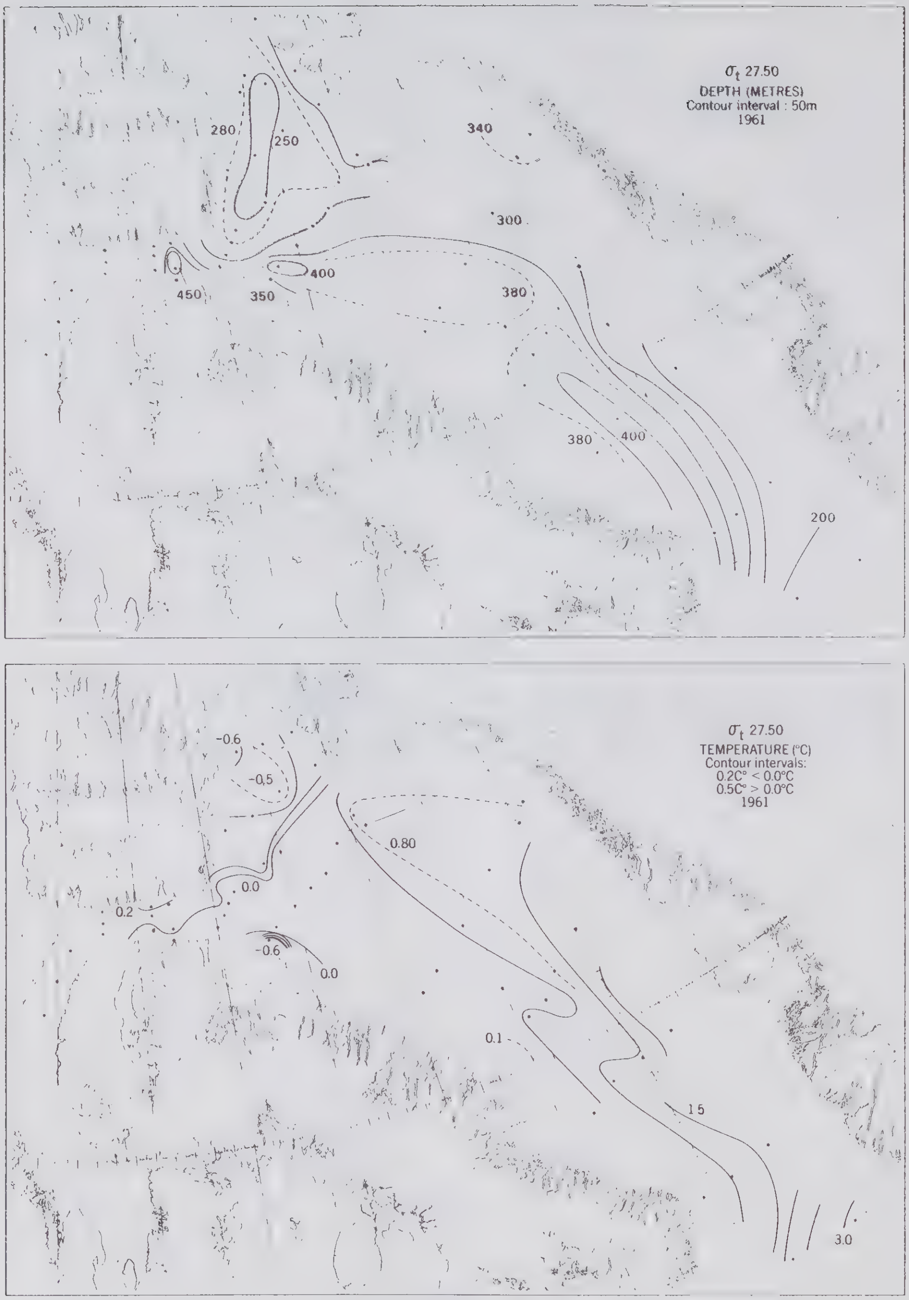

1000M
SALINITY (°/oo)
Contour interval: 0.01°/oo
1961
34.55
34.47


1000M
TEMPERATURE (°C)
Contour interval: 0.1C°
1961
0.45
0.6
0.45
0.1

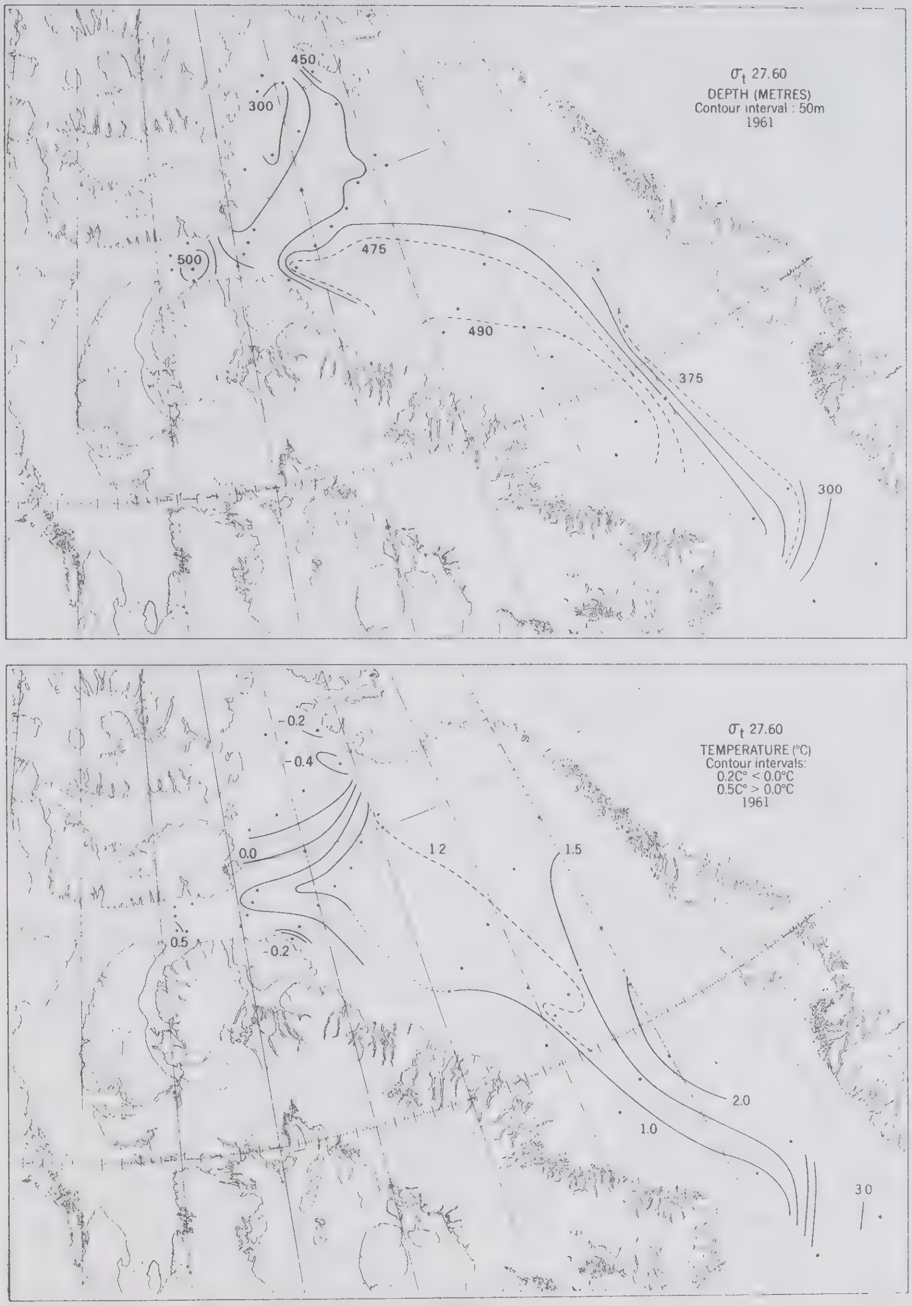
9 (o)

Figure 10

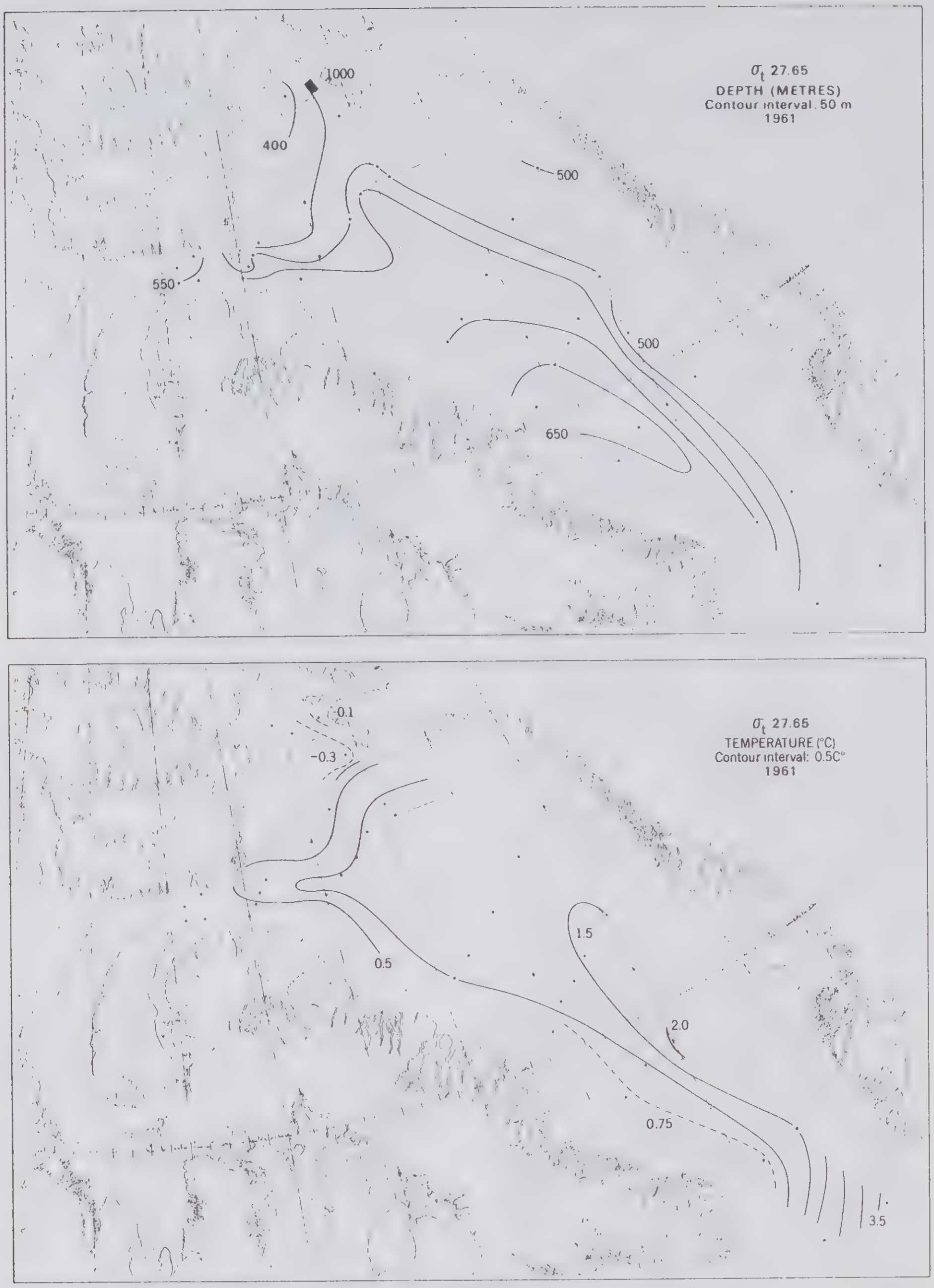
The distribution of depth and temperature on a σ_t surface during 1961 from data observed in "Labrador". (a) 27.50. (b) 27.60. (c) 27.65. (d) 27.70. (e) 27.72. (f) 27.74.


σ_t 27.50
DEPTH (METRES)
Contour interval : 50m
1961
280
250
340
300
450
350
400
380
380
400
200
σ_t 27.50
TEMPERATURE (°C)
Contour intervals:
0.2C° < 0.0°C
0.5C° > 0.0°C
1961
-0.6
-0.5
0.80
0.0
0.2
-0.6
0.0
0.1
1.5
3.0

10(a)


σt 27.60
DEPTH (METRES)
Contour interval : 50m
1961
450
300
500
475
490
375
300
σt 27.60
TEMPERATURE (°C)
Contour intervals:
0.2C° < 0.0°C
0.5C° > 0.0°C
1961
-0.2
-0.4
0.0
1.2
1.5
0.5
-0.2
2.0
1.0
3.0

10(b)



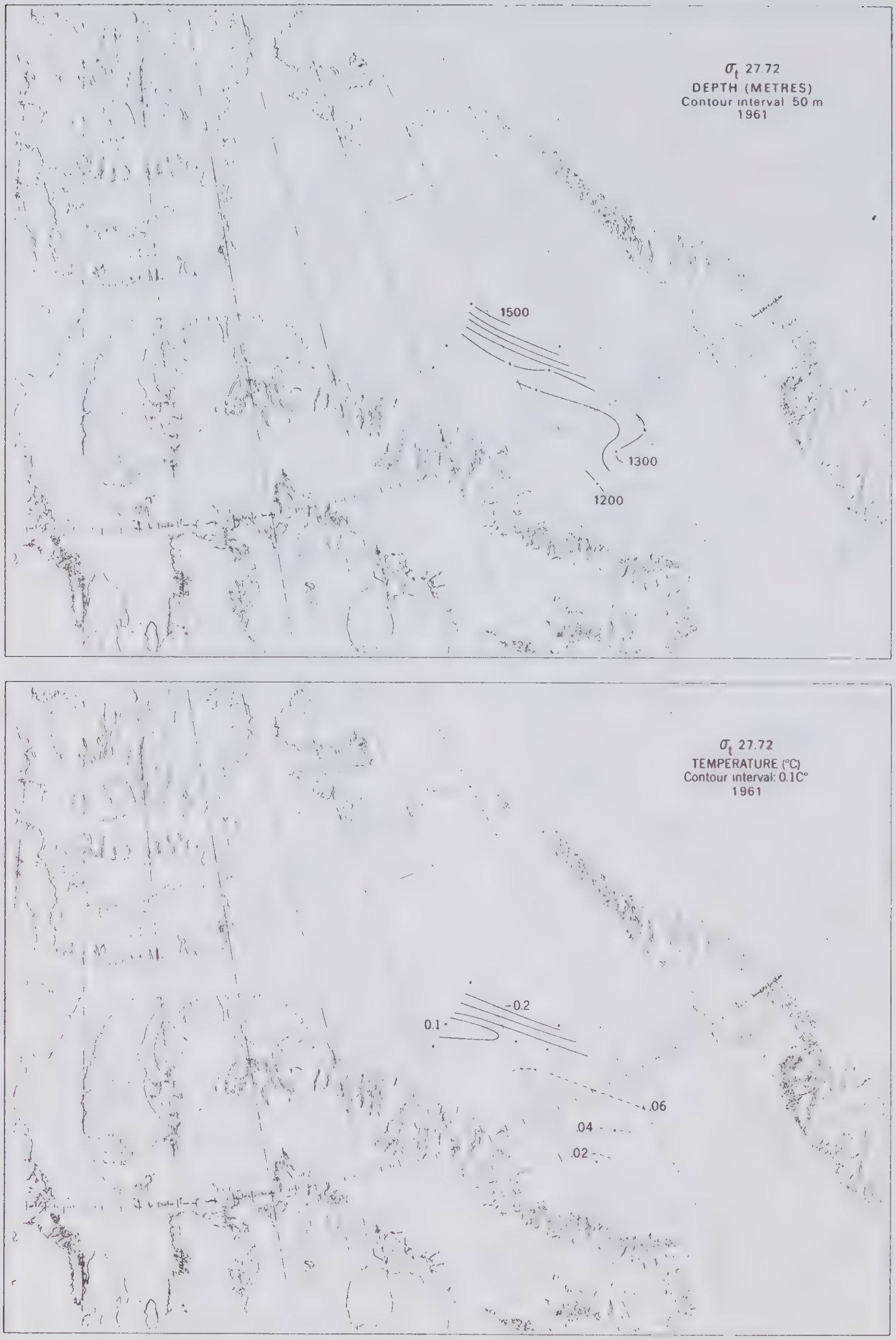
σt 27.65
DEPTH (METRES)
Contour interval 50 m
1961
1000
400
500
550
500
650
σt 27.65
TEMPERATURE (°C)
Contour interval: 0.5C°
1961
-0.1
-0.3
1.5
0.5
2.0
0.75
3.5

10(c)


σ_t 27.70
DEPTH (METRES)
Contour interval: 100 m
1961
600
1000
1200
1180
600


σ_t 27.70
TEMPERATURE (°C)
Contour interval: 0.1C°
1961
-0.3
0.0
0.5
0.1
40
20

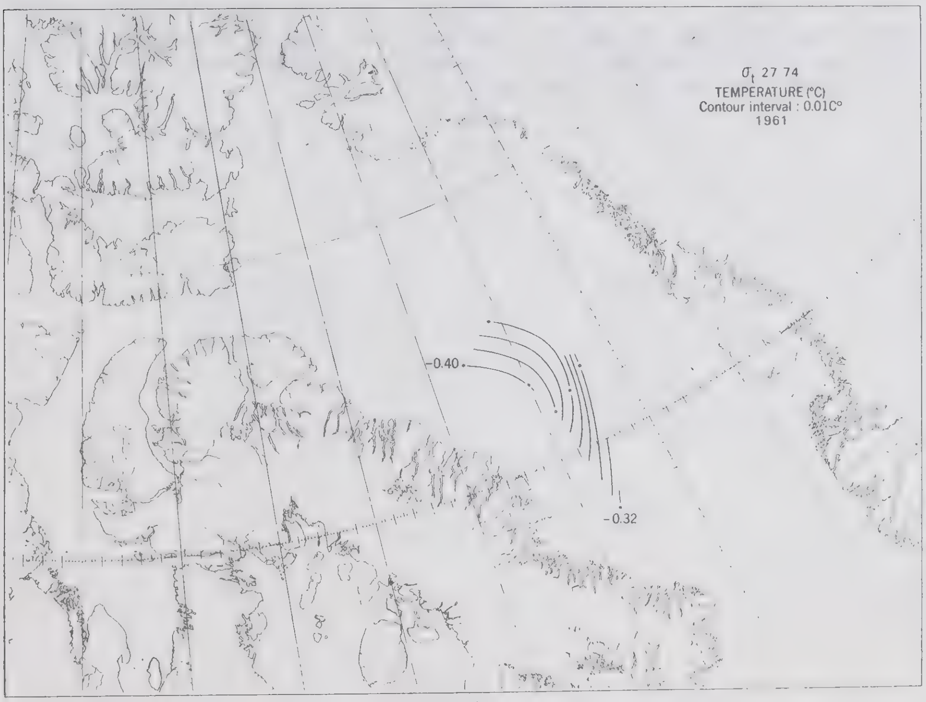
10(d)



σ_t 27.72
DEPTH (METRES)
Contour interval 50 m
1961
1500
1300
1200
σ_t 27.72
TEMPERATURE (°C)
Contour interval: 0.1C°
1961
-0.2
0.1
.06
.04
.02

10(e)

σ_t 27.74
DEPTH (METRES)
Contour interval: 100 m
1961
1800
1980
1750
1500

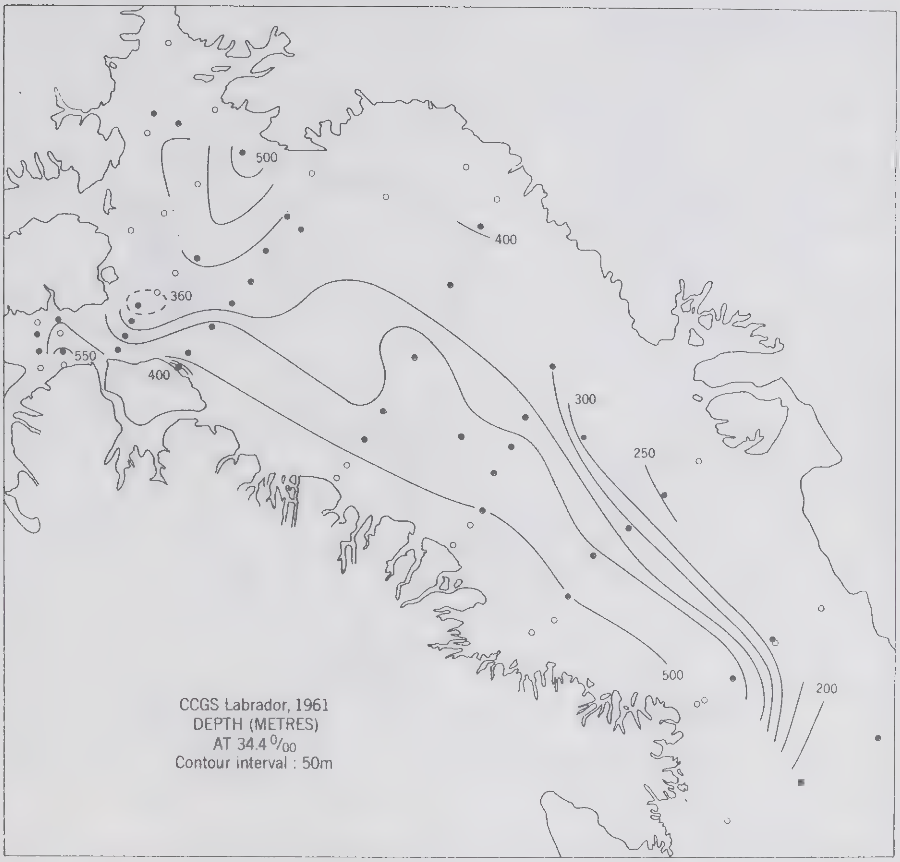


σ_t 27 74
TEMPERATURE (°C)
Contour interval : 0.01C°
1961
-0.40
-0.32

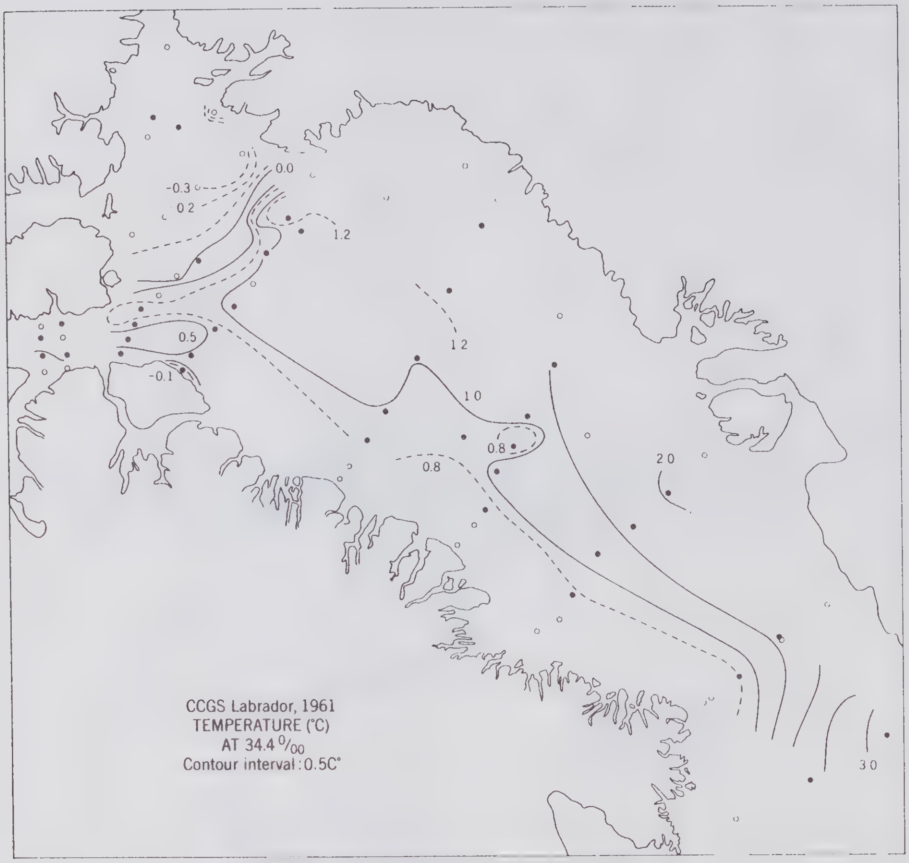
10(f)

Figure 11

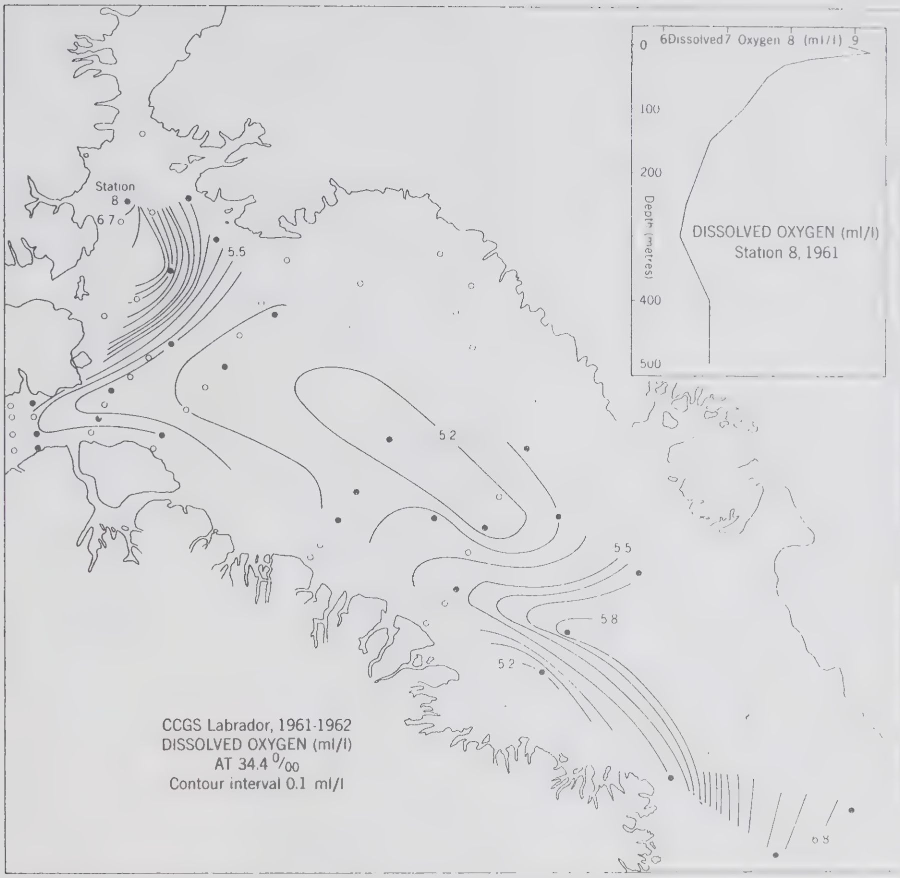
The distribution of depth, temperature and dissolved oxygen on the $34.4^{o}/oo$ surface. (a) Depth. (b) Temperature. (c) Dissolved oxygen (includes 1962 data).


500
400
360
550
400
300
250
500
200
CCGS Labrador, 1961
DEPTH (METRES)
AT 34.4 ‰
Contour interval : 50m

11(a)


CCGS Labrador, 1961
TEMPERATURE (°C)
AT 34.4 ‰
Contour interval: 0.5C°
0.0
-0.3
0.2
1.2
0.5
-0.1
1.2
1.0
0.8
0.8
2.0
3.0

11(b)

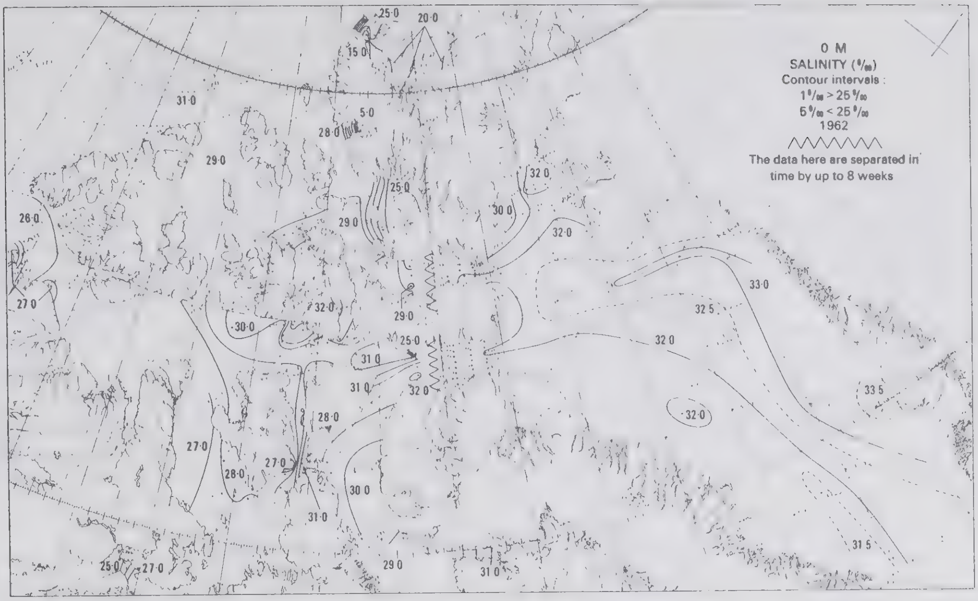

Station
8
6 7
5.5
5 2
5 5
5 8
5 2
6 8
CCGS Labrador, 1961-1962
DISSOLVED OXYGEN (ml/l)
AT 34.4 ‰
Contour interval 0.1 ml/l
Dissolved Oxygen (ml/l)
6
7
8
9
0
100
200
400
500
Depth (metres)
DISSOLVED OXYGEN (ml/l)
Station 8, 1961

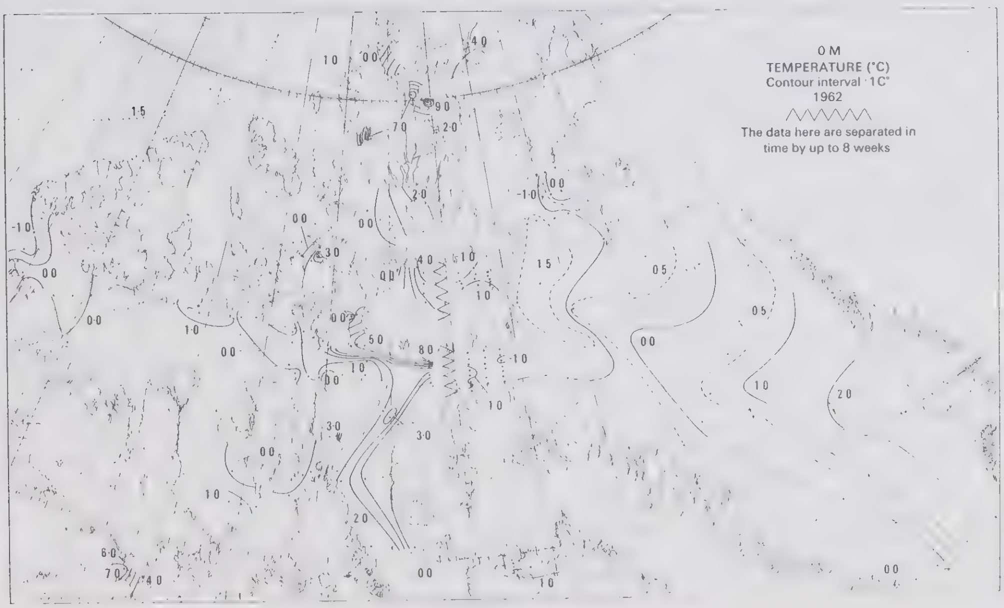
11(c)

PART B

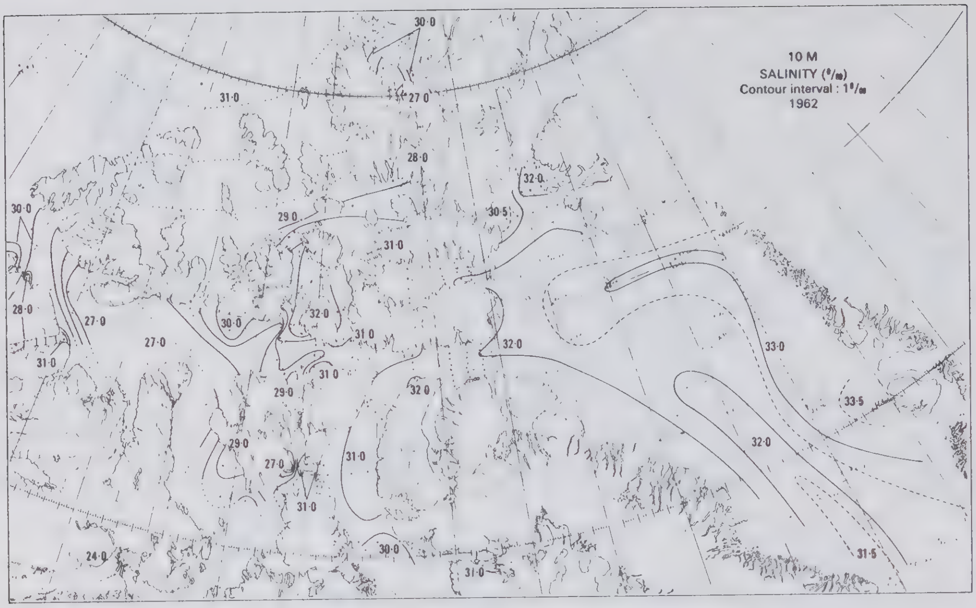
Figure 12

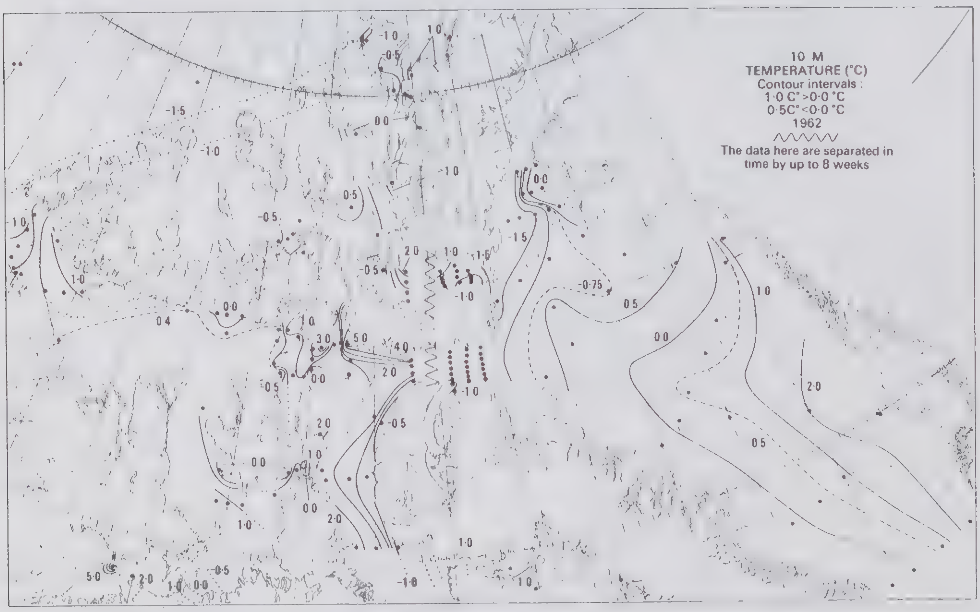
The distribution of salinity and temperature near the surface during 1962 from data observed from "John A. MacDonald", "Labrador", by the land-based parties and from the Ice Camps. (a) 0 m. (b) 10 m. (c) 20 m.


0 M
SALINITY (‰)
Contour intervals :
1‰ > 25‰
5‰ < 25‰
1962
The data here are separated in time by up to 8 weeks


0 M
TEMPERATURE (°C)
Contour interval 1C°
1962
The data here are separated in time by up to 8 weeks

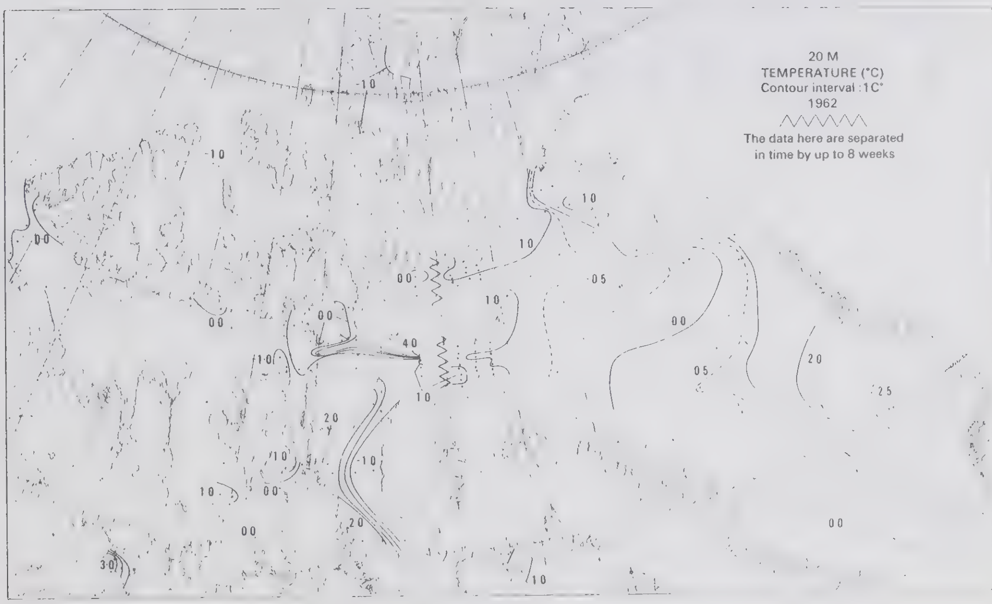
12(a)


10 M
SALINITY (‰)
Contour interval : 1‰
1962


10 M
TEMPERATURE (°C)
Contour intervals :
1·0 C°>0·0 °C
0·5C°<0·0 °C
1962
The data here are separated in
time by up to 8 weeks

12(b)

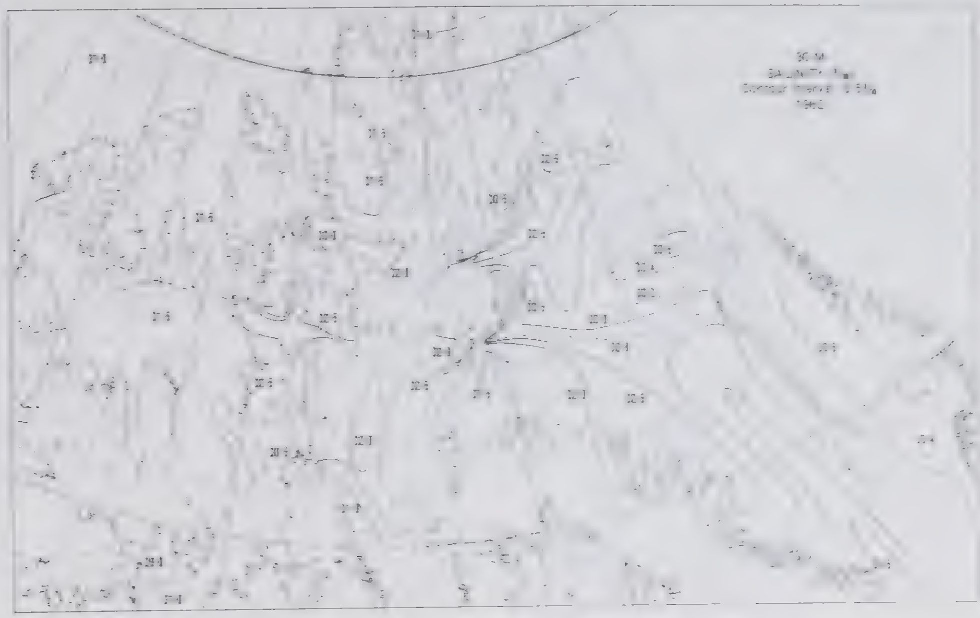

20 M
SALINITY (‰)
Contour interval : 1‰
1962

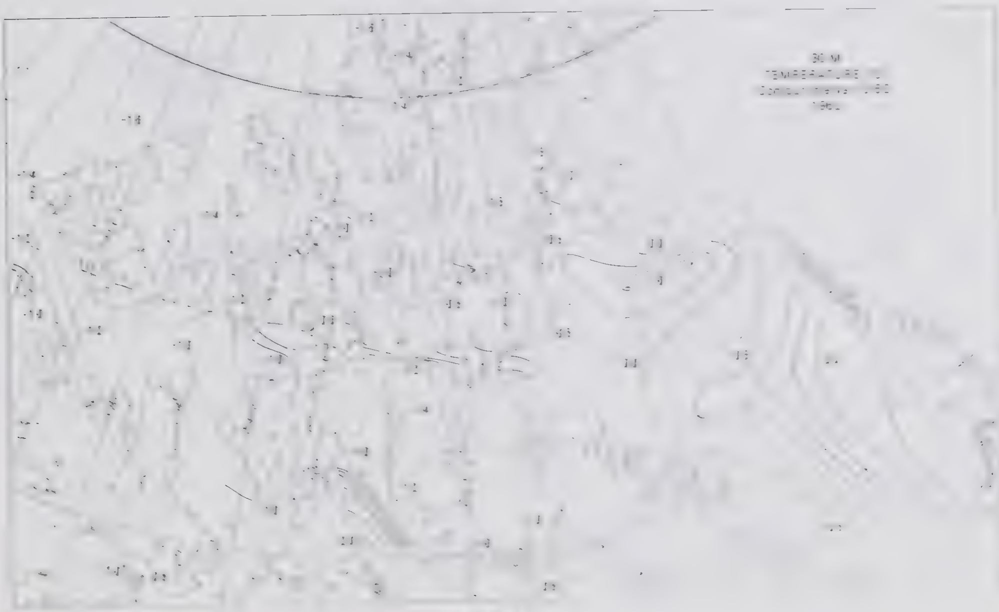

20 M
TEMPERATURE (°C)
Contour interval : 1C°
1962
The data here are separated
in time by up to 8 weeks

12(c)

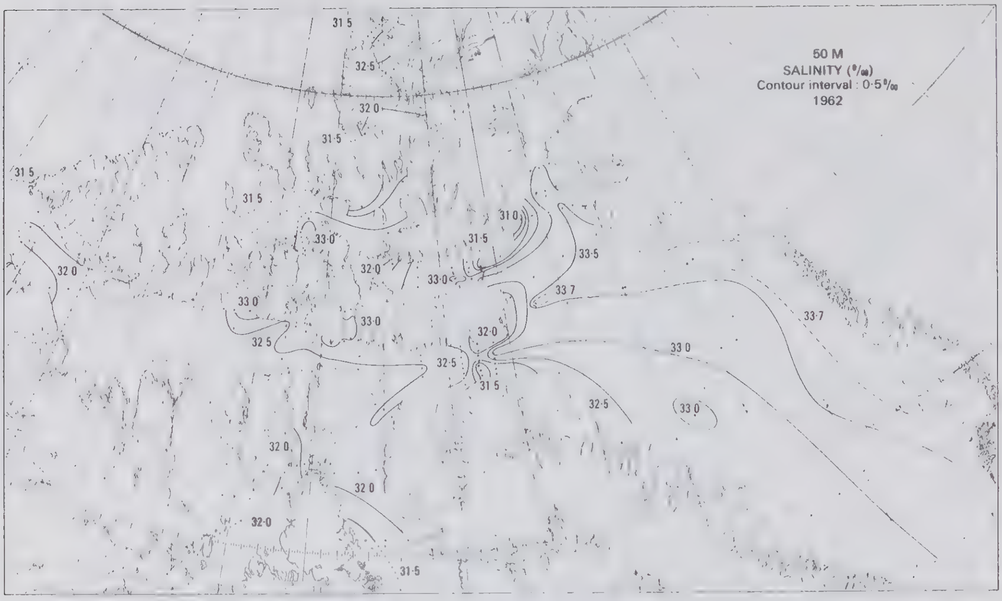
Figure 13

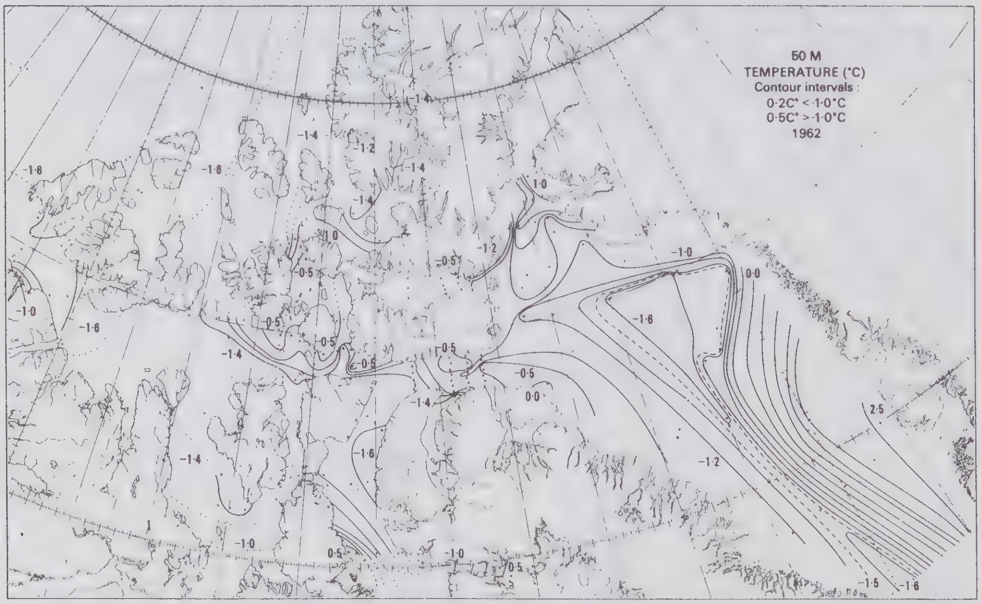
The distribution of salinity and temperature during 1962 from data observed on "John A. MacDonald", "Labrador", "Atka", and from the Ice Camps and by the land-based parties. On the distributions at 200 m and greater the hatched lines show the approximate limits of areas where the greatest depth in any section of the channel is less than the depth of the distribution. (a) 30 m. (b) 50 m. (c) 75 m. (d) 100 m. (e) 150 m. (f) 200 m. (g) 250 m. (h) 300 m. (i) 400 m. (j) 500 m. (k) 600 m. (l) 800 m. (m) 1000 m.



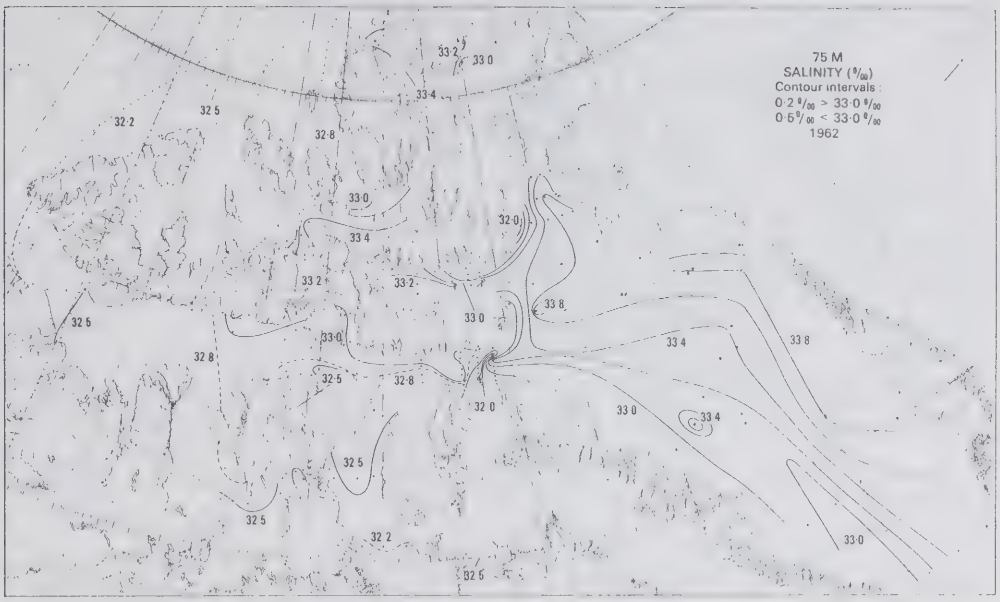


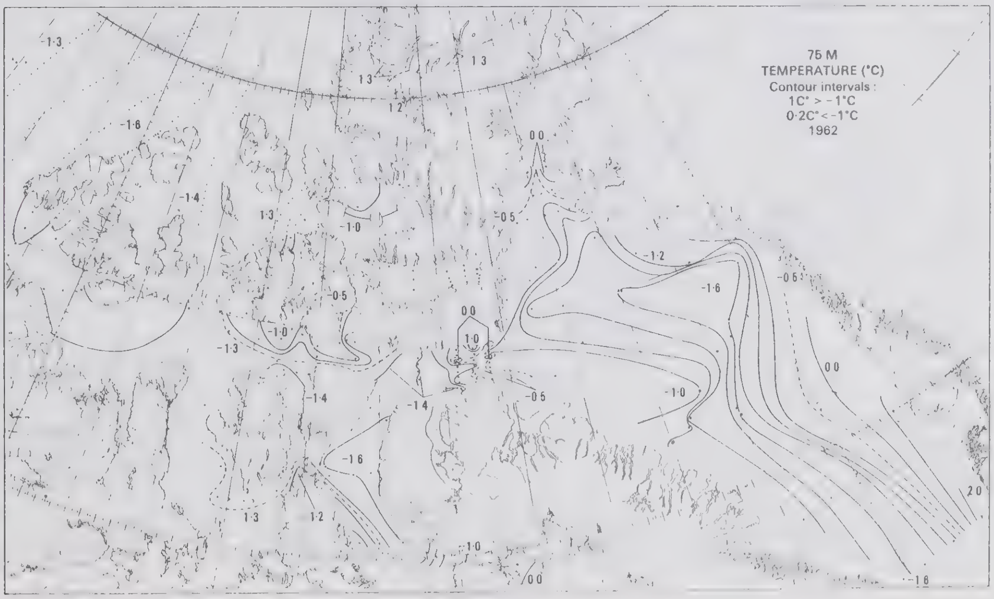
13(a)


50 M
SALINITY (‰)
Contour interval : 0·5 ‰
1962

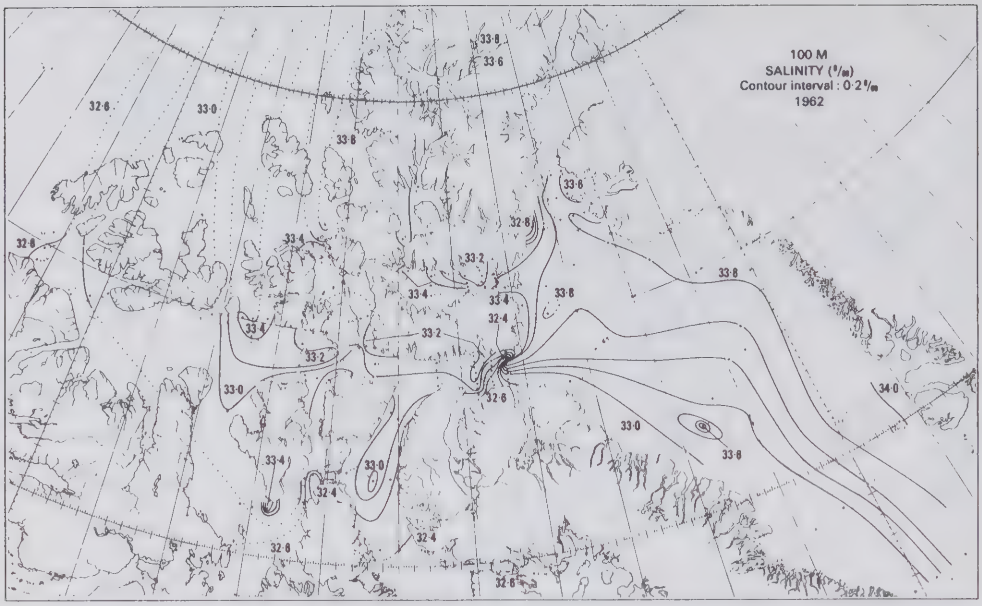

50 M
TEMPERATURE (°C)
Contour intervals :
0·2C° < -1·0°C
0·5C° > -1·0°C
1962

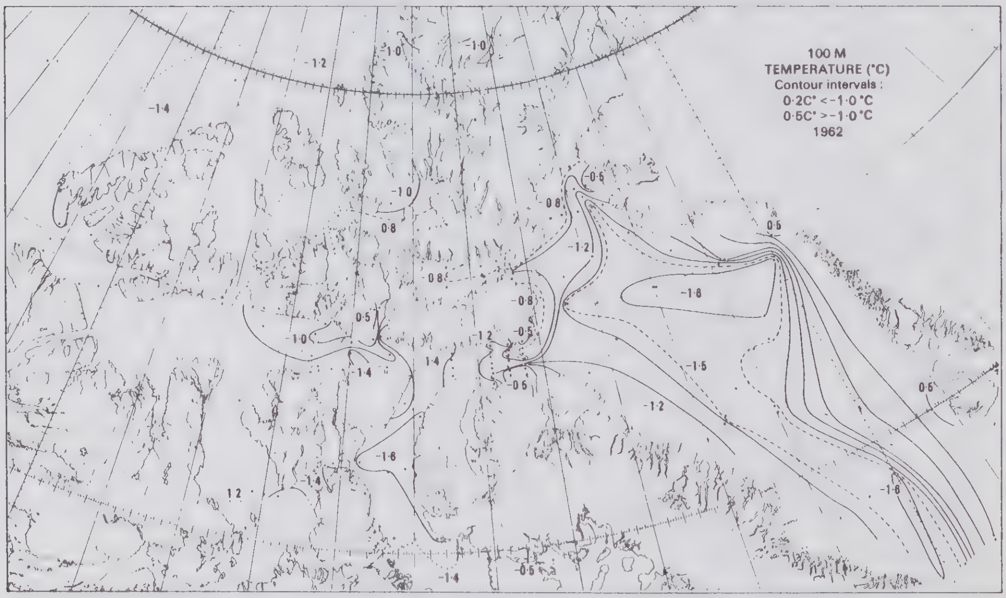
13(b)


75 M
SALINITY (‰)
Contour intervals:
0·2 ‰ > 33·0 ‰
0·5 ‰ < 33·0 ‰
1962

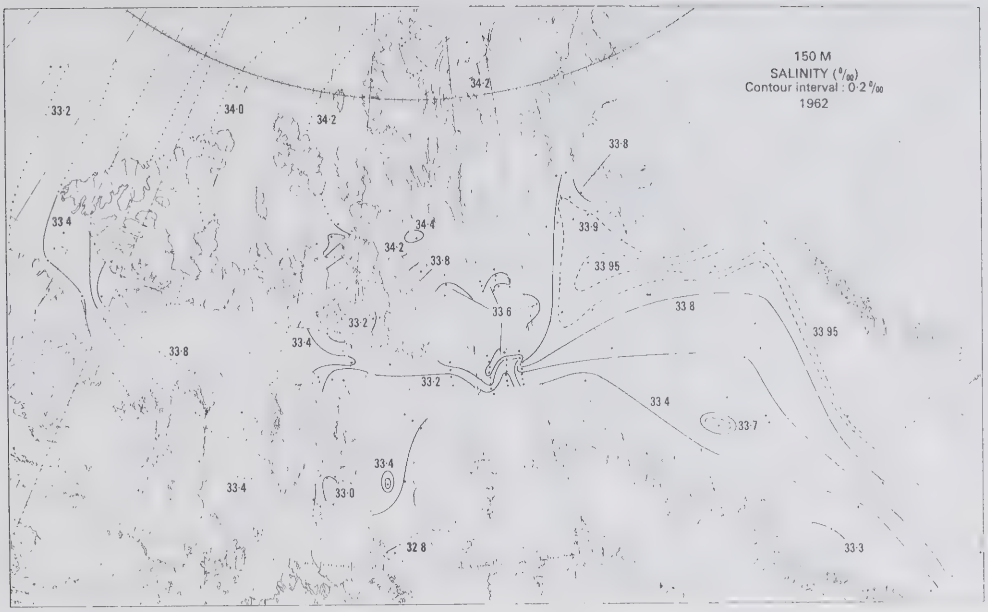

75 M
TEMPERATURE (°C)
Contour intervals:
1C° > −1°C
0·2C° < −1°C
1962

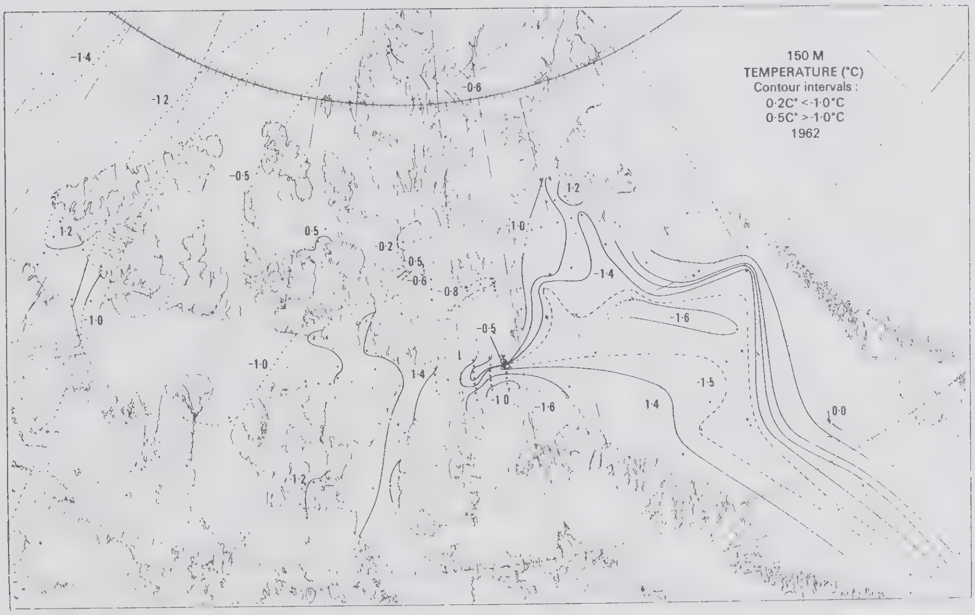
13(c)


100 M
SALINITY (‰)
Contour interval : 0·2‰
1962

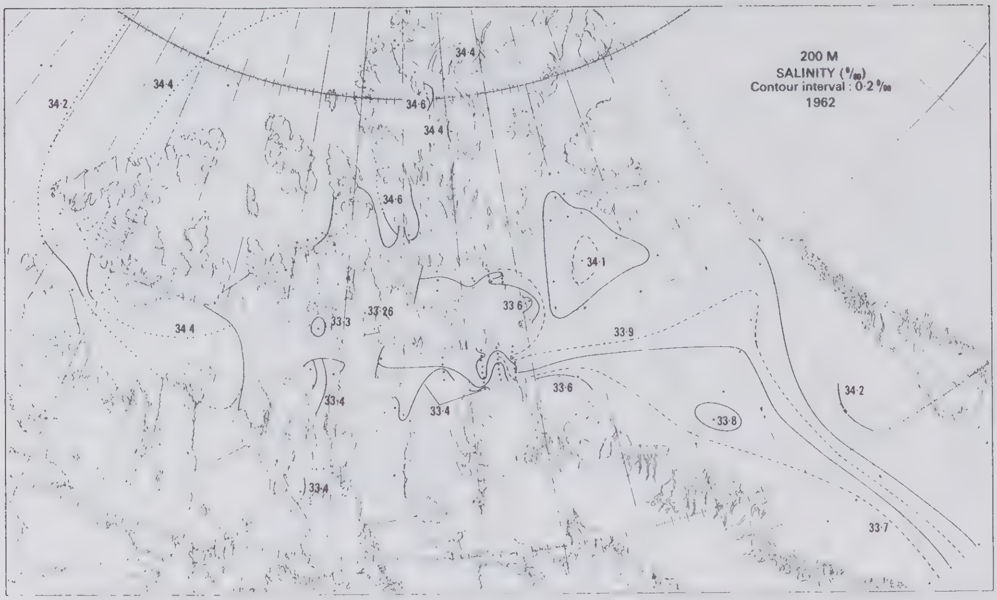

100 M
TEMPERATURE (°C)
Contour intervals :
0·2C° < −1·0 °C
0·5C° > −1·0 °C
1962

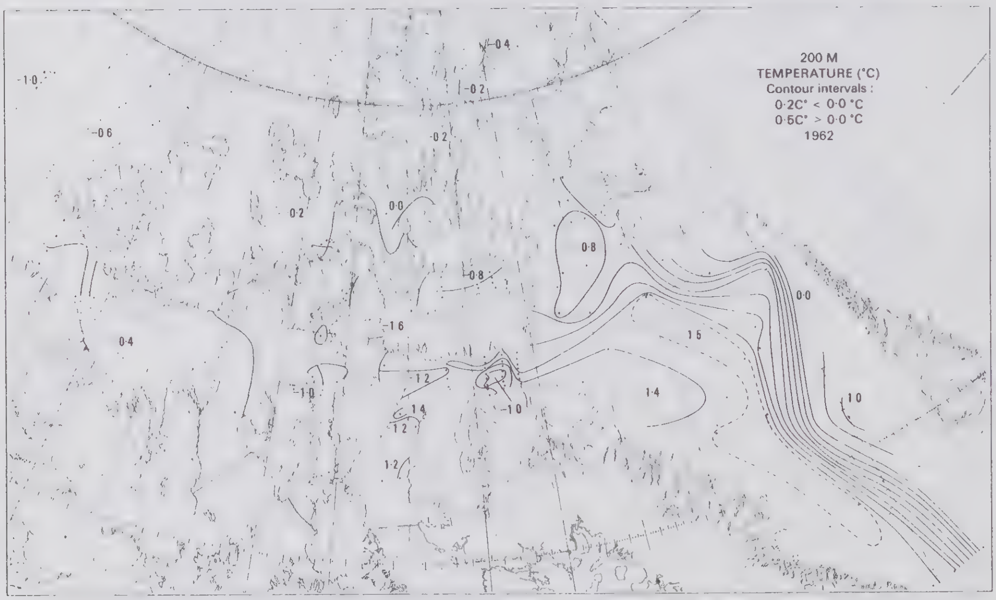
13 (d)


150 M
SALINITY (‰)
Contour interval : 0·2 ‰
1962
33·2
34·0
34·2
34·2
33·8
33·4
34·4
34·2
33·9
33·8
33 95
33·6
33·8
33·2
33·4
33·8
33 95
33·2
33 4
33·7
33·4
33·4
33·0
32 8
33·3


150 M
TEMPERATURE (°C)
Contour intervals :
0·2C° < -1·0°C
0·5C° > -1·0°C
1962
-1·4
-1·2
-0·6
-0·5
1·2
1·2
0·5
-1·0
0·2
0·5
-0·6
-0·8
-1·4
-1·6
-1·0
-0·5
-1·0
1·4
-1·5
-1·0
-1·6
1·4
0·0
1·2

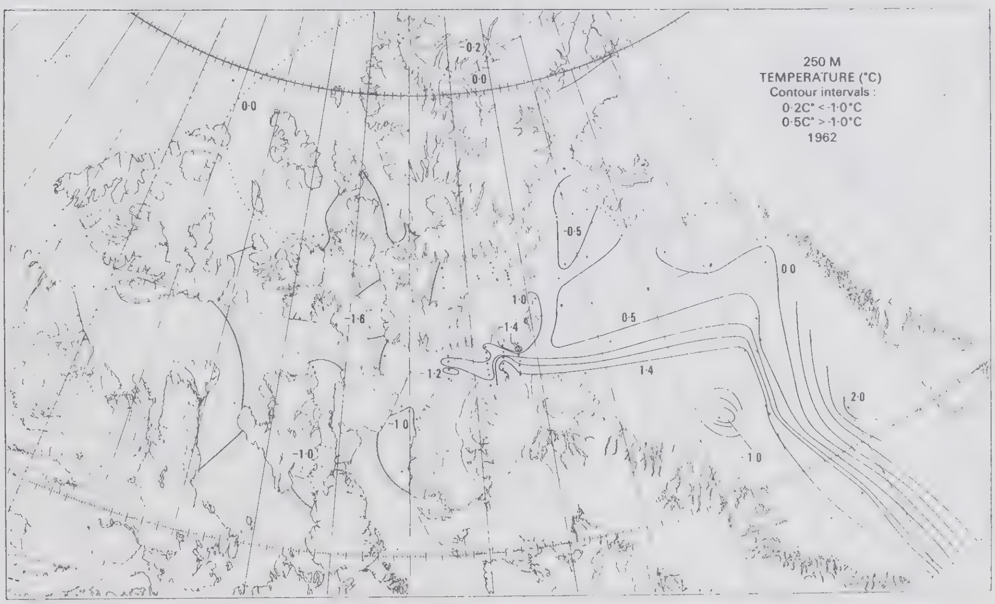
13(e)


200 M
SALINITY (‰)
Contour interval : 0·2 ‰
1962
34·4
34·2
34·6
34·1
33·3
33·26
33·6
33·9
33·4
34·2
33·8
33·7

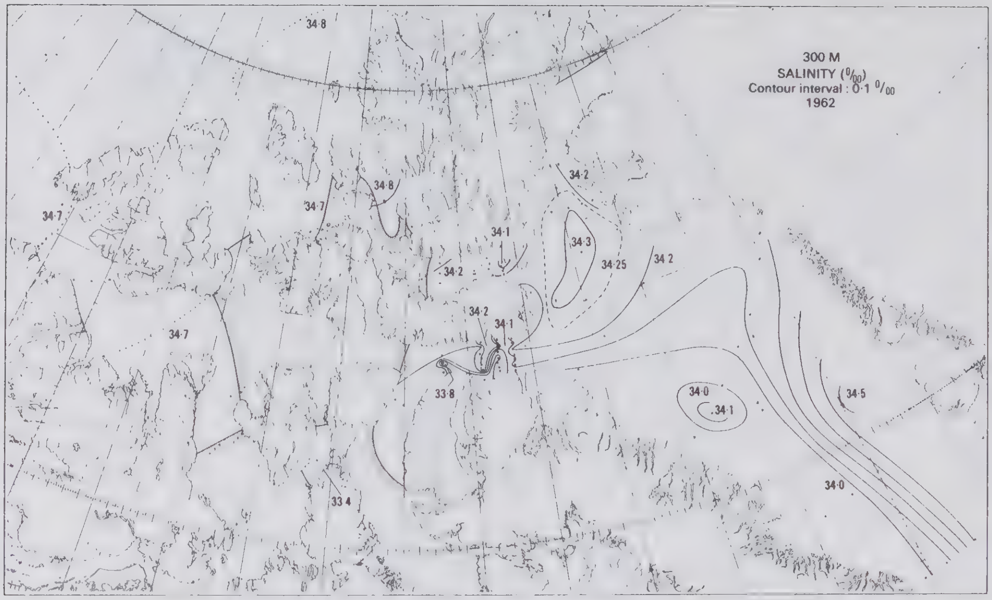

200 M
TEMPERATURE (°C)
Contour intervals :
0·2C° < 0·0 °C
0·5C° > 0·0 °C
1962
-1·0
-0·6
-0·4
-0·2
0·2
0·0
0·8
0·4
-1·6
1·5
1·4
1·0
1·2
-1·0
1·4

13(f)


250 M
SALINITY (‰)
Contour interval : 0·2 ‰
1962
34·4
34·6
34·6
34·4
34·1
34·2
34·0
34·0
33·4
34·0
34·1
34·1
33·9
33·8
33·8
33·6
34·2
33·9
34·0
33·4
33·6
33·8

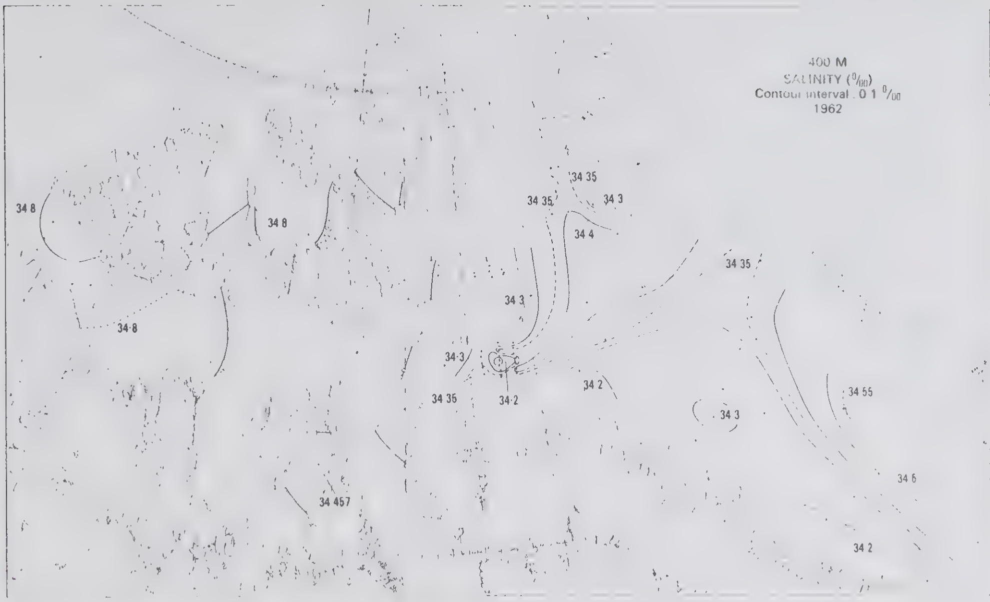

250 M
TEMPERATURE (°C)
Contour intervals :
0·2C° < -1·0°C
0·5C° > -1·0°C
1962
-0·2
0·0
0·0
-0·5
0·0
1·0
0·5
-1·6
-1·4
-1·2
1·4
2·0
-1·0
-1·0
1·0

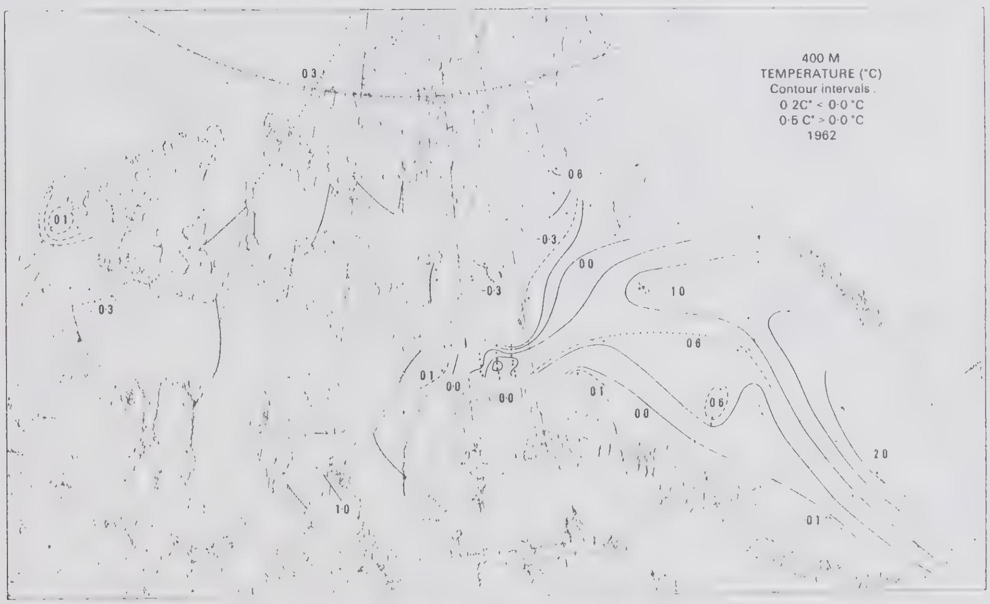
13 (g)


300 M
SALINITY (‰)
Contour interval : 0·1 ‰
1962
34·8
34·7
34·8
34·7
34·2
34·1
34·3
34·25
34 2
34·2
34·1
34·7
33·8
34·0
34·1
34·5
34·0
33 4

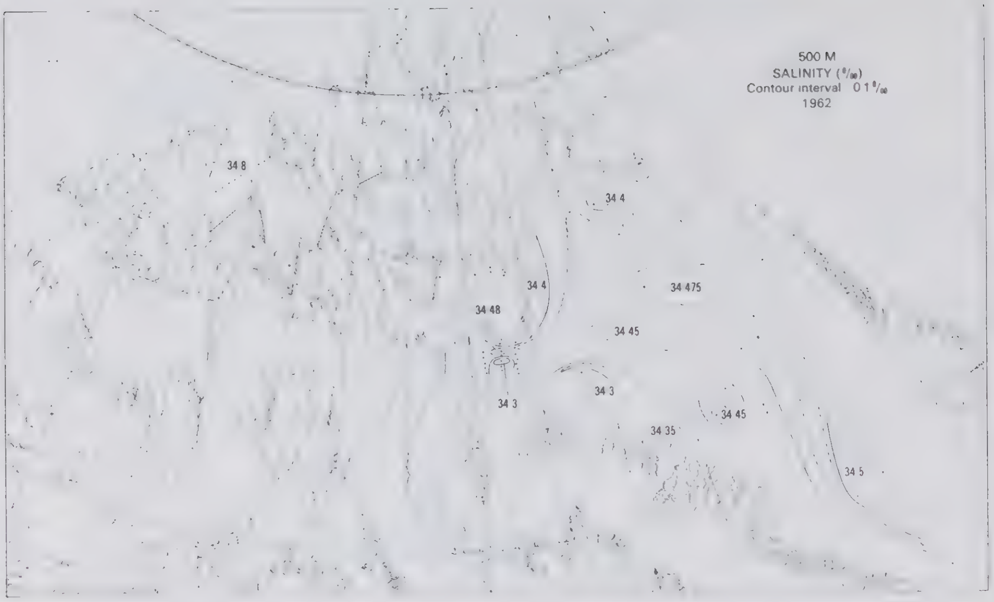

300 M
TEMPERATURE (°C)
Contour interval : 0·5C°
1962
0 2
0 2
0 0
0 2
0 2
0·5
0·2
0 0
0·5
0·5
0·2
0·5
1·0
1·0
1 0
0·5
1 0
2 0

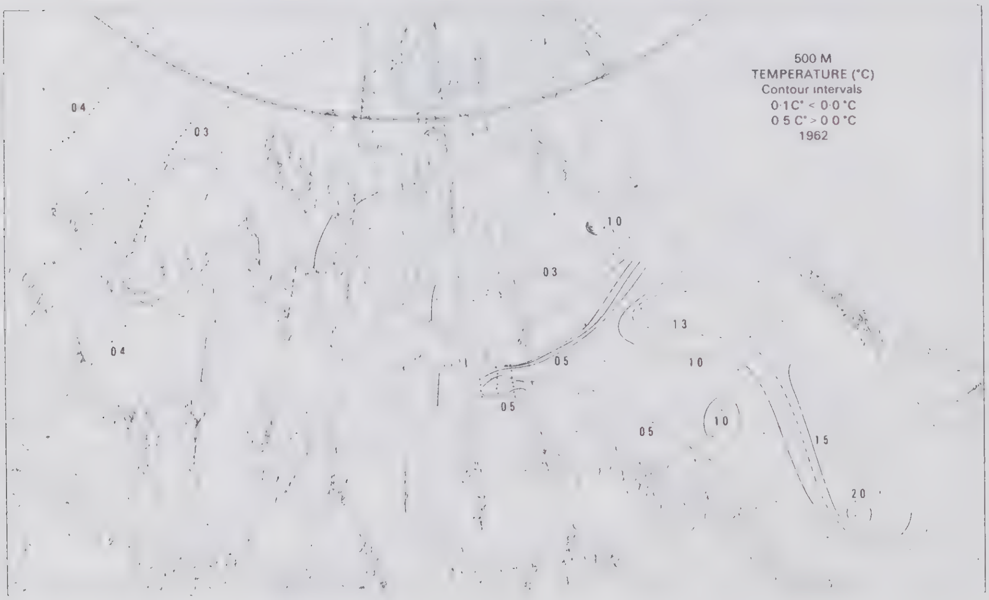
13(h)


400 M
SALINITY (‰)
Contour interval . 0 1 ‰
1962
34 8
34 8
34·8
34 35
34 35
34 3
34 4
34 35
34 3
34·3
34 35
34·2
34 2
34 55
34 3
34 6
34 457
34 2

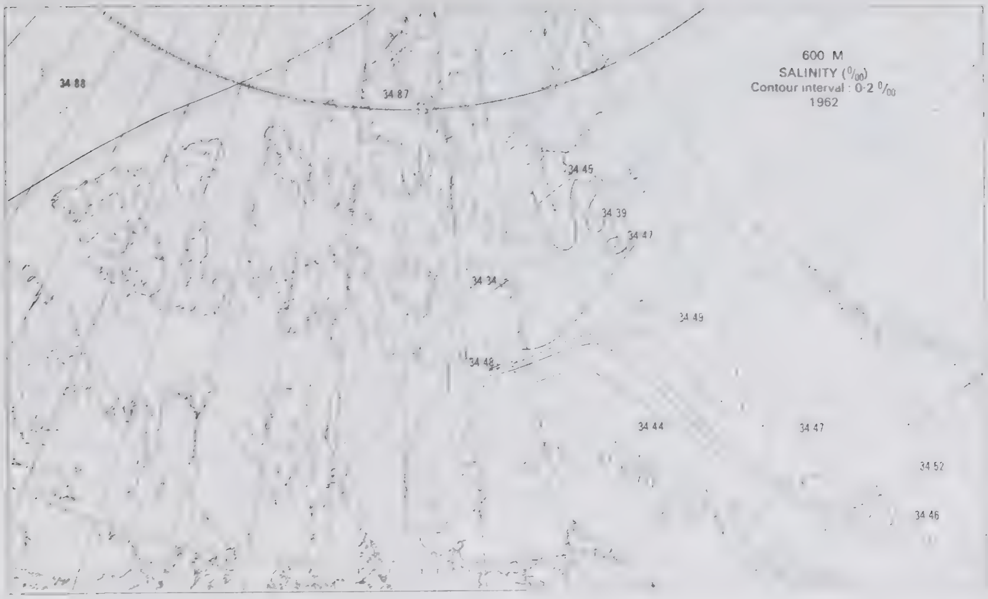

400 M
TEMPERATURE (°C)
Contour intervals .
0 2C° < 0·0 °C
0·5 C° > 0·0 °C
1962
0 3
0 6
0 1
-0·3
0 0
-0·3
1 0
0·3
0 6
0 1
0·0
0·0
0 1
0 0
0 6
2 0
1·0
0 1

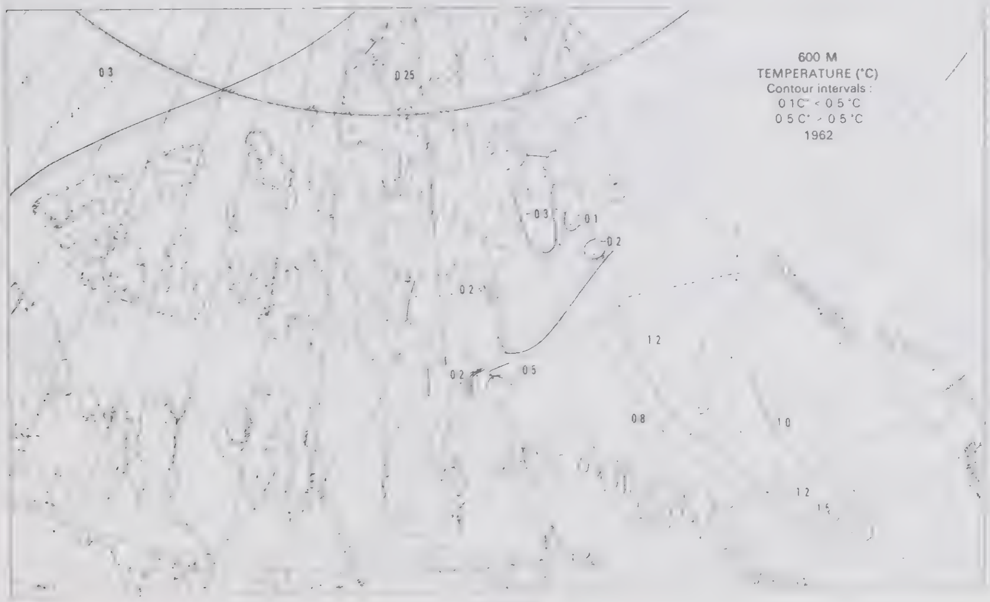
13(i)


500 M
SALINITY (‰)
Contour interval 0 1 ‰
1962
34 8
34 4
34 475
34 4
34 48
34 45
34 3
34 3
34 45
34 35
34 5

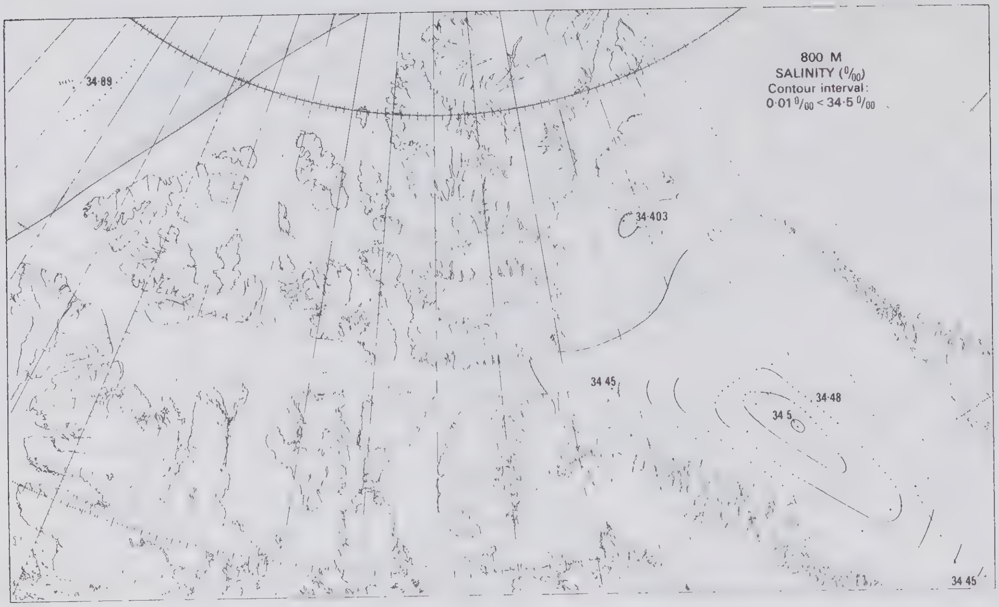

500 M
TEMPERATURE (°C)
Contour intervals
0·1 C° < 0·0 °C
0 5 C° > 0 0 °C
1962
0 4
0 3
1 0
0 3
1 3
0 4
0 5
1 0
0 5
1 0
0 5
1 5
2 0

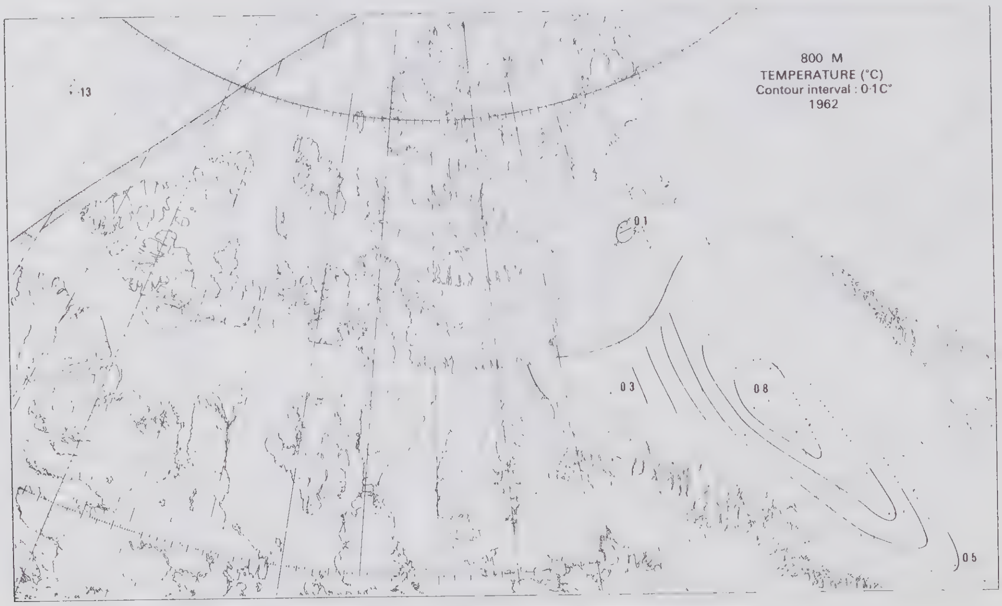
13(j)

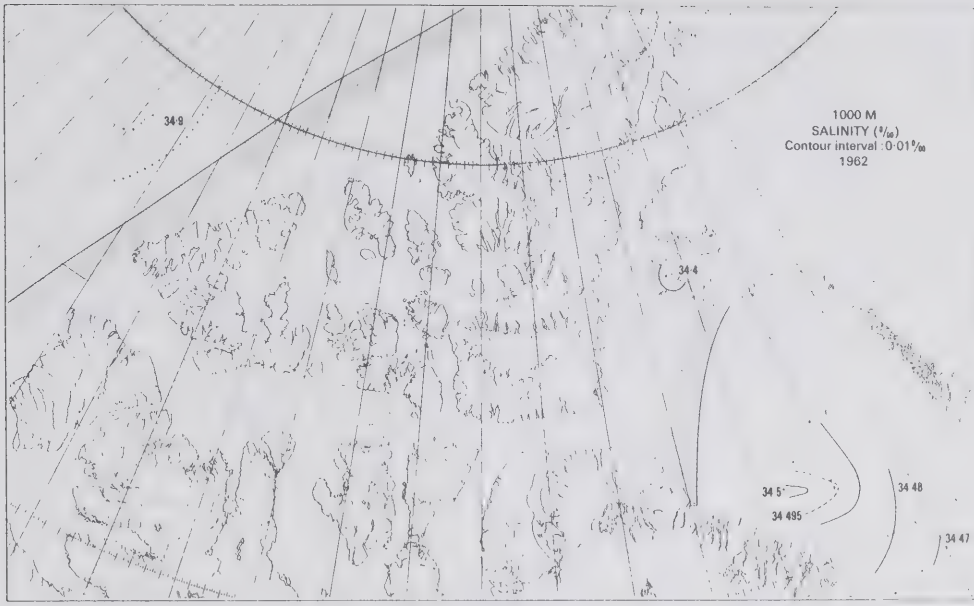

600 M
SALINITY (‰)
Contour interval : 0·2 ‰
1962
34.88
34.87
34.45
34.39
34.47
34.34
34.49
34.48
34.44
34.47
34.52
34.46

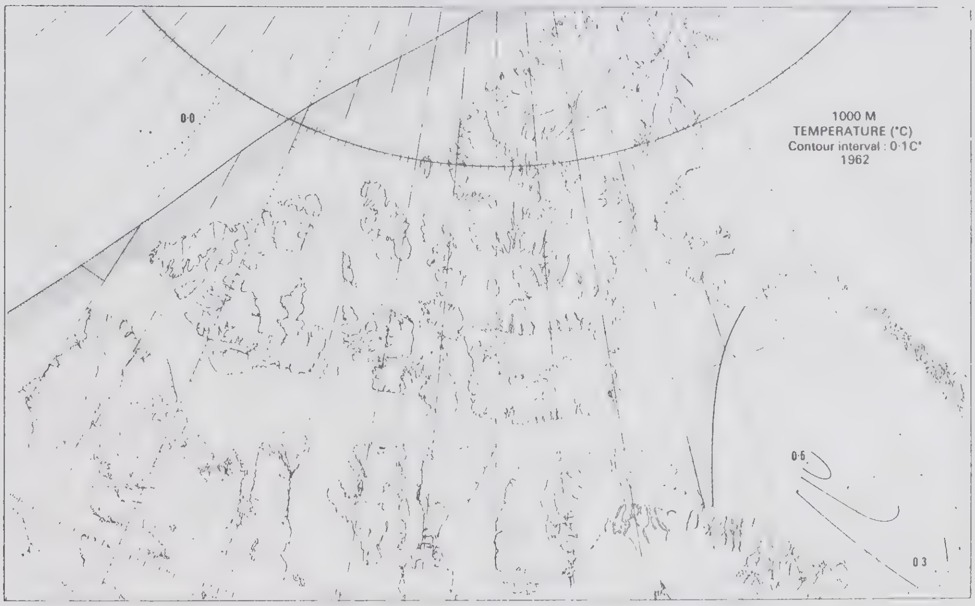

600 M
TEMPERATURE (°C)
Contour intervals :
0 1C° < 0 5 °C
0 5 C° > 0 5 °C
1962
0 3
0 25
-0 3
0 1
-0 2
0 2
1 2
0 2
0 5
0 8
1 0
1 2

13(k)


800 M
SALINITY ($^0/_{00}$)
Contour interval:
0·01 $^0/_{00}$ < 34·5 $^0/_{00}$
34·89
34·403
34 45
34·48
34 5
34 45


800 M
TEMPERATURE (°C)
Contour interval : 0·1C°
1962
·13
0 1
0 3
0 8
0 5

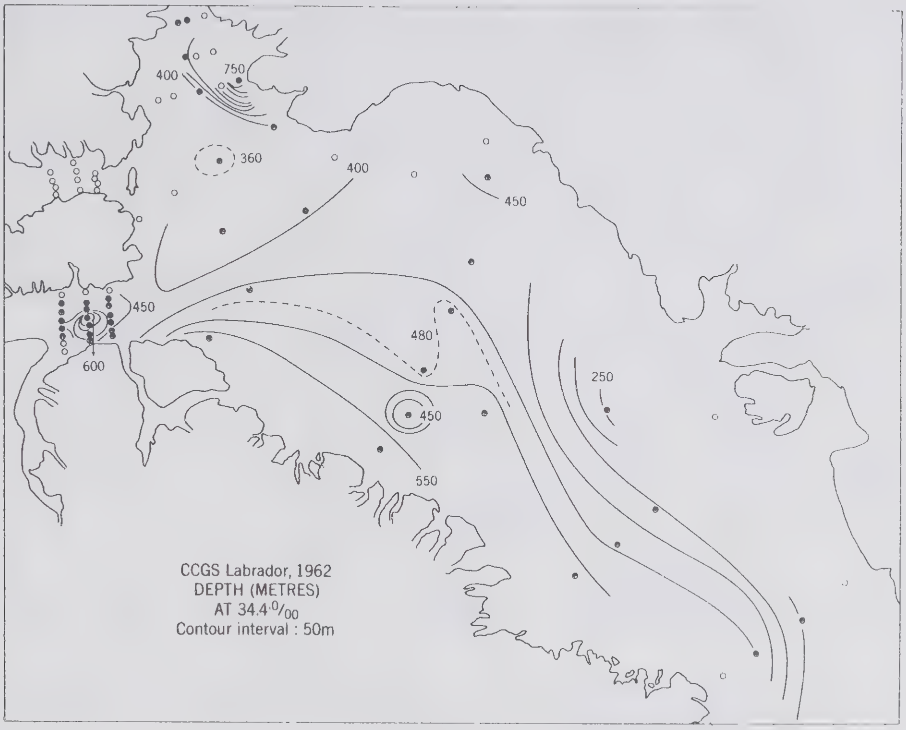

1000 M
SALINITY (‰)
Contour interval : 0·01‰
1962
34·9
34·4
34·5
34·495
34·48
34·47


1000 M
TEMPERATURE (°C)
Contour interval : 0·1C°
1962
0·0
0·5
0·3

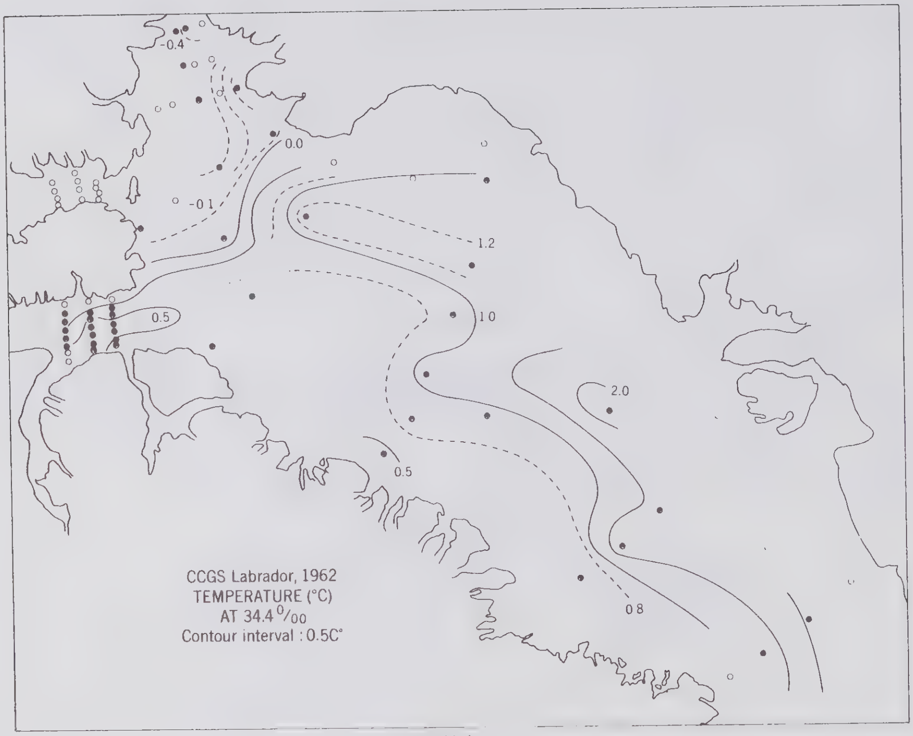
13(m)

Figure 14

The distribution of depth and temperature on the 34.4°/oo surface. (a) Depth. (b) Temperature.


400
750
360
400
450
450
480
600
250
450
550
CCGS Labrador, 1962
DEPTH (METRES)
AT 34.4 ‰
Contour interval : 50m

14(a)



-0.4
0.0
-0.1
1.2
0.5
1.0
2.0
0.5
0.8
CCGS Labrador, 1962
TEMPERATURE (°C)
AT 34.4 ‰
Contour interval : 0.5C°

14(b)

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No.22

Numerical values of the conical function $K_p(x)$ for a range of values of order p and argument x and their zeros and bend points

T.S. Murty, P.J. Richards and J.D. Taylor

1971

Marine Sciences Branch
Department of the Environment, Ottawa

Manuscript Report Series No.22

NUMERICAL VALUES OF THE CONICAL FUNCTION $K_p(X)$ FOR A RANGE OF VALUES OF ORDER P AND ARGUMENT X AND THEIR ZEROS AND BEND POINTS

T.S. Murty, P.J. Richards and J.D. Taylor

1971

0. Abstract

The conical functions $K_p(x)$ arise in many problems, e.g., in calculations of the effect of rotation on gravity mode frequencies in laboratory basins. Since the numerical values of these functions are not tabulated to cover all the ranges of interest, these were computed and tabulated here.

1. Introduction

The Legendre function $P_\nu(x)$ becomes a conical function $K_p(x)$ for $\nu = \frac{-1}{2} + ip$ where p is real. Usually these are referred to as Mehler's conical functions with the argument $x \geq 1$ (Hobson, 1931). Hobson gives the following infinite integral:

$$K_p(x) = \frac{\sqrt{2}.\ ch(p\ \pi)}{\pi} \int_0^\infty \frac{\cos(pt).dt}{(x+ch.t)^{1/2}} \tag{1}$$

However this formula was found to suffer severely from rounding errors for p>6 even when double precision is used. Hence the procedure described below is used.

2. Mathematical problem

For the range $1<x\leq 10$ for the argument and $0\leq p\leq 10$ for the order we used the following formula (Erdelyi, 1953).

$$P_\nu(x) = \frac{1}{\pi}\int_0^\pi (x+\sqrt{x^2-1}\cos t)^\nu\, dt \tag{2}$$

But $$K_p(x) = P_{\frac{-1}{2}+ip}(x) \tag{3}$$

Define $x + \sqrt{x^2-1}\cos t \equiv U$ (4)

Substituting (4) and (2) into (3) one gets

$$K_p(x) = \frac{1}{\pi}\int_0^{\pi} U^{-\frac{1}{2}+ip}\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{U^{ip}}{U^{1/2}}\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{\cos(p.\log U)}{U^{1/2}}\,dt \qquad (5)$$

The derivative of $K_p(x)$ is given by

$$K_p'(x) = \frac{1}{\pi}\int_0^{\pi} \frac{d}{dx} \quad \frac{\cos(p.\log U)}{U^{1/2}}\,dt$$

After some algebra this becomes

$$K_p'(x) = \frac{-1}{\pi}\int_0^{\pi} \frac{1}{U^{3/2}}\left(\frac{\cos(p.\log U)}{2} + p.\sin(p.\log U)\right)\left(1+\frac{x.\cos t}{\sqrt{x^2-1}}\right)dt \qquad (6)$$

3. Numerical procedure

The integrands of the expressions for $K_p(x)$ and $K_p'(x)$ are even functions of t at both t=0 and t=π. Hence the end corrections for the trapezoidal integration rule will be identically zero, as these corrections consist of odd differences only. The simple trapezoidal rule was used here to evaluate both $K_p(x)$ and $K_p'(x)$.

We found it necessary to use a different number of points for different ranges of the argument to obtain twelve

digits after the decimal point.

$1<x\leq 4$	no. of points	100
$4<x\leq 8$		200
$8<x\leq 10$		300

For x=1 the formula (6) for $K_p'(x)$ has a pole and hence cannot be used. In this case for $x\leq 1+10^{-10}$ the following formulae which are derived from the hypergeometric formula for $P_\nu(x)$ are used

$$K_p(x) = 1.0 \qquad (7)$$

$$K_p'(x) = -\left(\frac{p^2+0.25}{2}\right)$$

4. Results

The results are arranged in two sets of tables. The first table lists the value of the conical function $K_p(x)$ for $x = 1.0(0.4)9.8$ and for $p = 0.1(0.1)10.0$ to 8 decimal places. The second table lists the zeros, value of the derivative at the zeros, bend points and value of the function at the bend points, again to 8 decimal places.

Table 1

The value of the conical function $K_p(x)$ for $x = 1.0(0.4)9.8$ and for $p = 0.1(0.1)10.0$ to 8 decimal places.

P= 0.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.98735495D 00	1.2	0.97537407D 00	1.3	0.96399576D 00
1.4	0.95316646D 00	1.5	0.94283932D 00	1.6	0.93297313D 00	1.7	0.92353144D 00
1.8	0.91448189D 00	1.9	0.90579561D 00	2.0	0.89744673D 00	2.1	0.88941201D 00
2.2	0.88167046D 00	2.3	0.87420309D 00	2.4	0.86699265D 00	2.5	0.86002345D 00
2.6	0.85328114D 00	2.7	0.84675261D 00	2.8	0.84042580D 00	2.9	0.83428964D 00
3.0	0.82833391D 00	3.1	0.82254918D 00	3.2	0.81692674D 00	3.3	0.81145847D 00
3.4	0.80613686D 00	3.5	0.80095493D 00	3.6	0.79590614D 00	3.7	0.79098442D 00
3.8	0.78618408D 00	3.9	0.78149980D 00	4.0	0.77692658D 00	4.1	0.77245975D 00
4.2	0.76809490D 00	4.3	0.76382789D 00	4.4	0.75965483D 00	4.5	0.75557203D 00
4.6	0.75157602D 00	4.7	0.74766353D 00	4.8	0.74383145D 00	4.9	0.74007684D 00
5.0	0.73639692D 00	5.1	0.73278904D 00	5.2	0.72925070D 00	5.3	0.72577952D 00
5.4	0.72237322D 00	5.5	0.71902967D 00	5.6	0.71574679D 00	5.7	0.71252264D 00
5.8	0.70935535D 00	5.9	0.70624313D 00	6.0	0.70318428D 00	6.1	0.70017719D 00
6.2	0.69722028D 00	6.3	0.69431209D 00	6.4	0.69145116D 00	6.5	0.68863616D 00
6.6	0.68586576D 00	6.7	0.68313871D 00	6.8	0.68045380D 00	6.9	0.67780988D 00
7.0	0.67520585D 00	7.1	0.67264062D 00	7.2	0.67011317D 00	7.3	0.66762252D 00
7.4	0.66516771D 00	7.5	0.66274783D 00	7.6	0.66036200D 00	7.7	0.65800937D 00
7.8	0.65568911D 00	7.9	0.65340043D 00	8.0	0.65114259D 00	8.1	0.64891482D 00
8.2	0.64671643D 00	8.3	0.64454674D 00	8.4	0.64240506D 00	8.5	0.64029077D 00
8.6	0.63820324D 00	8.7	0.63614187D 00	8.8	0.63410607D 00	8.9	0.63209530D 00
9.0	0.63010899D 00	9.1	0.62814663D 00	9.2	0.62620770D 00	9.3	0.62429171D 00
9.4	0.62239817D 00	9.5	0.62052662D 00	9.6	0.61867661D 00	9.7	0.61684769D 00
9.8	0.61503944D 00	9.9	0.61325145D 00	10.0	0.61148332D 00		

P= 0.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.98590109D 00	1.2	0.97255246D 00	1.3	0.95988419D 00
1.4	0.94783552D 00	1.5	0.93635329D 00	1.6	0.92539079D 00	1.7	0.91490672D 00
1.8	0.90486440D 00	1.9	0.89523114D 00	2.0	0.88597764D 00	2.1	0.87707760D 00
2.2	0.86850727D 00	2.3	0.86024517D 00	2.4	0.85227179D 00	2.5	0.84456940D 00
2.6	0.83712178D 00	2.7	0.82991410D 00	2.8	0.82293276D 00	2.9	0.81616526D 00
3.0	0.80960006D 00	3.1	0.80322652D 00	3.2	0.79703479D 00	3.3	0.79101575D 00
3.4	0.78516092D 00	3.5	0.77946241D 00	3.6	0.77391288D 00	3.7	0.76850547D 00
3.8	0.76323379D 00	3.9	0.75809182D 00	4.0	0.75307397D 00	4.1	0.74817495D 00
4.2	0.74338981D 00	4.3	0.73871391D 00	4.4	0.73414284D 00	4.5	0.72967249D 00
4.6	0.72529893D 00	4.7	0.72101850D 00	4.8	0.71682769D 00	4.9	0.71272322D 00
5.0	0.70870195D 00	5.1	0.70476092D 00	5.2	0.70089732D 00	5.3	0.69710846D 00
5.4	0.69339182D 00	5.5	0.68974496D 00	5.6	0.68616560D 00	5.7	0.68265152D 00
5.8	0.67920065D 00	5.9	0.67581098D 00	6.0	0.67248061D 00	6.1	0.66920771D 00
6.2	0.66599055D 00	6.3	0.66282745D 00	6.4	0.65971683D 00	6.5	0.65665715D 00
6.6	0.65364695D 00	6.7	0.65068482D 00	6.8	0.64775942D 00	6.9	0.64489946D 00
7.0	0.64207368D 00	7.1	0.63929091D 00	7.2	0.63654998D 00	7.3	0.63384980D 00
7.4	0.63118930D 00	7.5	0.62856747D 00	7.6	0.62598331D 00	7.7	0.62343587D 00
7.8	0.62092425D 00	7.9	0.61844755D 00	8.0	0.61600494D 00	8.1	0.61359557D 00
8.2	0.61121867D 00	8.3	0.60887347D 00	8.4	0.60655922D 00	8.5	0.60427521D 00
8.6	0.60202075D 00	8.7	0.59979517D 00	8.8	0.59759781D 00	8.9	0.59542806D 00
9.0	0.59328531D 00	9.1	0.59116896D 00	9.2	0.58907845D 00	9.3	0.58701323D 00
9.4	0.58497275D 00	9.5	0.58295651D 00	9.6	0.58096399D 00	9.7	0.57899472D 00
9.8	0.57704821D 00	9.9	0.57512400D 00	10.0	0.57322165D 00		

P= 0.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.98348038D 00	1.2	0.96785888D 00	1.3	0.95305114D 00
1.4	0.93898390D 00	1.5	0.92559312D 00	1.6	0.91282252D 00	1.7	0.90062240D 00
1.8	0.88894865D 00	1.9	0.87776196D 00	2.0	0.86702713D 00	2.1	0.85671255D 00
2.2	0.84678973D 00	2.3	0.83723291D 00	2.4	0.82801869D 00	2.5	0.81912581D 00
2.6	0.81053486D 00	2.7	0.80222807D 00	2.8	0.79418918D 00	2.9	0.78640322D 00
3.0	0.77885641D 00	3.1	0.77153603D 00	3.2	0.76443033D 00	3.3	0.75752839D 00
3.4	0.75082012D 00	3.5	0.74429610D 00	3.6	0.73794758D 00	3.7	0.73176638D 00
3.8	0.72574489D 00	3.9	0.71987596D 00	4.0	0.71415291D 00	4.1	0.70856948D 00
4.2	0.70311976D 00	4.3	0.69779823D 00	4.4	0.69259967D 00	4.5	0.68751918D 00
4.6	0.68255212D 00	4.7	0.67769412D 00	4.8	0.67294104D 00	4.9	0.66828897D 00
5.0	0.66373420D 00	5.1	0.65927322D 00	5.2	0.65490270D 00	5.3	0.65061948D 00
5.4	0.64642053D 00	5.5	0.64230301D 00	5.6	0.63826419D 00	5.7	0.63430148D 00
5.8	0.63041240D 00	5.9	0.62659460D 00	6.0	0.62284582D 00	6.1	0.61916391D 00
6.2	0.61554682D 00	6.3	0.61199259D 00	6.4	0.60849933D 00	6.5	0.60506524D 00
6.6	0.60168860D 00	6.7	0.59836774D 00	6.8	0.59510110D 00	6.9	0.59188713D 00
7.0	0.58872439D 00	7.1	0.58561146D 00	7.2	0.58254700D 00	7.3	0.57952971D 00
7.4	0.57655834D 00	7.5	0.57363169D 00	7.6	0.57074861D 00	7.7	0.56790797D 00
7.8	0.56510871D 00	7.9	0.56234979D 00	8.0	0.55963021D 00	8.1	0.55694900D 00
8.2	0.55430525D 00	8.3	0.55169804D 00	8.4	0.54912652D 00	8.5	0.54658984D 00
8.6	0.54408720D 00	8.7	0.54161781D 00	8.8	0.53918091D 00	8.9	0.53677577D 00
9.0	0.53440168D 00	9.1	0.53205796D 00	9.2	0.52974393D 00	9.3	0.52745895D 00
9.4	0.52520240D 00	9.5	0.52297367D 00	9.6	0.52077217D 00	9.7	0.51859733D 00
9.8	0.51644861D 00	9.9	0.51432546D 00	10.0	0.51222736D 00		

P= 0.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.98009639D 00	1.2	0.96130693D 00	1.3	0.94352585D 00
1.4	0.92666133D 00	1.5	0.91063321D 00	1.6	0.89537108D 00	1.7	0.88081283D 00
1.8	0.86690335D 00	1.9	0.85359358D 00	2.0	0.84083964D 00	2.1	0.82860214D 00
2.2	0.81684561D 00	2.3	0.80553797D 00	2.4	0.79465016D 00	2.5	0.78415576D 00
2.6	0.77403065D 00	2.7	0.76425281D 00	2.8	0.75480207D 00	2.9	0.74565988D 00
3.0	0.73680919D 00	3.1	0.72823425D 00	3.2	0.71992052D 00	3.3	0.71185455D 00
3.4	0.70402383D 00	3.5	0.69641677D 00	3.6	0.68902256D 00	3.7	0.68183113D 00
3.8	0.67483310D 00	3.9	0.66801967D 00	4.0	0.66138263D 00	4.1	0.65491426D 00
4.2	0.64860732D 00	4.3	0.64245503D 00	4.4	0.63645098D 00	4.5	0.63058915D 00
4.6	0.62486387D 00	4.7	0.61926977D 00	4.8	0.61380180D 00	4.9	0.60845515D 00
5.0	0.60322531D 00	5.1	0.59810797D 00	5.2	0.59309906D 00	5.3	0.58819472D 00
5.4	0.58339127D 00	5.5	0.57868524D 00	5.6	0.57407329D 00	5.7	0.56955227D 00
5.8	0.56511917D 00	5.9	0.56077111D 00	6.0	0.55650536D 00	6.1	0.55231930D 00
6.2	0.54821044D 00	6.3	0.54417638D 00	6.4	0.54021485D 00	6.5	0.53632366D 00
6.6	0.53250071D 00	6.7	0.52874400D 00	6.8	0.52505161D 00	6.9	0.52142170D 00
7.0	0.51785249D 00	7.1	0.51434228D 00	7.2	0.51088945D 00	7.3	0.50749242D 00
7.4	0.50414970D 00	7.5	0.50085981D 00	7.6	0.49762138D 00	7.7	0.49443305D 00
7.8	0.49129353D 00	7.9	0.48820158D 00	8.0	0.48515599D 00	8.1	0.48215559D 00
8.2	0.47919927D 00	8.3	0.47628596D 00	8.4	0.47341460D 00	8.5	0.47058419D 00
8.6	0.46779375D 00	8.7	0.46504235D 00	8.8	0.46232908D 00	8.9	0.45965305D 00
9.0	0.45701341D 00	9.1	0.45440934D 00	9.2	0.45184004D 00	9.3	0.44930474D 00
9.4	0.44680269D 00	9.5	0.44433316D 00	9.6	0.44189546D 00	9.7	0.43948889D 00
9.8	0.43711281D 00	9.9	0.43476656D 00	10.0	0.43244952D 00		

P= 0.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.97575410D 00	1.2	0.95291560D 00	1.3	0.93134905D 00
1.4	0.91093698D 00	1.5	0.89157693D 00	1.6	0.87317905D 00	1.7	0.85566409D 00
1.8	0.83896186D 00	1.9	0.82300992D 00	2.0	0.80775248D 00	2.1	0.79313951D 00
2.2	0.77912599D 00	2.3	0.76567126D 00	2.4	0.75273851D 00	2.5	0.74029429D 00
2.6	0.72830813D 00	2.7	0.71675223D 00	2.8	0.70560109D 00	2.9	0.69483137D 00
3.0	0.68442156D 00	3.1	0.67435185D 00	3.2	0.66460397D 00	3.3	0.65516098D 00
3.4	0.64600721D 00	3.5	0.63712811D 00	3.6	0.62851013D 00	3.7	0.62014068D 00
3.8	0.61200799D 00	3.9	0.60410107D 00	4.0	0.59640966D 00	4.1	0.58892413D 00
4.2	0.58163547D 00	4.3	0.57453518D 00	4.4	0.56761532D 00	4.5	0.56086838D 00
4.6	0.55428731D 00	4.7	0.54786544D 00	4.8	0.54159649D 00	4.9	0.53547451D 00
5.0	0.52949389D 00	5.1	0.52364930D 00	5.2	0.51793570D 00	5.3	0.51234832D 00
5.4	0.50688261D 00	5.5	0.50153426D 00	5.6	0.49629918D 00	5.7	0.49117346D 00
5.8	0.48615339D 00	5.9	0.48123544D 00	6.0	0.47641623D 00	6.1	0.47169255D 00
6.2	0.46706133D 00	6.3	0.46251963D 00	6.4	0.45806465D 00	6.5	0.45369370D 00
6.6	0.44940422D 00	6.7	0.44519376D 00	6.8	0.44105995D 00	6.9	0.43700055D 00
7.0	0.43301338D 00	7.1	0.42909638D 00	7.2	0.42524754D 00	7.3	0.42146495D 00
7.4	0.41774678D 00	7.5	0.41409124D 00	7.6	0.41049665D 00	7.7	0.40696137D 00
7.8	0.40348381D 00	7.9	0.40006246D 00	8.0	0.39669586D 00	8.1	0.39338260D 00
8.2	0.39012132D 00	8.3	0.38691070D 00	8.4	0.38374949D 00	8.5	0.38063646D 00
8.6	0.37757043D 00	8.7	0.37455027D 00	8.8	0.37157486D 00	8.9	0.36864316D 00
9.0	0.36575412D 00	9.1	0.36290676D 00	9.2	0.36010011D 00	9.3	0.35733324D 00
9.4	0.35460525D 00	9.5	0.35191525D 00	9.6	0.34926241D 00	9.7	0.34664591D 00
9.8	0.34406494D 00	9.9	0.34151874D 00	10.0	0.33900655D 00		

P= 0.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.97045990D 00	1.2	0.94270921D 00	1.3	0.91657280D 00
1.4	0.89189904D 00	1.5	0.86855583D 00	1.6	0.84642748D 00	1.7	0.82541208D 00
1.8	0.80541950D 00	1.9	0.78636961D 00	2.0	0.76819096D 00	2.1	0.75081952D 00
2.2	0.73419774D 00	2.3	0.71827373D 00	2.4	0.70300054D 00	2.5	0.68833557D 00
2.6	0.67424006D 00	2.7	0.66067865D 00	2.8	0.64761903D 00	2.9	0.63503154D 00
3.0	0.62288897D 00	3.1	0.61116626D 00	3.2	0.59984028D 00	3.3	0.58888969D 00
3.4	0.57829470D 00	3.5	0.56803697D 00	3.6	0.55809946D 00	3.7	0.54846632D 00
3.8	0.53912278D 00	3.9	0.53005506D 00	4.0	0.52125027D 00	4.1	0.51269636D 00
4.2	0.50438205D 00	4.3	0.49629674D 00	4.4	0.48843048D 00	4.5	0.48077392D 00
4.6	0.47331826D 00	4.7	0.46605521D 00	4.8	0.45897693D 00	4.9	0.45207606D 00
5.0	0.44534560D 00	5.1	0.43877896D 00	5.2	0.43236989D 00	5.3	0.42611245D 00
5.4	0.42000105D 00	5.5	0.41403034D 00	5.6	0.40819527D 00	5.7	0.40249102D 00
5.8	0.39691301D 00	5.9	0.39145688D 00	6.0	0.38611850D 00	6.1	0.38089389D 00
6.2	0.37577928D 00	6.3	0.37077108D 00	6.4	0.36586584D 00	6.5	0.36106027D 00
6.6	0.35635123D 00	6.7	0.35173571D 00	6.8	0.34721083D 00	6.9	0.34277384D 00
7.0	0.33842209D 00	7.1	0.33415305D 00	7.2	0.32996428D 00	7.3	0.32585347D 00
7.4	0.32181836D 00	7.5	0.31785680D 00	7.6	0.31396673D 00	7.7	0.31014616D 00
7.8	0.30639317D 00	7.9	0.30270594D 00	8.0	0.29908267D 00	8.1	0.29552168D 00
8.2	0.29202130D 00	8.3	0.28857997D 00	8.4	0.28519615D 00	8.5	0.28186836D 00
8.6	0.27859520D 00	8.7	0.27537527D 00	8.8	0.27220726D 00	8.9	0.26908988D 00
9.0	0.26602190D 00	9.1	0.26300213D 00	9.2	0.26002940D 00	9.3	0.25710259D 00
9.4	0.25422063D 00	9.5	0.25138247D 00	9.6	0.24858709D 00	9.7	0.24583351D 00
9.8	0.24312078D 00	9.9	0.24044798D 00	10.0	0.23781421D 00		

P= 0.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.96422161D 00	1.2	0.93071730D 00	1.3	0.89926020D 00
1.4	0.86965416D 00	1.5	0.84172860D 00	1.6	0.81533430D 00	1.7	0.79034007D 00
1.8	0.76663001D 00	1.9	0.74410129D 00	2.0	0.72266227D 00	2.1	0.70223102D 00
2.2	0.68273396D 00	2.3	0.66410487D 00	2.4	0.64628388D 00	2.5	0.62921677D 00
2.6	0.61285427D 00	2.7	0.59715149D 00	2.8	0.58206745D 00	2.9	0.56756461D 00
3.0	0.55360854D 00	3.1	0.54016761D 00	3.2	0.52721264D 00	3.3	0.51471673D 00
3.4	0.50265500D 00	3.5	0.49100440D 00	3.6	0.47974353D 00	3.7	0.46885254D 00
3.8	0.45831292D 00	3.9	0.44810744D 00	4.0	0.43822000D 00	4.1	0.42863558D 00
4.2	0.41934010D 00	4.3	0.41032038D 00	4.4	0.40156405D 00	4.5	0.39305950D 00
4.6	0.38479580D 00	4.7	0.37676268D 00	4.8	0.36895047D 00	4.9	0.36135002D 00
5.0	0.35395272D 00	5.1	0.34675042D 00	5.2	0.33973543D 00	5.3	0.33290045D 00
5.4	0.32623858D 00	5.5	0.31974327D 00	5.6	0.31340831D 00	5.7	0.30722780D 00
5.8	0.30119613D 00	5.9	0.29530798D 00	6.0	0.28955827D 00	6.1	0.28394218D 00
6.2	0.27845510D 00	6.3	0.27309264D 00	6.4	0.26785062D 00	6.5	0.26272503D 00
6.6	0.25771207D 00	6.7	0.25280808D 00	6.8	0.24800958D 00	6.9	0.24331322D 00
7.0	0.23871581D 00	7.1	0.23421429D 00	7.2	0.22980571D 00	7.3	0.22548728D 00
7.4	0.22125629D 00	7.5	0.21711015D 00	7.6	0.21304638D 00	7.7	0.20906258D 00
7.8	0.20515647D 00	7.9	0.20132582D 00	8.0	0.19756853D 00	8.1	0.19388255D 00
8.2	0.19026591D 00	8.3	0.18671672D 00	8.4	0.18323316D 00	8.5	0.17981347D 00
8.6	0.17645595D 00	8.7	0.17315897D 00	8.8	0.16992095D 00	8.9	0.16674037D 00
9.0	0.16361577D 00	9.1	0.16054572D 00	9.2	0.15752886D 00	9.3	0.15456385D 00
9.4	0.15164943D 00	9.5	0.14878434D 00	9.6	0.14596740D 00	9.7	0.14319745D 00
9.8	0.14047335D 00	9.9	0.13779404D 00	10.0	0.13515846D 00		

P= 0.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.95704841D 00	1.2	0.91697458D 00	1.3	0.87948512D 00
1.4	0.84432682D 00	1.5	0.81127985D 00	1.6	0.78015229D 00	1.7	0.75077567D 00
1.8	0.72300143D 00	1.9	0.69669797D 00	2.0	0.67174823D 00	2.1	0.64804770D 00
2.2	0.62550271D 00	2.3	0.60402905D 00	2.4	0.58355077D 00	2.5	0.56399913D 00
2.6	0.54531181D 00	2.7	0.52743211D 00	2.8	0.51030831D 00	2.9	0.49389317D 00
3.0	0.47814340D 00	3.1	0.46301925D 00	3.2	0.44848418D 00	3.3	0.43450449D 00
3.4	0.42104908D 00	3.5	0.40808918D 00	3.6	0.39559814D 00	3.7	0.38355122D 00
3.8	0.37192544D 00	3.9	0.36069940D 00	4.0	0.34985317D 00	4.1	0.33936813D 00
4.2	0.32922690D 00	4.3	0.31941321D 00	4.4	0.30991181D 00	4.5	0.30070840D 00
4.6	0.29178956D 00	4.7	0.28314269D 00	4.8	0.27475590D 00	4.9	0.26661802D 00
5.0	0.25871851D 00	5.1	0.25104743D 00	5.2	0.24359538D 00	5.3	0.23635348D 00
5.4	0.22931333D 00	5.5	0.22246699D 00	5.6	0.21580690D 00	5.7	0.20932594D 00
5.8	0.20301730D 00	5.9	0.19687455D 00	6.0	0.19089157D 00	6.1	0.18506253D 00
6.2	0.17938189D 00	6.3	0.17384437D 00	6.4	0.16844494D 00	6.5	0.16317881D 00
6.6	0.15804140D 00	6.7	0.15302834D 00	6.8	0.14813547D 00	6.9	0.14335879D 00
7.0	0.13869449D 00	7.1	0.13413893D 00	7.2	0.12968862D 00	7.3	0.12534020D 00
7.4	0.12109048D 00	7.5	0.11693640D 00	7.6	0.11287499D 00	7.7	0.10890344D 00
7.8	0.10501904D 00	7.9	0.10121917D 00	8.0	0.97501347D-01	8.1	0.93863155D-01
8.2	0.90302285D-01	8.3	0.86816511D-01	8.4	0.83403694D-01	8.5	0.80061774D-01
8.6	0.76788766D-01	8.7	0.73582760D-01	8.8	0.70441913D-01	8.9	0.67364450D-01
9.0	0.64348659D-01	9.1	0.61392887D-01	9.2	0.58495542D-01	9.3	0.55655085D-01
9.4	0.52870029D-01	9.5	0.50138940D-01	9.6	0.47460432D-01	9.7	0.44833164D-01
9.8	0.42255840D-01	9.9	0.39727209D-01	10.0	0.37246057D-01		

P= 0.90

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.94895085D 00	1.2	0.90152079D 00	1.3	0.85733183D 00
1.4	0.81605849D 00	1.5	0.77741874D 00	1.6	0.74116682D 00	1.7	0.70708749D 00
1.8	0.67499126D 00	1.9	0.64471065D 00	2.0	0.61609695D 00	2.1	0.58901761D 00
2.2	0.56335412D 00	2.3	0.53900007D 00	2.4	0.51585970D 00	2.5	0.49384652D 00
2.6	0.47288219D 00	2.7	0.45289563D 00	2.8	0.43382209D 00	2.9	0.41560252D 00
3.0	0.39818292D 00	3.1	0.38151379D 00	3.2	0.36554971D 00	3.3	0.35024884D 00
3.4	0.33557267D 00	3.5	0.32148559D 00	3.6	0.30795471D 00	3.7	0.29494951D 00
3.8	0.28244173D 00	3.9	0.27040507D 00	4.0	0.25881506D 00	4.1	0.24764892D 00
4.2	0.23688538D 00	4.3	0.22650458D 00	4.4	0.21648793D 00	4.5	0.20681804D 00
4.6	0.19747861D 00	4.7	0.18845431D 00	4.8	0.17973079D 00	4.9	0.17129451D 00
5.0	0.16313275D 00	5.1	0.15523353D 00	5.2	0.14758552D 00	5.3	0.14017806D 00
5.4	0.13300108D 00	5.5	0.12604502D 00	5.6	0.11930089D 00	5.7	0.11276013D 00
5.8	0.10641466D 00	5.9	0.10025679D 00	6.0	0.94279255D-01	6.1	0.88475124D-01
6.2	0.82837829D-01	6.3	0.77361118D-01	6.4	0.72039045D-01	6.5	0.66865945D-01
6.6	0.61836425D-01	6.7	0.56945342D-01	6.8	0.52187791D-01	6.9	0.47559090D-01
7.0	0.43054771D-01	7.1	0.38670565D-01	7.2	0.34402388D-01	7.3	0.30246340D-01
7.4	0.26198685D-01	7.5	0.22255849D-01	7.6	0.18414409D-01	7.7	0.14671084D-01
7.8	0.11022732D-01	7.9	0.74663367D-02	8.0	0.39990058D-02	8.1	0.61796325D-03
8.2	-0.26794566D-02	8.3	-0.58958141D-02	8.4	-0.90335698D-02	8.5	-0.12095089D-01
8.6	-0.15082646D-01	8.7	-0.17998428D-01	8.8	-0.20844543D-01	8.9	-0.23623017D-01
9.0	-0.26335801D-01	9.1	-0.28984778D-01	9.2	-0.31571759D-01	9.3	-0.34098491D-01
9.4	-0.36566660D-01	9.5	-0.38977890D-01	9.6	-0.41333749D-01	9.7	-0.43635752D-01
9.8	-0.45885359D-01	9.9	-0.48083981D-01	10.0	-0.50232982D-01		

P= 1.00

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.93994086D 00	1.2	0.88440056D 00	1.3	0.83289463D 00
1.4	0.78500682D 00	1.5	0.74037730D 00	1.6	0.69869324D 00	1.7	0.65968122D 00
1.8	0.62310111D 00	1.9	0.58874111D 00	2.0	0.55641355D 00	2.1	0.52595156D 00
2.2	0.49720620D 00	2.3	0.47004408D 00	2.4	0.44434535D 00	2.5	0.42000202D 00
2.6	0.39691649D 00	2.7	0.37500033D 00	2.8	0.35417321D 00	2.9	0.33436198D 00
3.0	0.31549988D 00	3.1	0.29752586D 00	3.2	0.28038397D 00	3.3	0.26402282D 00
3.4	0.24839515D 00	3.5	0.23345740D 00	3.6	0.21916934D 00	3.7	0.20549381D 00
3.8	0.19239636D 00	3.9	0.17984506D 00	4.0	0.16781024D 00	4.1	0.15626432D 00
4.2	0.14518161D 00	4.3	0.13453815D 00	4.4	0.12431159D 00	4.5	0.11448101D 00
4.6	0.10502686D 00	4.7	0.95930819D-01	4.8	0.87175704D-01	4.9	0.78745390D-01
5.0	0.70624724D-01	5.1	0.62799455D-01	5.2	0.55256167D-01	5.3	0.47982223D-01
5.4	0.40965703D-01	5.5	0.34195360D-01	5.6	0.27660570D-01	5.7	0.21351289D-01
5.8	0.15258017D-01	5.9	0.93717610D-02	6.0	0.36840008D-02	6.1	-0.18133410D-02
6.2	-0.71279266D-02	6.3	-0.12267030D-01	6.4	-0.17237560D-01	6.5	-0.22046084D-01
6.6	-0.26698847D-01	6.7	-0.31201793D-01	6.8	-0.35560580D-01	6.9	-0.39780599D-01
7.0	-0.43866989D-01	7.1	-0.47824651D-01	7.2	-0.51658261D-01	7.3	-0.55372283D-01
7.4	-0.58970983D-01	7.5	-0.62458434D-01	7.6	-0.65838532D-01	7.7	-0.69115005D-01
7.8	-0.72291417D-01	7.9	-0.75371184D-01	8.0	-0.78357573D-01	8.1	-0.81253719D-01
8.2	-0.84062624D-01	8.3	-0.86787168D-01	8.4	-0.89430113D-01	8.5	-0.91994110D-01
8.6	-0.94481705D-01	8.7	-0.96895342D-01	8.8	-0.99237370D-01	8.9	-0.10151004D 00
9.0	-0.10371554D 00	9.1	-0.10585593D 00	9.2	-0.10793324D 00	9.3	-0.10994939D 00
9.4	-0.11190625D 00	9.5	-0.11380560D 00	9.6	-0.11564918D 00	9.7	-0.11743863D 00
9.8	-0.11917558D 00	9.9	-0.12086155D 00	10.0	-0.12249805D 00		

P= 1.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.93003168D 00	1.2	0.86566331D 00	1.3	0.80627734D 00
1.4	0.75134459D 00	1.5	0.70040873D 00	1.6	0.65307403D 00	1.7	0.60899547D 00
1.8	0.56787081D 00	1.9	0.52943406D 00	2.0	0.49345016D 00	2.1	0.45971057D 00
2.2	0.42802959D 00	2.3	0.39824132D 00	2.4	0.37019704D 00	2.5	0.34376304D 00
2.6	0.31881879D 00	2.7	0.29525529D 00	2.8	0.27297378D 00	2.9	0.25188452D 00
3.0	0.23190579D 00	3.1	0.21296303D 00	3.2	0.19498805D 00	3.3	0.17791838D 00
3.4	0.16169667D 00	3.5	0.14627017D 00	3.6	0.13159032D 00	3.7	0.11761230D 00
3.8	0.10429469D 00	3.9	0.91599154D-01	4.0	0.79490178D-01	4.1	0.67934794D-01
4.2	0.56902365D-01	4.3	0.46364385D-01	4.4	0.36294301D-01	4.5	0.26667343D-01
4.6	0.17460384D-01	4.7	0.86518050D-02	4.8	0.22137729D-03	4.9	-0.78498492D-02
5.0	-0.15579642D-01	5.1	-0.22984676D-01	5.2	-0.30080616D-01	5.3	-0.36882191D-01
5.4	-0.43403258D-01	5.5	-0.49656872D-01	5.6	-0.55655336D-01	5.7	-0.61410256D-01
5.8	-0.66932588D-01	5.9	-0.72232683D-01	6.0	-0.77320326D-01	6.1	-0.82204776D-01
6.2	-0.86894798D-01	6.3	-0.91398692D-01	6.4	-0.95724331D-01	6.5	-0.99879179D-01
6.6	-0.10387032D 00	6.7	-0.10770449D 00	6.8	-0.11138807D 00	6.9	-0.11492714D 00
7.0	-0.11832749D 00	7.1	-0.12159462D 00	7.2	-0.12473377D 00	7.3	-0.12774993D 00
7.4	-0.13064786D 00	7.5	-0.13343210D 00	7.6	-0.13610697D 00	7.7	-0.13867662D 00
7.8	-0.14114498D 00	7.9	-0.14351583D 00	8.0	-0.14579277D 00	8.1	-0.14797924D 00
8.2	-0.15007855D 00	8.3	-0.15209385D 00	8.4	-0.15402815D 00	8.5	-0.15588435D 00
8.6	-0.15766522D 00	8.7	-0.15937341D 00	8.8	-0.16101147D 00	8.9	-0.16258185D 00
9.0	-0.16408688D 00	9.1	-0.16552881D 00	9.2	-0.16690980D 00	9.3	-0.16823192D 00
9.4	-0.16949717D 00	9.5	-0.17070745D 00	9.6	-0.17186460D 00	9.7	-0.17297041D 00
9.8	-0.17402655D 00	9.9	-0.17503468D 00	10.0	-0.17599636D 00		

P= 1.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.91923788D 00	1.2	0.84536304D 00	1.3	0.77759289D 00
1.4	0.71525870D 00	1.5	0.65778541D 00	1.6	0.60467570D 00	1.7	0.55549718D 00
1.8	0.50987208D 00	1.9	0.46746882D 00	2.0	0.42799518D 00	2.1	0.39119259D 00
2.2	0.35683139D 00	2.3	0.32470694D 00	2.4	0.29463630D 00	2.5	0.26645539D 00
2.6	0.24001666D 00	2.7	0.21518707D 00	2.8	0.19184634D 00	2.9	0.16988548D 00
3.0	0.14920547D 00	3.1	0.12971619D 00	3.2	0.11133545D 00	3.3	0.93988104D-01
3.4	0.77605368D-01	3.5	0.62124134D-01	3.6	0.47486419D-01	3.7	0.33638852D-01
3.8	0.20532241D-01	3.9	0.81211672D-02	4.0	-0.36363567D-02	4.1	-0.14779203D-01
4.2	-0.25343409D-01	4.3	-0.35362420D-01	4.4	-0.44867324D-01	4.5	-0.53887040D-01
4.6	-0.62448510D-01	4.7	-0.70576857D-01	4.8	-0.78295530D-01	4.9	-0.85626445D-01
5.0	-0.92590098D-01	5.1	-0.99205680D-01	5.2	-0.10549117D 00	5.3	-0.11146345D 00
5.4	-0.11713834D 00	5.5	-0.12253072D 00	5.6	-0.12765458D 00	5.7	-0.13252308D 00
5.8	-0.13714862D 00	5.9	-0.14154288D 00	6.0	-0.14571689D 00	6.1	-0.14968103D 00
6.2	-0.15344513D 00	6.3	-0.15701847D 00	6.4	-0.16040982D 00	6.5	-0.16362748D 00
6.6	-0.16667932D 00	6.7	-0.16957278D 00	6.8	-0.17231493D 00	6.9	-0.17491244D 00
7.0	-0.17737168D 00	7.1	-0.17969868D 00	7.2	-0.18189916D 00	7.3	-0.18397856D 00
7.4	-0.18594206D 00	7.5	-0.18779459D 00	7.6	-0.18954082D 00	7.7	-0.19118522D 00
7.8	-0.19273203D 00	7.9	-0.19418531D 00	8.0	-0.19554891D 00	8.1	-0.19682652D 00
8.2	-0.19802165D 00	8.3	-0.19913764D 00	8.4	-0.20017771D 00	8.5	-0.20114490D 00
8.6	-0.20204214D 00	8.7	-0.20287222D 00	8.8	-0.20363781D 00	8.9	-0.20434146D 00
9.0	-0.20498562D 00	9.1	-0.20557263D 00	9.2	-0.20610473D 00	9.3	-0.20658405D 00
9.4	-0.20701266D 00	9.5	-0.20739252D 00	9.6	-0.20772553D 00	9.7	-0.20801348D 00
9.8	-0.20825812D 00	9.9	-0.20846112D 00	10.0	-0.20862406D 00		

P= 1.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.90757531D 00	1.2	0.82355820D 00	1.3	0.74696272D 00
1.4	0.67694894D 00	1.5	0.61279683D 00	1.6	0.55388551D 00	1.7	0.49967680D 00
1.8	0.44970189D 00	1.9	0.40355051D 00	2.0	0.36086215D 00	2.1	0.32131872D 00
2.2	0.28463854D 00	2.3	0.25057133D 00	2.4	0.21889397D 00	2.5	0.18940694D 00
2.6	0.16193132D 00	2.7	0.13630623D 00	2.8	0.11238665D 00	2.9	0.90041517D-01
3.0	0.69152124D-01	3.1	0.49610719D-01	3.2	0.31319289D-01	3.3	0.14188507D-01
3.4	-0.18631942D-02	3.5	-0.16910428D-01	3.6	-0.31021562D-01	3.7	-0.44259347D-01
3.8	-0.56681465D-01	3.9	-0.68341021D-01	4.0	-0.79286974D-01	4.1	-0.89564525D-01
4.2	-0.99215457D-01	4.3	-0.10827845D 00	4.4	-0.11678934D 00	4.5	-0.12478139D 00
4.6	-0.13228550D 00	4.7	-0.13933037D 00	4.8	-0.14594277D 00	4.9	-0.15214760D 00
5.0	-0.15796810D 00	5.1	-0.16342597D 00	5.2	-0.16854148D 00	5.3	-0.17333361D 00
5.4	-0.17782009D 00	5.5	-0.18201756D 00	5.6	-0.18594161D 00	5.7	-0.18960687D 00
5.8	-0.19302707D 00	5.9	-0.19621513D 00	6.0	-0.19918319D 00	6.1	-0.20194266D 00
6.2	-0.20450431D 00	6.3	-0.20687826D 00	6.4	-0.20907407D 00	6.5	-0.21110076D 00
6.6	-0.21296683D 00	6.7	-0.21468032D 00	6.8	-0.21624882D 00	6.9	-0.21767950D 00
7.0	-0.21897917D 00	7.1	-0.22015425D 00	7.2	-0.22121082D 00	7.3	-0.22215465D 00
7.4	-0.22299120D 00	7.5	-0.22372567D 00	7.6	-0.22436295D 00	7.7	-0.22490772D 00
7.8	-0.22536441D 00	7.9	-0.22573723D 00	8.0	-0.22603017D 00	8.1	-0.22624703D 00
8.2	-0.22639143D 00	8.3	-0.22646681D 00	8.4	-0.22647643D 00	8.5	-0.22642341D 00
8.6	-0.22631072D 00	8.7	-0.22614118D 00	8.8	-0.22591748D 00	8.9	-0.22564219D 00
9.0	-0.22531775D 00	9.1	-0.22494650D 00	9.2	-0.22453066D 00	9.3	-0.22407235D 00
9.4	-0.22357361D 00	9.5	-0.22303637D 00	9.6	-0.22246247D 00	9.7	-0.22185369D 00
9.8	-0.22121170D 00	9.9	-0.22053812D 00	10.0	-0.21983450D 00		

P= 1.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.89506111D 00	1.2	0.80031148D 00	1.3	0.71451622D 00
1.4	0.63662683D 00	1.5	0.56574739D 00	1.6	0.50110793D 00	1.7	0.44204318D 00
1.8	0.38797547D 00	1.9	0.33840095D 00	2.0	0.29287822D 00	2.1	0.25101910D 00
2.2	0.21248094D 00	2.3	0.17696021D 00	2.4	0.14418719D 00	2.5	0.11392147D 00
2.6	0.85948101D-01	2.7	0.60074441D-01	2.8	0.36127357D-01	2.9	0.13950883D-01
3.0	-0.65958035D-02	3.1	-0.25640134D-01	3.2	-0.43297656D-01	3.3	-0.59673351D-01
3.4	-0.74862781D-01	3.5	-0.88953084D-01	3.6	-0.10202386D 00	3.7	-0.11414793D 00
3.8	-0.12539203D 00	3.9	-0.13581741D 00	4.0	-0.14548034D 00	4.1	-0.15443263D 00
4.2	-0.16272201D 00	4.3	-0.17039251D 00	4.4	-0.17748483D 00	4.5	-0.18403658D 00
4.6	-0.19008261D 00	4.7	-0.19565519D 00	4.8	-0.20078429D 00	4.9	-0.20549774D 00
5.0	-0.20982140D 00	5.1	-0.21377934D 00	5.2	-0.21739398D 00	5.3	-0.22068620D 00
5.4	-0.22367552D 00	5.5	-0.22638011D 00	5.6	-0.22881700D 00	5.7	-0.23100208D 00
5.8	-0.23295023D 00	5.9	-0.23467539D 00	6.0	-0.23619062D 00	6.1	-0.23750814D 00
6.2	-0.23863947D 00	6.3	-0.23959536D 00	6.4	-0.24038597D 00	6.5	-0.24102079D 00
6.6	-0.24150880D 00	6.7	-0.24185841D 00	6.8	-0.24207756D 00	6.9	-0.24217371D 00
7.0	-0.24215391D 00	7.1	-0.24202478D 00	7.2	-0.24179260D 00	7.3	-0.24146326D 00
7.4	-0.24104234D 00	7.5	-0.24053511D 00	7.6	-0.23994654D 00	7.7	-0.23928133D 00
7.8	-0.23854392D 00	7.9	-0.23773853D 00	8.0	-0.23686912D 00	8.1	-0.23593947D 00
8.2	-0.23495313D 00	8.3	-0.23391348D 00	8.4	-0.23282372D 00	8.5	-0.23168687D 00
8.6	-0.23050581D 00	8.7	-0.22928326D 00	8.8	-0.22802180D 00	8.9	-0.22672388D 00
9.0	-0.22539182D 00	9.1	-0.22402783D 00	9.2	-0.22263399D 00	9.3	-0.22121230D 00
9.4	-0.21976463D 00	9.5	-0.21829278D 00	9.6	-0.21679844D 00	9.7	-0.21528324D 00
9.8	-0.21374871D 00	9.9	-0.21219631D 00	10.0	-0.21062741D 00		

P= 1.10

P= 1.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.88171362D 00	1.2	0.77568960D 00	1.3	0.68039016D 00
1.4	0.59451423D 00	1.5	0.51695409D 00	1.6	0.44676106D 00	1.7	0.38311835D 00
1.8	0.32531924D 00	1.9	0.27274940D 00	2.0	0.22487257D 00	2.1	0.18121876D 00
2.2	0.14137455D 00	2.3	0.10497498D 00	2.4	0.71696863D-01	2.5	0.41253112D-01
2.6	0.13387970D-01	2.7	-0.12127026D-01	2.8	-0.35496475D-01	2.9	-0.56904051D-01
3.0	-0.76515025D-01	3.1	-0.94478424D-01	3.2	-0.11092891D 00	3.3	-0.12598841D 00
3.4	-0.13976751D 00	3.5	-0.15236669D 00	3.6	-0.16387743D 00	3.7	-0.17438311D 00
3.8	-0.18395987D 00	3.9	-0.19267733D 00	4.0	-0.20059926D 00	4.1	-0.20778410D 00
4.2	-0.21428554D 00	4.3	-0.22015291D 00	4.4	-0.22543163D 00	4.5	-0.23016353D 00
4.6	-0.23438720D 00	4.7	-0.23813829D 00	4.8	-0.24144974D 00	4.9	-0.24435200D 00
5.0	-0.24687331D 00	5.1	-0.24903981D 00	5.2	-0.25087576D 00	5.3	-0.25240370D 00
5.4	-0.25364455D 00	5.5	-0.25461776D 00	5.6	-0.25534145D 00	5.7	-0.25583248D 00
5.8	-0.25610656D 00	5.9	-0.25617833D 00	6.0	-0.25606146D 00	6.1	-0.25576872D 00
6.2	-0.25531201D 00	6.3	-0.25470248D 00	6.4	-0.25395054D 00	6.5	-0.25306593D 00
6.6	-0.25205777D 00	6.7	-0.25093460D 00	6.8	-0.24970442D 00	6.9	-0.24837472D 00
7.0	-0.24695254D 00	7.1	-0.24544447D 00	7.2	-0.24385670D 00	7.3	-0.24219502D 00
7.4	-0.24046490D 00	7.5	-0.23867146D 00	7.6	-0.23681950D 00	7.7	-0.23491355D 00
7.8	-0.23295785D 00	7.9	-0.23095641D 00	8.0	-0.22891298D 00	8.1	-0.22683109D 00
8.2	-0.22471407D 00	8.3	-0.22256504D 00	8.4	-0.22038696D 00	8.5	-0.21818259D 00
8.6	-0.21595453D 00	8.7	-0.21370526D 00	8.8	-0.21143707D 00	8.9	-0.20915214D 00
9.0	-0.20685253D 00	9.1	-0.20454015D 00	9.2	-0.20221684D 00	9.3	-0.19988429D 00
9.4	-0.19754412D 00	9.5	-0.19519784D 00	9.6	-0.19284688D 00	9.7	-0.19049258D 00
9.8	-0.18813621D 00	9.9	-0.18577895D 00	10.0	-0.18342193D 00		

P= 1.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.86755240D 00	1.2	0.74976313D 00	1.3	0.64472796D 00
1.4	0.55084202D 00	1.5	0.46674408D 00	1.6	0.39127283D 00	1.7	0.32343217D 00
1.8	0.26236348D 00	1.9	0.20732322D 00	2.0	0.15766479D 00	2.1	0.11282361D 00
2.2	0.72304923D-01	2.3	0.35673645D-01	2.4	0.25459192D-02	2.5	-0.27417942D-01
2.6	-0.54519404D-01	2.7	-0.79026734D-01	2.8	-0.10117926D 00	2.9	-0.12119102D 00
3.0	-0.13925385D 00	3.1	-0.15554007D 00	3.2	-0.17020479D 00	3.3	-0.18338787D 00
3.4	-0.19521570D 00	3.5	-0.20580266D 00	3.6	-0.21525243D 00	3.7	-0.22365917D 00
3.8	-0.23110850D 00	3.9	-0.23767839D 00	4.0	-0.24343993D 00	4.1	-0.24845806D 00
4.2	-0.25279214D 00	4.3	-0.25649649D 00	4.4	-0.25962090D 00	4.5	-0.26221106D 00
4.6	-0.26430889D 00	4.7	-0.26595296D 00	4.8	-0.26717875D 00	4.9	-0.26801891D 00
5.0	-0.26850357D 00	5.1	-0.26866049D 00	5.2	-0.26851533D 00	5.3	-0.26809177D 00
5.4	-0.26741171D 00	5.5	-0.26649542D 00	5.6	-0.26536167D 00	5.7	-0.26402781D 00
5.8	-0.26250997D 00	5.9	-0.26082307D 00	6.0	-0.25898097D 00	6.1	-0.25699655D 00
6.2	-0.25488173D 00	6.3	-0.25264763D 00	6.4	-0.25030456D 00	6.5	-0.24786212D 00
6.6	-0.24532922D 00	6.7	-0.24271417D 00	6.8	-0.24002469D 00	6.9	-0.23726798D 00
7.0	-0.23445074D 00	7.1	-0.23157920D 00	7.2	-0.22865918D 00	7.3	-0.22569608D 00
7.4	-0.22269495D 00	7.5	-0.21966050D 00	7.6	-0.21659710D 00	7.7	-0.21350883D 00
7.8	-0.21039950D 00	7.9	-0.20727265D 00	8.0	-0.20413159D 00	8.1	-0.20097938D 00
8.2	-0.19781890D 00	8.3	-0.19465282D 00	8.4	-0.19148362D 00	8.5	-0.18831361D 00
8.6	-0.18514494D 00	8.7	-0.18197963D 00	8.8	-0.17881953D 00	8.9	-0.17566637D 00
9.0	-0.17252175D 00	9.1	-0.16938718D 00	9.2	-0.16626403D 00	9.3	-0.16315358D 00
9.4	-0.16005702D 00	9.5	-0.15697546D 00	9.6	-0.15390990D 00	9.7	-0.15086128D 00
9.8	-0.14783047D 00	9.9	-0.14481826D 00	10.0	-0.14182538D 00		

P= 1.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.85259820D 00	1.2	0.72260621D 00	1.3	0.60767910D 00
1.4	0.50584864D 00	1.5	0.41545224D 00	1.6	0.33507723D 00	1.7	0.26351686D 00
1.8	0.19973516D 00	1.9	0.14283866D 00	2.0	0.92053513D-01	2.1	0.46706827D-01
2.2	0.62113196D-02	2.3	-0.29947339D-01	2.4	-0.62221047D-01	2.5	-0.91008052D-01
2.6	-0.11666032D 00	2.7	-0.13948974D 00	2.8	-0.15977339D 00	2.9	-0.17775801D 00
3.0	-0.19366382D 00	3.1	-0.20768776D 00	3.2	-0.22000633D 00	3.3	-0.23077800D 00
3.4	-0.24014533D 00	3.5	-0.24823673D 00	3.6	-0.25516812D 00	3.7	-0.26104424D 00
3.8	-0.26595992D 00	3.9	-0.27000111D 00	4.0	-0.27324579D 00	4.1	-0.27576485D 00
4.2	-0.27762276D 00	4.3	-0.27887824D 00	4.4	-0.27958485D 00	4.5	-0.27979143D 00
4.6	-0.27954263D 00	4.7	-0.27887925D 00	4.8	-0.27783864D 00	4.9	-0.27645499D 00
5.0	-0.27475965D 00	5.1	-0.27278136D 00	5.2	-0.27054649D 00	5.3	-0.26807929D 00
5.4	-0.26540199D 00	5.5	-0.26253505D 00	5.6	-0.25949731D 00	5.7	-0.25630605D 00
5.8	-0.25297724D 00	5.9	-0.24952553D 00	6.0	-0.24596444D 00	6.1	-0.24230643D 00
6.2	-0.23856297D 00	6.3	-0.23474465D 00	6.4	-0.23086121D 00	6.5	-0.22692165D 00
6.6	-0.22293424D 00	6.7	-0.21890664D 00	6.8	-0.21484589D 00	6.9	-0.21075849D 00
7.0	-0.20665041D 00	7.1	-0.20252718D 00	7.2	-0.19839388D 00	7.3	-0.19425518D 00
7.4	-0.19011540D 00	7.5	-0.18597849D 00	7.6	-0.18184810D 00	7.7	-0.17772759D 00
7.8	-0.17362001D 00	7.9	-0.16952820D 00	8.0	-0.16545474D 00	8.1	-0.16140200D 00
8.2	-0.15737215D 00	8.3	-0.15336715D 00	8.4	-0.14938881D 00	8.5	-0.14543877D 00
8.6	-0.14151852D 00	8.7	-0.13762942D 00	8.8	-0.13377267D 00	8.9	-0.12994938D 00
9.0	-0.12616053D 00	9.1	-0.12240702D 00	9.2	-0.11868961D 00	9.3	-0.11500902D 00
9.4	-0.11136586D 00	9.5	-0.10776065D 00	9.6	-0.10419387D 00	9.7	-0.10066591D 00
9.8	-0.97177110D-01	9.9	-0.93727739D-01	10.0	-0.90318025D-01		

P= 1.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.83687290D 00	1.2	0.69429638D 00	1.3	0.56939837D 00
1.4	0.45977867D 00	1.5	0.36341860D 00	1.6	0.27861041D 00	1.7	0.20390162D 00
1.8	0.13805074D 00	1.9	0.79991752D-01	2.0	0.28805399D-01	2.1	-0.16304123D-01
2.2	-0.56028291D-01	2.3	-0.90969985D-01	2.4	-0.12165653D 00	2.5	-0.14855053D 00
2.6	-0.17205898D 00	2.7	-0.19254087D 00	2.8	-0.21031362D 00	2.9	-0.22565859D 00
3.0	-0.23882567D 00	3.1	-0.25003729D 00	3.2	-0.25949182D 00	3.3	-0.26736647D 00
3.4	-0.27381985D 00	3.5	-0.27899414D 00	3.6	-0.28301698D 00	3.7	-0.28600313D 00
3.8	-0.28805589D 00	3.9	-0.28926841D 00	4.0	-0.28972474D 00	4.1	-0.28950081D 00
4.2	-0.28866532D 00	4.3	-0.28728044D 00	4.4	-0.28540255D 00	4.5	-0.28308273D 00
4.6	-0.28036738D 00	4.7	-0.27729864D 00	4.8	-0.27391481D 00	4.9	-0.27025070D 00
5.0	-0.26633803D 00	5.1	-0.26220564D 00	5.2	-0.25787981D 00	5.3	-0.25338448D 00
5.4	-0.24874149D 00	5.5	-0.24397071D 00	5.6	-0.23909028D 00	5.7	-0.23411675D 00
5.8	-0.22906518D 00	5.9	-0.22394934D 00	6.0	-0.21878176D 00	6.1	-0.21357386D 00
6.2	-0.20833607D 00	6.3	-0.20307787D 00	6.4	-0.19780789D 00	6.5	-0.19253400D 00
6.6	-0.18726336D 00	6.7	-0.18200246D 00	6.8	-0.17675720D 00	6.9	-0.17153293D 00
7.0	-0.16633450D 00	7.1	-0.16116630D 00	7.2	-0.15603228D 00	7.3	-0.15093603D 00
7.4	-0.14588074D 00	7.5	-0.14086930D 00	7.6	-0.13590429D 00	7.7	-0.13098802D 00
7.8	-0.12612252D 00	7.9	-0.12130961D 00	8.0	-0.11655088D 00	8.1	-0.11184773D 00
8.2	-0.10720137D 00	8.3	-0.10261283D 00	8.4	-0.98083019D-01	8.5	-0.93612670D-01
8.6	-0.89202404D-01	8.7	-0.84852718D-01	8.8	-0.80563997D-01	8.9	-0.76336527D-01
9.0	-0.72170501D-01	9.1	-0.68066028D-01	9.2	-0.64023138D-01	9.3	-0.60041791D-01
9.4	-0.56121880D-01	9.5	-0.52263242D-01	9.6	-0.48465656D-01	9.7	-0.44728855D-01
9.8	-0.41052526D-01	9.9	-0.37436313D-01	10.0	-0.33879827D-01		

P= 1.90

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.82039948D 00	1.2	0.66491428D 00	1.3	0.53004515D 00
1.4	0.41288127D 00	1.5	0.31098580D 00	1.6	0.22230681D 00	1.7	0.14510721D 00
1.8	0.77909152D-01	1.9	0.19449620D-01	2.0	-0.31355350D-01	2.1	-0.75439700D-01
2.2	-0.11361104D 00	2.3	-0.14657009D 00	2.4	-0.17492677D 00	2.5	-0.19921347D 00
2.6	-0.21989626D 00	2.7	-0.23738413D 00	2.8	-0.25203689D 00	2.9	-0.26417175D 00
3.0	-0.27406901D 00	3.1	-0.28197678D 00	3.2	-0.28811512D 00	3.3	-0.29267954D 00
3.4	-0.29584401D 00	3.5	-0.29776358D 00	3.6	-0.29857660D 00	3.7	-0.29840672D 00
3.8	-0.29736456D 00	3.9	-0.29554918D 00	4.0	-0.29304941D 00	4.1	-0.28994497D 00
4.2	-0.28630746D 00	4.3	-0.28220122D 00	4.4	-0.27768415D 00	4.5	-0.27280834D 00
4.6	-0.26762071D 00	4.7	-0.26216351D 00	4.8	-0.25647484D 00	4.9	-0.25058901D 00
5.0	-0.24453699D 00	5.1	-0.23834668D 00	5.2	-0.23204323D 00	5.3	-0.22564933D 00
5.4	-0.21918542D 00	5.5	-0.21266992D 00	5.6	-0.20611941D 00	5.7	-0.19954883D 00
5.8	-0.19297159D 00	5.9	-0.18639977D 00	6.0	-0.17984418D 00	6.1	-0.17331453D 00
6.2	-0.16681952D 00	6.3	-0.16036690D 00	6.4	-0.15396359D 00	6.5	-0.14761576D 00
6.6	-0.14132886D 00	6.7	-0.13510773D 00	6.8	-0.12895661D 00	6.9	-0.12287925D 00
7.0	-0.11687891D 00	7.1	-0.11095839D 00	7.2	-0.10512015D 00	7.3	-0.99366241D-01
7.4	-0.93698420D-01	7.5	-0.88118135D-01	7.6	-0.82626568D-01	7.7	-0.77224657D-01
7.8	-0.71913115D-01	7.9	-0.66692457D-01	8.0	-0.61563013D-01	8.1	-0.56524949D-01
8.2	-0.51578279D-01	8.3	-0.46722885D-01	8.4	-0.41958523D-01	8.5	-0.37284840D-01
8.6	-0.32701385D-01	8.7	-0.28207616D-01	8.8	-0.23802911D-01	8.9	-0.19486577D-01
9.0	-0.15257858D-01	9.1	-0.11115939D-01	9.2	-0.70599567D-02	9.3	-0.30890018D-02
9.4	0.79787312D-03	9.5	0.46016495D-02	9.6	0.83233385D-02	9.7	0.11963976D-01
9.8	0.15524621D-01	9.9	0.19006348D-01	10.0	0.22410249D-01		

P= 2.00

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.80320198D 00	1.2	0.63454340D 00	1.3	0.48978266D 00
1.4	0.36540867D 00	1.5	0.25849651D 00	1.6	0.16659537D 00	1.7	0.87640751D-01
1.8	0.19885037D-01	1.9	-0.38157829D-01	2.0	-0.87756765D-01	2.1	-0.12999946D 00
2.2	-0.16582159D 00	2.3	-0.19603074D 00	2.4	-0.22132606D 00	2.5	-0.24231460D 00
2.6	-0.25952479D 00	2.7	-0.27341779D 00	2.8	-0.28439699D 00	2.9	-0.29281597D 00
3.0	-0.29898527D 00	3.1	-0.30317813D 00	3.2	-0.30563540D 00	3.3	-0.30656968D 00
3.4	-0.30616892D 00	3.5	-0.30459948D 00	3.6	-0.30200882D 00	3.7	-0.29852774D 00
3.8	-0.29427242D 00	3.9	-0.28934612D 00	4.0	-0.28384069D 00	4.1	-0.27783786D 00
4.2	-0.27141045D 00	4.3	-0.26462330D 00	4.4	-0.25753420D 00	4.5	-0.25019467D 00
4.6	-0.24265061D 00	4.7	-0.23494294D 00	4.8	-0.22710811D 00	4.9	-0.21917859D 00
5.0	-0.21118330D 00	5.1	-0.20314794D 00	5.2	-0.19509536D 00	5.3	-0.18704585D 00
5.4	-0.17901738D 00	5.5	-0.17102588D 00	5.6	-0.16308540D 00	5.7	-0.15520833D 00
5.8	-0.14740555D 00	5.9	-0.13968660D 00	6.0	-0.13205981D 00	6.1	-0.12453241D 00
6.2	-0.11711063D 00	6.3	-0.10979985D 00	6.4	-0.10260462D 00	6.5	-0.95528797D-01
6.6	-0.88575581D-01	6.7	-0.81747594D-01	6.8	-0.75046940D-01	6.9	-0.68475253D-01
7.0	-0.62033745D-01	7.1	-0.55723253D-01	7.2	-0.49544276D-01	7.3	-0.43497008D-01
7.4	-0.37581376D-01	7.5	-0.31797062D-01	7.6	-0.26143534D-01	7.7	-0.20620071D-01
7.8	-0.15225777D-01	7.9	-0.99596108D-02	8.0	-0.48203951D-02	8.1	0.19316253D-03
8.2	0.50824569D-02	8.3	0.98489715D-02	8.4	0.14494267D-01	8.5	0.19019970D-01
8.6	0.23427763D-01	8.7	0.27719378D-01	8.8	0.31896583D-01	8.9	0.35961180D-01
9.0	0.39914999D-01	9.1	0.43759885D-01	9.2	0.47497701D-01	9.3	0.51130318D-01
9.4	0.54659612D-01	9.5	0.58087462D-01	9.6	0.61415744D-01	9.7	0.64646328D-01
9.8	0.67781076D-01	9.9	0.70821841D-01	10.0	0.73770461D-01		

P= 2.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.78530549D 00	1.2	0.60326983D 00	1.3	0.44877719D 00
1.4	0.31761467D 00	1.5	0.20629088D 00	1.6	0.11189570D 00	1.7	0.31990599D-01
1.8	-0.35477669D-01	1.9	-0.92245649D-01	2.0	-0.13978807D 00	2.1	-0.17936206D 00
2.2	-0.21204297D 00	2.3	-0.23875366D 00	2.4	-0.26028844D 00	2.5	-0.27733288D 00
2.6	-0.29048017D 00	2.7	-0.30024480D 00	2.8	-0.30707391D 00	2.9	-0.31135691D 00
3.0	-0.31343353D 00	3.1	-0.31360065D 00	3.2	-0.31211807D 00	3.3	-0.30921347D 00
3.4	-0.30508658D 00	3.5	-0.29991283D 00	3.6	-0.29384643D 00	3.7	-0.28702302D 00
3.8	-0.27956200D 00	3.9	-0.27156850D 00	4.0	-0.26313516D 00	4.1	-0.25434354D 00
4.2	-0.24526553D 00	4.3	-0.23596442D 00	4.4	-0.22649594D 00	4.5	-0.21690913D 00
4.6	-0.20724707D 00	4.7	-0.19754763D 00	4.8	-0.18784398D 00	4.9	-0.17816518D 00
5.0	-0.16853661D 00	5.1	-0.15898038D 00	5.2	-0.14951570D 00	5.3	-0.14015922D 00
5.4	-0.13092526D 00	5.5	-0.12182613D 00	5.6	-0.11287229D 00	5.7	-0.10407260D 00
5.8	-0.95434471D-01	5.9	-0.86964045D-01	6.0	-0.78666310D-01	6.1	-0.70545247D-01
6.2	-0.62603935D-01	6.3	-0.54844653D-01	6.4	-0.47268971D-01	6.5	-0.39877832D-01
6.6	-0.32671622D-01	6.7	-0.25650236D-01	6.8	-0.18813136D-01	6.9	-0.12159404D-01
7.0	-0.56877883D-02	7.1	0.60325699D-03	7.2	0.67155306D-02	7.3	0.12651053D-01
7.4	0.18412035D-01	7.5	0.24000851D-01	7.6	0.29420016D-01	7.7	0.34672161D-01
7.8	0.39760017D-01	7.9	0.44686395D-01	8.0	0.49454171D-01	8.1	0.54066273D-01
8.2	0.58525667D-01	8.3	0.62835348D-01	8.4	0.66998327D-01	8.5	0.71017626D-01
8.6	0.74896268D-01	8.7	0.78637269D-01	8.8	0.82243635D-01	8.9	0.85718352D-01
9.0	0.89064387D-01	9.1	0.92284679D-01	9.2	0.95382139D-01	9.3	0.98359642D-01
9.4	0.10122003D 00	9.5	0.10396610D 00	9.6	0.10660063D 00	9.7	0.10912633D 00
9.8	0.11154587D 00	9.9	0.11386190D 00	10.0	0.11607699D 00		

P= 2.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.76673608D 00	1.2	0.57118198D 00	1.3	0.40719734D 00
1.4	0.26975301D 00	1.5	0.15470399D 00	1.6	0.58614465D-01	1.7	-0.21378484D-01
1.8	-0.87671769D-01	1.9	-0.14228094D 00	2.0	-0.18690768D 00	2.1	-0.22299391D 00
2.2	-0.25176561D 00	2.3	-0.27426838D 00	2.4	-0.29139647D 00	2.5	-0.30391669D 00
2.6	-0.31248810D 00	2.7	-0.31767839D 00	2.8	-0.31997747D 00	2.9	-0.31980887D 00
3.0	-0.31753928D 00	3.1	-0.31348665D 00	3.2	-0.30792693D 00	3.3	-0.30109991D 00
3.4	-0.29321407D 00	3.5	-0.28445083D 00	3.6	-0.27496809D 00	3.7	-0.26490332D 00
3.8	-0.25437620D 00	3.9	-0.24349091D 00	4.0	-0.23233807D 00	4.1	-0.22099647D 00
4.2	-0.20953453D 00	4.3	-0.19801160D 00	4.4	-0.18647905D 00	4.5	-0.17498127D 00
4.6	-0.16355648D 00	4.7	-0.15223754D 00	4.8	-0.14105253D 00	4.9	-0.13002537D 00
5.0	-0.11917630D 00	5.1	-0.10852234D 00	5.2	-0.98077626D-01	5.3	-0.87853826D-01
5.4	-0.77860388D-01	5.5	-0.68104826D-01	5.6	-0.58592950D-01	5.7	-0.49329079D-01
5.8	-0.40316219D-01	5.9	-0.31556229D-01	6.0	-0.23049966D-01	6.1	-0.14797409D-01
6.2	-0.67977784D-02	6.3	0.95037095D-03	6.4	0.84490550D-02	6.5	0.15700784D-01
6.6	0.22708491D-01	6.7	0.29475472D-01	6.8	0.36005328D-01	6.9	0.42301919D-01
7.0	0.48369316D-01	7.1	0.54211769D-01	7.2	0.59833669D-01	7.3	0.65239517D-01
7.4	0.70433901D-01	7.5	0.75421469D-01	7.6	0.80206912D-01	7.7	0.84794942D-01
7.8	0.89190279D-01	7.9	0.93397636D-01	8.0	0.97421705D-01	8.1	0.10126715D 00
8.2	0.10493859D 00	8.3	0.10844061D 00	8.4	0.11177773D 00	8.5	0.11495441D 00
8.6	0.11797504D 00	8.7	0.12084396D 00	8.8	0.12356542D 00	8.9	0.12614359D 00
9.0	0.12858259D 00	9.1	0.13088643D 00	9.2	0.13305904D 00	9.3	0.13510430D 00
9.4	0.13702596D 00	9.5	0.13882774D 00	9.6	0.14051322D 00	9.7	0.14208594D 00
9.8	0.14354934D 00	9.9	0.14490677D 00	10.0	0.14616153D 00		

P= 2.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.74752076D 00	1.2	0.53837031D 00	1.3	0.36521323D 00
1.4	0.22207591D 00	1.5	0.10406337D 00	1.6	0.71418326D-02	1.7	-0.72029912D-01
1.8	-0.13623235D 00	1.9	-0.18778923D 00	2.0	-0.22865009D 00	2.1	-0.26045637D 00
2.2	-0.28459448D 00	2.3	-0.30223888D 00	2.4	-0.31438707D 00	2.5	-0.32188831D 00
2.6	-0.32546711D 00	2.7	-0.32574277D 00	2.8	-0.32324550D 00	2.9	-0.31842989D 00
3.0	-0.31168610D 00	3.1	-0.30334939D 00	3.2	-0.29370796D 00	3.3	-0.28300973D 00
3.4	-0.27146799D 00	3.5	-0.25926623D 00	3.6	-0.24656225D 00	3.7	-0.23349165D 00
3.8	-0.22017087D 00	3.9	-0.20669972D 00	4.0	-0.19316363D 00	4.1	-0.17963552D 00
4.2	-0.16617750D 00	4.3	-0.15284221D 00	4.4	-0.13967413D 00	4.5	-0.12671060D 00
4.6	-0.11398276D 00	4.7	-0.10151634D 00	4.8	-0.89332404D-01	4.9	-0.77447911D-01
5.0	-0.65876278D-01	5.1	-0.54627838D-01	5.2	-0.43710249D-01	5.3	-0.33128847D-01
5.4	-0.22886963D-01	5.5	-0.12986191D-01	5.6	-0.34266339D-02	5.7	0.57928921D-02
5.8	0.14674670D-01	5.9	0.23221922D-01	6.0	0.31438664D-01	6.1	0.39329588D-01
6.2	0.46899945D-01	6.3	0.54155454D-01	6.4	0.61102219D-01	6.5	0.67746651D-01
6.6	0.74095408D-01	6.7	0.80155334D-01	6.8	0.85933416D-01	6.9	0.91436737D-01
7.0	0.96672437D-01	7.1	0.10164769D 00	7.2	0.10636965D 00	7.3	0.11084548D 00
7.4	0.11508226D 00	7.5	0.11908703D 00	7.6	0.12286674D 00	7.7	0.12642826D 00
7.8	0.12977833D 00	7.9	0.13292361D 00	8.0	0.13587061D 00	8.1	0.13862575D 00
8.2	0.14119528D 00	8.3	0.14358534D 00	8.4	0.14580192D 00	8.5	0.14785088D 00
8.6	0.14973794D 00	8.7	0.15146866D 00	8.8	0.15304848D 00	8.9	0.15448269D 00
9.0	0.15577644D 00	9.1	0.15693473D 00	9.2	0.15796244D 00	9.3	0.15886430D 00
9.4	0.15964492D 00	9.5	0.16030876D 00	9.6	0.16086016D 00	9.7	0.16130333D 00
9.8	0.16164236D 00	9.9	0.16188122D 00	10.0	0.16202373D 00		

P= 2.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.72768744D 00	1.2	0.50492699D 00	1.3	0.32299568D 00
1.4	0.17483250D 00	1.5	0.54686594D-01	1.6	-0.42151864D-01	1.7	-0.11955953D 00
1.8	-0.18074179D 00	1.9	-0.22835992D 00	2.0	-0.26463167D 00	2.1	-0.29141137D 00
2.2	-0.31025414D 00	2.3	-0.32246768D 00	2.4	-0.32915422D 00	2.5	-0.33124478D 00
2.6	-0.32952713D 00	2.7	-0.32466885D 00	2.8	-0.31723637D 00	2.9	-0.30771070D 00
3.0	-0.29650064D 00	3.1	-0.28395367D 00	3.2	-0.27036521D 00	3.3	-0.25598630D 00
3.4	-0.24103012D 00	3.5	-0.22567750D 00	3.6	-0.21008159D 00	3.7	-0.19437181D 00
3.8	-0.17865724D 00	3.9	-0.16302947D 00	4.0	-0.14756509D 00	4.1	-0.13232776D 00
4.2	-0.11737007D 00	4.3	-0.10273504D 00	4.4	-0.88457484D-01	4.5	-0.74565156D-01
4.6	-0.61079744D-01	4.7	-0.48017725D-01	4.8	-0.35391109D-01	4.9	-0.23208081D-01
5.0	-0.11473554D-01	5.1	-0.18965547D-03	5.2	0.10643860D-01	5.3	0.21029245D-01
5.4	0.30970447D-01	5.5	0.40472835D-01	5.6	0.49542962D-01	5.7	0.58188362D-01
5.8	0.66417370D-01	5.9	0.74238968D-01	6.0	0.81662652D-01	6.1	0.88698318D-01
6.2	0.95356158D-01	6.3	0.10164658D 00	6.4	0.10758012D 00	6.5	0.11316740D 00
6.6	0.11841906D 00	6.7	0.12334570D 00	6.8	0.12795788D 00	6.9	0.13226602D 00
7.0	0.13628045D 00	7.1	0.14001133D 00	7.2	0.14346865D 00	7.3	0.14666221D 00
7.4	0.14960160D 00	7.5	0.15229623D 00	7.6	0.15475525D 00	7.7	0.15698761D 00
7.8	0.15900203D 00	7.9	0.16080699D 00	8.0	0.16241074D 00	8.1	0.16382129D 00
8.2	0.16504642D 00	8.3	0.16609369D 00	8.4	0.16697041D 00	8.5	0.16768368D 00
8.6	0.16824036D 00	8.7	0.16864711D 00	8.8	0.16891035D 00	8.9	0.16903632D 00
9.0	0.16903103D 00	9.1	0.16890029D 00	9.2	0.16864971D 00	9.3	0.16828472D 00
9.4	0.16781056D 00	9.5	0.16723229D 00	9.6	0.16655477D 00	9.7	0.16578271D 00
9.8	0.16492065D 00	9.9	0.16397296D 00	10.0	0.16294386D 00		

P= 2.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.70726489D 00	1.2	0.47094569D 00	1.3	0.28071546D 00
1.4	0.12826733D 00	1.5	0.68789495D-02	1.6	-0.88919289D-01	1.7	-0.16359947D 00
1.8	-0.22083412D 00	1.9	-0.26365101D 00	2.0	-0.29455512D 00	2.1	-0.31562536D 00
2.2	-0.32859175D 00	2.3	-0.33489739D 00	2.4	-0.33574854D 00	2.5	-0.33215512D 00
2.6	-0.32496384D 00	2.7	-0.31488531D 00	2.8	-0.30251626D 00	2.9	-0.28835792D 00
3.0	-0.27283130D 00	3.1	-0.25628978D 00	3.2	-0.23902970D 00	3.3	-0.22129915D 00
3.4	-0.20330539D 00	3.5	-0.18522108D 00	3.6	-0.16718949D 00	3.7	-0.14932894D 00
3.8	-0.13173655D 00	3.9	-0.11449143D 00	4.0	-0.97657355D-01	4.1	-0.81285080D-01
4.2	-0.65414293D-01	4.3	-0.50075286D-01	4.4	-0.35290383D-01	4.5	-0.21075160D-01
4.6	-0.74394921D-02	4.7	0.56115528D-02	4.8	0.18076944D-01	4.9	0.29959031D-01
5.0	0.41262979D-01	5.1	0.51996281D-01	5.2	0.62168347D-01	5.3	0.71790143D-01
5.4	0.80873887D-01	5.5	0.89432788D-01	5.6	0.97480820D-01	5.7	0.10503253D 00
5.8	0.11210288D 00	5.9	0.11870709D 00	6.0	0.12486056D 00	6.1	0.13057871D 00
6.2	0.13587696D 00	6.3	0.14077061D 00	6.4	0.14527479D 00	6.5	0.14940445D 00
6.6	0.15317425D 00	6.7	0.15659861D 00	6.8	0.15969159D 00	6.9	0.16246697D 00
7.0	0.16493815D 00	7.1	0.16711817D 00	7.2	0.16901973D 00	7.3	0.17065512D 00
7.4	0.17203629D 00	7.5	0.17317480D 00	7.6	0.17408182D 00	7.7	0.17476816D 00
7.8	0.17524428D 00	7.9	0.17552026D 00	8.0	0.17560581D 00	8.1	0.17551032D 00
8.2	0.17524282D 00	8.3	0.17481201D 00	8.4	0.17422627D 00	8.5	0.17349365D 00
8.6	0.17262189D 00	8.7	0.17161846D 00	8.8	0.17049050D 00	8.9	0.16924490D 00
9.0	0.16788825D 00	9.1	0.16642690D 00	9.2	0.16486693D 00	9.3	0.16321417D 00
9.4	0.16147423D 00	9.5	0.15965246D 00	9.6	0.15775402D 00	9.7	0.15578384D 00
9.8	0.15374663D 00	9.9	0.15164694D 00	10.0	0.14948908D 00		

P= 2.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.68628268D 00	1.2	0.43652121D 00	1.3	0.23854247D 00
1.4	0.82618897D-01	1.5	-0.39068772D-01	1.6	-0.13283902D 00	1.7	-0.20382165D 00
1.8	-0.25619881D 00	1.9	-0.29339269D 00	2.0	-0.31821266D 00	2.1	-0.33297153D 00
2.2	-0.33957783D 00	2.3	-0.33960963D 00	2.4	-0.33437380D 00	2.5	-0.32495388D 00
2.6	-0.31224890D 00	2.7	-0.29700507D 00	2.8	-0.27984158D 00	2.9	-0.26127193D 00
3.0	-0.24172139D 00	3.1	-0.22154153D 00	3.2	-0.20102215D 00	3.3	-0.18040135D 00
3.4	-0.15987379D 00	3.5	-0.13959768D 00	3.6	-0.11970060D 00	3.7	-0.10028438D 00
3.8	-0.81429242D-01	3.9	-0.63197221D-01	4.0	-0.45635132D-01	4.1	-0.28777011D-01
4.2	-0.12646217D-01	4.3	0.27428077D-02	4.4	0.17383035D-01	4.5	0.31273584D-01
4.6	0.44418642D-01	4.7	0.56826552D-01	4.8	0.68509038D-01	4.9	0.79480547D-01
5.0	0.89757691D-01	5.1	0.99358777D-01	5.2	0.10830341D 00	5.3	0.11661214D 00
5.4	0.12430621D 00	5.5	0.13140729D 00	5.6	0.13793727D 00	5.7	0.14391811D 00
5.8	0.14937173D 00	5.9	0.15431981D 00	6.0	0.15878378D 00	6.1	0.16278470D 00
6.2	0.16634321D 00	6.3	0.16947948D 00	6.4	0.17221315D 00	6.5	0.17456335D 00
6.6	0.17654862D 00	6.7	0.17818696D 00	6.8	0.17949575D 00	6.9	0.18049179D 00
7.0	0.18119132D 00	7.1	0.18160994D 00	7.2	0.18176273D 00	7.3	0.18166414D 00
7.4	0.18132810D 00	7.5	0.18076797D 00	7.6	0.17999657D 00	7.7	0.17902620D 00
7.8	0.17786866D 00	7.9	0.17653523D 00	8.0	0.17503671D 00	8.1	0.17338347D 00
8.2	0.17158538D 00	8.3	0.16965190D 00	8.4	0.16759207D 00	8.5	0.16541452D 00
8.6	0.16312749D 00	8.7	0.16073883D 00	8.8	0.15825606D 00	8.9	0.15568633D 00
9.0	0.15303644D 00	9.1	0.15031291D 00	9.2	0.14752192D 00	9.3	0.14466936D 00
9.4	0.14176085D 00	9.5	0.13880171D 00	9.6	0.13579703D 00	9.7	0.13275164D 00
9.8	0.12967012D 00	9.9	0.12655683D 00	10.0	0.12341591D 00		

P= 2.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.66477116D 00	1.2	0.40174922D 00	1.3	0.19664497D 00
1.4	0.38118238D-01	1.5	-0.82882375D-01	1.6	-0.17361823D 00	1.7	-0.23994076D 00
1.8	-0.28658380D 00	1.9	-0.31739003D 00	2.0	-0.33548787D 00	2.1	-0.34343034D 00
2.2	-0.34330500D 00	2.3	-0.33682130D 00	2.4	-0.32538041D 00	2.5	-0.31013139D 00
2.6	-0.29201646D 00	2.7	-0.27180769D 00	2.8	-0.25013692D 00	2.9	-0.22752012D 00
3.0	-0.20437740D 00	3.1	-0.18104942D 00	3.2	-0.15781096D 00	3.3	-0.13488210D 00
3.4	-0.11243748D 00	3.5	-0.90614028D-01	3.6	-0.69517349D-01	3.7	-0.49227067D-01
3.8	-0.29801282D-01	3.9	-0.11280296D-01	4.0	0.63102735D-02	4.1	0.22956879D-01
4.2	0.38655856D-01	4.3	0.53411593D-01	4.4	0.67234983D-01	4.5	0.80142145D-01
4.6	0.92153331D-01	4.7	0.10329203D 00	4.8	0.11358420D 00	4.9	0.12305764D 00
5.0	0.13174146D 00	5.1	0.13966565D 00	5.2	0.14686069D 00	5.3	0.15335727D 00
5.4	0.15918606D 00	5.5	0.16437745D 00	5.6	0.16896144D 00	5.7	0.17296749D 00
5.8	0.17642440D 00	5.9	0.17936027D 00	6.0	0.18180239D 00	6.1	0.18377721D 00
6.2	0.18531032D 00	6.3	0.18642642D 00	6.4	0.18714932D 00	6.5	0.18750191D 00
6.6	0.18750618D 00	6.7	0.18718324D 00	6.8	0.18655331D 00	6.9	0.18563574D 00
7.0	0.18444904D 00	7.1	0.18301090D 00	7.2	0.18133819D 00	7.3	0.17944703D 00
7.4	0.17735277D 00	7.5	0.17507002D 00	7.6	0.17261269D 00	7.7	0.16999404D 00
7.8	0.16722665D 00	7.9	0.16432247D 00	8.0	0.16129287D 00	8.1	0.15814863D 00
8.2	0.15489997D 00	8.3	0.15155659D 00	8.4	0.14812769D 00	8.5	0.14462195D 00
8.6	0.14104763D 00	8.7	0.13741252D 00	8.8	0.13372399D 00	8.9	0.12998901D 00
9.0	0.12621417D 00	9.1	0.12240567D 00	9.2	0.11856939D 00	9.3	0.11471085D 00
9.4	0.11083527D 00	9.5	0.10694755D 00	9.6	0.10305232D 00	9.7	0.99153922D-01
9.8	0.95256437D-01	9.9	0.91363702D-01	10.0	0.87479314D-01		

P= 2.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.64276136D 00	1.2	0.36672596D 00	1.3	0.15518879D 00
1.4	-0.50124519D-02	1.5	-0.12430621D 00	1.6	-0.21099503D 00	1.7	-0.27171681D 00
1.8	-0.31179778D 00	1.9	-0.33552466D 00	2.0	-0.34635616D 00	2.1	-0.34708830D 00
2.2	-0.33998449D 00	2.3	-0.32687846D 00	2.4	-0.30925608D 00	2.5	-0.28832083D 00
2.6	-0.26504621D 00	2.7	-0.24021816D 00	2.8	-0.21446920D 00	2.9	-0.18830628D 00
3.0	-0.16213339D 00	3.1	-0.13627007D 00	3.2	-0.11096652D 00	3.3	-0.86416018D-01
3.4	-0.62765204D-01	3.5	-0.40122446D-01	3.6	-0.18564825D-01	3.7	0.18561219D-02
3.8	0.21109641D-01	3.9	0.39181817D-01	4.0	0.56072310D-01	4.1	0.71791639D-01
4.2	0.86358945D-01	4.3	0.99800122D-01	4.4	0.11214628D 00	4.5	0.12343246D 00
4.6	0.13369660D 00	4.7	0.14297866D 00	4.8	0.15131988D 00	4.9	0.15876225D 00
5.0	0.16534799D 00	5.1	0.17111919D 00	5.2	0.17611748D 00	5.3	0.18038381D 00
5.4	0.18395821D 00	5.5	0.18687972D 00	5.6	0.18918617D 00	5.7	0.19091420D 00
5.8	0.19209912D 00	5.9	0.19277494D 00	6.0	0.19297429D 00	6.1	0.19272846D 00
6.2	0.19206736D 00	6.3	0.19101958D 00	6.4	0.18961240D 00	6.5	0.18787180D 00
6.6	0.18582247D 00	6.7	0.18348794D 00	6.8	0.18089049D 00	6.9	0.17805129D 00
7.0	0.17499041D 00	7.1	0.17172683D 00	7.2	0.16827853D 00	7.3	0.16466250D 00
7.4	0.16089479D 00	7.5	0.15699056D 00	7.6	0.15296410D 00	7.7	0.14882891D 00
7.8	0.14459766D 00	7.9	0.14028232D 00	8.0	0.13589414D 00	8.1	0.13144367D 00
8.2	0.12694086D 00	8.3	0.12239502D 00	8.4	0.11781488D 00	8.5	0.11320865D 00
8.6	0.10858398D 00	8.7	0.10394806D 00	8.8	0.99307591D-01	8.9	0.94668832D-01
9.0	0.90037625D-01	9.1	0.85419413D-01	9.2	0.80819258D-01	9.3	0.76241868D-01
9.4	0.71691614D-01	9.5	0.67172545D-01	9.6	0.62688409D-01	9.7	0.58242669D-01
9.8	0.53838519D-01	9.9	0.49478898D-01	10.0	0.45166503D-01		

P= 2.90

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.62028501D 00	1.2	0.33154789D 00	1.3	0.11433660D 00
1.4	-0.46561116D-01	1.5	-0.16310496D 00	1.6	-0.24474060D 00	1.7	-0.29895718D 00
1.8	-0.33171181D 00	1.9	-0.34775529D 00	2.0	-0.35088388D 00	2.1	-0.34413504D 00
2.2	-0.32994070D 00	2.3	-0.31024786D 00	2.4	-0.28661389D 00	2.5	-0.26028236D 00
2.6	-0.23224345D 00	2.7	-0.20328245D 00	2.8	-0.17401868D 00	2.9	-0.14493686D 00
3.0	-0.11641261D 00	3.1	-0.88732986D-01	3.2	-0.62113302D-01	3.3	-0.36710739D-01
3.4	-0.12635512D-01	3.5	0.10040022D-01	3.6	0.31273915D-01	3.7	0.51048518D-01
3.8	0.69365478D-01	3.9	0.86241641D-01	4.0	0.10170569D 00	4.1	0.11579539D 00
4.2	0.12855535D 00	4.3	0.14003517D 00	4.4	0.15028794D 00	4.5	0.15936906D 00
4.6	0.16733520D 00	4.7	0.17424355D 00	4.8	0.18015119D 00	4.9	0.18511453D 00
5.0	0.18918900D 00	5.1	0.19242867D 00	5.2	0.19488607D 00	5.3	0.19661198D 00
5.4	0.19765535D 00	5.5	0.19806317D 00	5.6	0.19788047D 00	5.7	0.19715025D 00
5.8	0.19591353D 00	5.9	0.19420934D 00	6.0	0.19207475D 00	6.1	0.18954491D 00
6.2	0.18665311D 00	6.3	0.18343086D 00	6.4	0.17990788D 00	6.5	0.17611224D 00
6.6	0.17207037D 00	6.7	0.16780717D 00	6.8	0.16334605D 00	6.9	0.15870899D 00
7.0	0.15391664D 00	7.1	0.14898834D 00	7.2	0.14394223D 00	7.3	0.13879527D 00
7.4	0.13356333D 00	7.5	0.12826121D 00	7.6	0.12290276D 00	7.7	0.11750085D 00
7.8	0.11206748D 00	7.9	0.10661382D 00	8.0	0.10115024D 00	8.1	0.95686355D-01
8.2	0.90231091D-01	8.3	0.84792698D-01	8.4	0.79378801D-01	8.5	0.73996435D-01
8.6	0.68652080D-01	8.7	0.63351693D-01	8.8	0.58100737D-01	8.9	0.52904214D-01
9.0	0.47766692D-01	9.1	0.42692330D-01	9.2	0.37684901D-01	9.3	0.32747823D-01
9.4	0.27884173D-01	9.5	0.23096712D-01	9.6	0.18387907D-01	9.7	0.13759944D-01
9.8	0.92147502D-02	9.9	0.47540111D-02	10.0	0.37918310D-03		

P= 3.00

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.59737443D 00	1.2	0.29631147D 00	1.3	0.74247142D-01
1.4	-0.86327079D-01	1.5	-0.19906529D 00	1.6	-0.27466092D 00	1.7	-0.32151809D 00
1.8	-0.34626005D 00	1.9	-0.35411733D 00	2.0	-0.34922606D 00	2.1	-0.33485874D 00
2.2	-0.31360384D 00	2.3	-0.28750629D 00	2.4	-0.25817796D 00	2.5	-0.22688491D 00
2.6	-0.19461655D 00	2.7	-0.16214071D 00	2.8	-0.13004763D 00	2.9	-0.98785192D-01
3.0	-0.68687169D-01	3.1	-0.39996038D-01	3.2	-0.12881330D-01	3.3	0.12545531D-01
3.4	0.36219222D-01	3.5	0.58110805D-01	3.6	0.78219855D-01	3.7	0.96568083D-01
3.8	0.11319417D 00	3.9	0.12814959D 00	4.0	0.14149522D 00	4.1	0.15329866D 00
4.2	0.16363197D 00	4.3	0.17257001D 00	4.4	0.18018895D 00	4.5	0.18656524D 00
4.6	0.19177468D 00	4.7	0.19589181D 00	4.8	0.19898933D 00	4.9	0.20113775D 00
5.0	0.20240511D 00	5.1	0.20285677D 00	5.2	0.20255526D 00	5.3	0.20156027D 00
5.4	0.19992856D 00	5.5	0.19771397D 00	5.6	0.19496749D 00	5.7	0.19173729D 00
5.8	0.18806879D 00	5.9	0.18400475D 00	6.0	0.17958535D 00	6.1	0.17484832D 00
6.2	0.16982899D 00	6.3	0.16456043D 00	6.4	0.15907357D 00	6.5	0.15339726D 00
6.6	0.14755840D 00	6.7	0.14158204D 00	6.8	0.13549148D 00	6.9	0.12930834D 00
7.0	0.12305269D 00	7.1	0.11674312D 00	7.2	0.11039683D 00	7.3	0.10402970D 00
7.4	0.97656373D-01	7.5	0.91290342D-01	7.6	0.84944007D-01	7.7	0.78628745D-01
7.8	0.72354976D-01	7.9	0.66132220D-01	8.0	0.59969156D-01	8.1	0.53873676D-01
8.2	0.47852937D-01	8.3	0.41913405D-01	8.4	0.36060904D-01	8.5	0.30300657D-01
8.6	0.24637327D-01	8.7	0.19075056D-01	8.8	0.13617498D-01	8.9	0.82678531D-02
9.0	0.30289010D-02	9.1	-0.20969710D-02	9.2	-0.71077401D-02	9.3	-0.12001722D-01
9.4	-0.16777544D-01	9.5	-0.21434128D-01	9.6	-0.25970662D-01	9.7	-0.30386583D-01
9.8	-0.34681559D-01	9.9	-0.38855470D-01	10.0	-0.42908392D-01		

P= 3.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.57406248D 00	1.2	0.26111276D 00	1.3	0.35074539D-01
1.4	-0.12412220D 00	1.5	-0.23199727D 00	1.6	-0.30059821D 00	1.7	-0.33930563D 00
1.8	-0.35543984D 00	1.9	-0.35472136D 00	2.0	-0.34162291D 00	2.1	-0.31964007D 00
2.2	-0.29150075D 00	2.3	-0.25932797D 00	2.4	-0.22476713D 00	2.5	-0.18908583D 00
2.6	-0.15325249D 00	2.7	-0.11799848D 00	2.8	-0.83867470D-01	2.9	-0.51254501D-01
3.0	-0.20437256D-01	3.1	0.83990552D-02	3.2	0.35141784D-01	3.3	0.59734911D-01
3.4	0.82166287D-01	3.5	0.10245742D 00	3.6	0.12065530D 00	3.7	0.13682584D 00
3.8	0.15104870D 00	3.9	0.16341310D 00	4.0	0.17401449D 00	4.1	0.18295200D 00
4.2	0.19032638D 00	4.3	0.19623838D 00	4.4	0.20078751D 00	4.5	0.20407115D 00
4.6	0.20618382D 00	4.7	0.20721666D 00	4.8	0.20725711D 00	4.9	0.20638864D 00
5.0	0.20469063D 00	5.1	0.20223831D 00	5.2	0.19910275D 00	5.3	0.19535093D 00
5.4	0.19104579D 00	5.5	0.18624635D 00	5.6	0.18100787D 00	5.7	0.17538194D 00
5.8	0.16941668D 00	5.9	0.16315687D 00	6.0	0.15664411D 00	6.1	0.14991701D 00
6.2	0.14301131D 00	6.3	0.13596007D 00	6.4	0.12879381D 00	6.5	0.12154065D 00
6.6	0.11422648D 00	6.7	0.10687505D 00	6.8	0.99508160D-01	6.9	0.92145739D-01
7.0	0.84805987D-01	7.1	0.77505484D-01	7.2	0.70259296D-01	7.3	0.63081077D-01
7.4	0.55983166D-01	7.5	0.48976676D-01	7.6	0.42071581D-01	7.7	0.35276795D-01
7.8	0.28600247D-01	7.9	0.22048953D-01	8.0	0.15629083D-01	8.1	0.93460200D-02
8.2	0.32044210D-02	8.3	-0.27917285D-02	8.4	-0.86390642D-02	8.5	-0.14334795D-01
8.6	-0.19876659D-01	8.7	-0.25262879D-01	8.8	-0.30492129D-01	8.9	-0.35563492D-01
9.0	-0.40476430D-01	9.1	-0.45230750D-01	9.2	-0.49826577D-01	9.3	-0.54264324D-01
9.4	-0.58544669D-01	9.5	-0.62668529D-01	9.6	-0.66637038D-01	9.7	-0.70451529D-01
9.8	-0.74113513D-01	9.9	-0.77624659D-01	10.0	-0.80986783D-01		

P= 3.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.55038255D 00	1.2	0.22604721D 00	1.3	-0.30324279D-02
1.4	-0.15977194D 00	1.5	-0.26173563D 00	1.6	-0.32243198D 00	1.7	-0.35227615D 00
1.8	-0.35931089D 00	1.9	-0.34975067D 00	2.0	-0.32839502D 00	2.1	-0.29894462D 00
2.2	-0.26424422D 00	2.3	-0.22647035D 00	2.4	-0.18727695D 00	2.5	-0.14790895D 00
2.6	-0.10929097D 00	2.7	-0.72096997D-01	2.8	-0.36805133D-01	2.9	-0.37407659D-02
3.0	0.26889331D-01	3.1	0.54970045D-01	3.2	0.80456595D-01	3.3	0.10335764D 00
3.4	0.12372194D 00	3.5	0.14162775D 00	3.6	0.15717453D 00	3.7	0.17047638D 00
3.8	0.18165687D 00	3.9	0.19084503D 00	4.0	0.19817228D 00	4.1	0.20376998D 00
4.2	0.20776765D 00	4.3	0.21029163D 00	4.4	0.21146403D 00	4.5	0.21140209D 00
4.6	0.21021765D 00	4.7	0.20801691D 00	4.8	0.20490025D 00	4.9	0.20096218D 00
5.0	0.19629141D 00	5.1	0.19097092D 00	5.2	0.18507816D 00	5.3	0.17868520D 00
5.4	0.17185895D 00	5.5	0.16466142D 00	5.6	0.15714991D 00	5.7	0.14937731D 00
5.8	0.14139227D 00	5.9	0.13323953D 00	6.0	0.12496008D 00	6.1	0.11659143D 00
6.2	0.10816784D 00	6.3	0.99720481D-01	6.4	0.91277707D-01	6.5	0.82865196D-01
6.6	0.74506159D-01	6.7	0.66221511D-01	6.8	0.58030029D-01	6.9	0.49948516D-01
7.0	0.41991940D-01	7.1	0.34173576D-01	7.2	0.26505130D-01	7.3	0.18996864D-01
7.4	0.11657702D-01	7.5	0.44953437D-02	7.6	-0.24836452D-02	7.7	-0.92737369D-02
7.8	-0.15870358D-01	7.9	-0.22269810D-01	8.0	-0.28469195D-01	8.1	-0.34466348D-01
8.2	-0.40259773D-01	8.3	-0.45848580D-01	8.4	-0.51232439D-01	8.5	-0.56411519D-01
8.6	-0.61386447D-01	8.7	-0.66158264D-01	8.8	-0.70728382D-01	8.9	-0.75098546D-01
9.0	-0.79270805D-01	9.1	-0.83247474D-01	9.2	-0.87031107D-01	9.3	-0.90624469D-01
9.4	-0.94030513D-01	9.5	-0.97252353D-01	9.6	-0.10029325D 00	9.7	-0.10315658D 00
9.8	-0.10584582D 00	9.9	-0.10836456D 00	10.0	-0.11071643D 00		

P= 3.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.52636848D 00	1.2	0.19120930D 00	1.3	-0.39930969D-01
1.4	-0.19311636D 00	1.5	-0.28814082D 00	1.6	-0.34007978D 00	1.7	-0.36043616D 00
1.8	-0.35799383D 00	1.9	-0.33945777D 00	2.0	-0.30993746D 00	2.1	-0.27331407D 00
2.2	-0.23252089D 00	2.3	-0.18975844D 00	2.4	-0.14666034D 00	2.5	-0.10442151D 00
2.6	-0.63897708D-01	2.7	-0.25682899D-01	2.8	0.98303187D-02	2.9	0.42403598D-01
3.0	0.71915964D-01	3.1	0.98334974D-01	3.2	0.12169415D 00	3.3	0.14207535D 00
3.4	0.15959500D 00	3.5	0.17439338D 00	3.6	0.18662632D 00	3.7	0.19645880D 00
3.8	0.20406003D 00	3.9	0.20959972D 00	4.0	0.21324529D 00	4.1	0.21515983D 00
4.2	0.21550058D 00	4.3	0.21441795D 00	4.4	0.21205481D 00	4.5	0.20854611D 00
4.6	0.20401868D 00	4.7	0.19859123D 00	4.8	0.19237444D 00	4.9	0.18547112D 00
5.0	0.17797651D 00	5.1	0.16997857D 00	5.2	0.16155827D 00	5.3	0.15279003D 00
5.4	0.14374197D 00	5.5	0.13447639D 00	5.6	0.12505002D 00	5.7	0.11551447D 00
5.8	0.10591650D 00	5.9	0.96298390D-01	6.0	0.86698244D-01	6.1	0.77150288D-01
6.2	0.67685163D-01	6.3	0.58330191D-01	6.4	0.49109628D-01	6.5	0.40044906D-01
6.6	0.31154855D-01	6.7	0.22455914D-01	6.8	0.13962328D-01	6.9	0.56863270D-02
7.0	-0.23617025D-02	7.1	-0.10173058D-01	7.2	-0.17740575D-01	7.3	-0.25058493D-01
7.4	-0.32122326D-01	7.5	-0.38928749D-01	7.6	-0.45475486D-01	7.7	-0.51761216D-01
7.8	-0.57785474D-01	7.9	-0.63548573D-01	8.0	-0.69051520D-01	8.1	-0.74295945D-01
8.2	-0.79284040D-01	8.3	-0.84018489D-01	8.4	-0.88502417D-01	8.5	-0.92739341D-01
8.6	-0.96733116D-01	8.7	-0.10048790D 00	8.8	-0.10400810D 00	8.9	-0.10729835D 00
9.0	-0.11036348D 00	9.1	-0.11320847D 00	9.2	-0.11583843D 00	9.3	-0.11825859D 00
9.4	-0.12047423D 00	9.5	-0.12249074D 00	9.6	-0.12431351D 00	9.7	-0.12594798D 00
9.8	-0.12739959D 00	9.9	-0.12867378D 00	10.0	-0.12977598D 00		

P= 3.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.50205449D 00	1.2	0.15669228D 00	1.3	-0.75484921D-01
1.4	-0.22401097D 00	1.5	-0.31109986D 00	1.6	-0.35349755D 00	1.7	-0.36384168D 00
1.8	-0.35166793D 00	1.9	-0.32416010D 00	2.0	-0.28671284D 00	2.1	-0.24335627D 00
2.2	-0.19707794D 00	2.3	-0.15006812D 00	2.4	-0.10390741D 00	2.5	-0.59710614D-01
2.6	-0.18237563D-01	2.7	0.20021537D-01	2.8	0.54780124D-01	2.9	0.85904295D-01
3.0	0.11337337D 00	3.1	0.13724939D 00	3.2	0.15765364D 00	3.3	0.17474848D 00
3.4	0.18872345D 00	3.5	0.19978467D 00	3.6	0.20814674D 00	3.7	0.21402675D 00
3.8	0.21763975D 00	3.9	0.21919546D 00	4.0	0.21889598D 00	4.1	0.21693417D 00
4.2	0.21349263D 00	4.3	0.20874306D 00	4.4	0.20284606D 00	4.5	0.19595105D 00
4.6	0.18819647D 00	4.7	0.17971007D 00	4.8	0.17060927D 00	4.9	0.16100169D 00
5.0	0.15098557D 00	5.1	0.14065037D 00	5.2	0.13007723D 00	5.3	0.11933958D 00
5.4	0.10850359D 00	5.5	0.97628715D-01	5.6	0.86768190D-01	5.7	0.75969480D-01
5.8	0.65274740D-01	5.9	0.54721223D-01	6.0	0.44341689D-01	6.1	0.34164767D-01
6.2	0.24215311D-01	6.3	0.14514718D-01	6.4	0.50812369D-02	6.5	-0.40697547D-02
6.6	-0.12925489D-01	6.7	-0.21475565D-01	6.8	-0.29711730D-01	6.9	-0.37627670D-01
7.0	-0.45218819D-01	7.1	-0.52482189D-01	7.2	-0.59416207D-01	7.3	-0.66020567D-01
7.4	-0.72296095D-01	7.5	-0.78244624D-01	7.6	-0.83868885D-01	7.7	-0.89172397D-01
7.8	-0.94159373D-01	7.9	-0.98834638D-01	8.0	-0.10320354D 00	8.1	-0.10727190D 00
8.2	-0.11104592D 00	8.3	-0.11453212D 00	8.4	-0.11773733D 00	8.5	-0.12066858D 00
8.6	-0.12333310D 00	8.7	-0.12573827D 00	8.8	-0.12789155D 00	8.9	-0.12980051D 00
9.0	-0.13147274D 00	9.1	-0.13291587D 00	9.2	-0.13413751D 00	9.3	-0.13514524D 00
9.4	-0.13594662D 00	9.5	-0.13654914D 00	9.6	-0.13696019D 00	9.7	-0.13718711D 00
9.8	-0.13723711D 00	9.9	-0.13711731D 00	10.0	-0.13683471D 00		

P= 3.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.47747515D 00	1.2	0.12258785D 00	1.3	-0.10956535D 00
1.4	-0.25232756D 00	1.5	-0.33052694D 00	1.6	-0.36267960D 00	1.7	-0.36259704D 00
1.8	-0.34056819D 00	1.9	-0.30423476D 00	2.0	-0.25924338D 00	2.1	-0.20973426D 00
2.2	-0.15870903D 00	2.3	-0.10830872D 00	2.4	-0.60024775D-01	2.5	-0.14859428D-01
2.6	0.26551899D-01	2.7	0.63847192D-01	2.8	0.96866436D-01	2.9	0.12559704D 00
3.0	0.15013216D 00	3.1	0.17063894D 00	3.2	0.18733434D 00	3.3	0.20046690D 00
3.4	0.21030298D 00	3.5	0.21711656D 00	3.6	0.22118163D 00	3.7	0.22276681D 00
3.8	0.22213146D 00	3.9	0.21952299D 00	4.0	0.21517522D 00	4.1	0.20930737D 00
4.2	0.20212355D 00	4.3	0.19381275D 00	4.4	0.18454902D 00	4.5	0.17449190D 00
4.6	0.16378705D 00	4.7	0.15256684D 00	4.8	0.14095114D 00	4.9	0.12904807D 00
5.0	0.11695477D 00	5.1	0.10475814D 00	5.2	0.92535659D-01	5.3	0.80356026D-01
5.4	0.68279915D-01	5.5	0.56360609D-01	5.6	0.44644627D-01	5.7	0.33172309D-01
5.8	0.21978361D-01	5.9	0.11092369D-01	6.0	0.53927189D-03	6.1	-0.96602026D-02
6.2	-0.19489128D-01	6.3	-0.28934005D-01	6.4	-0.37984420D-01	6.5	-0.46632723D-01
6.6	-0.54873744D-01	6.7	-0.62704518D-01	6.8	-0.70124051D-01	6.9	-0.77133088D-01
7.0	-0.83733920D-01	7.1	-0.89930189D-01	7.2	-0.95726729D-01	7.3	-0.10112941D 00
7.4	-0.10614498D 00	7.5	-0.11078100D 00	7.6	-0.11504564D 00	7.7	-0.11894764D 00
7.8	-0.12249623D 00	7.9	-0.12570097D 00	8.0	-0.12857175D 00	8.1	-0.13111869D 00
8.2	-0.13335207D 00	8.3	-0.13528232D 00	8.4	-0.13691991D 00	8.5	-0.13827536D 00
8.6	-0.13935915D 00	8.7	-0.14018175D 00	8.8	-0.14075353D 00	8.9	-0.14108477D 00
9.0	-0.14118563D 00	9.1	-0.14106613D 00	9.2	-0.14073610D 00	9.3	-0.14020524D 00
9.4	-0.13948302D 00	9.5	-0.13857874D 00	9.6	-0.13750147D 00	9.7	-0.13626009D 00
9.8	-0.13486325D 00	9.9	-0.13331936D 00	10.0	-0.13163662D 00		

P= 3.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.45266532D 00	1.2	0.88985924D-01	1.3	-0.14205111D 00
1.4	-0.27795482D 00	1.5	-0.34636385D 00	1.6	-0.36765829D 00	1.7	-0.35685320D 00
1.8	-0.32498177D 00	1.9	-0.28011260D 00	2.0	-0.22810220D 00	2.1	-0.17315461D 00
2.2	-0.11823942D 00	2.3	-0.65405140D-01	2.4	-0.16014878D-01	2.5	0.29076006D-01
2.6	0.69385798D-01	2.7	0.10470511D 00	2.8	0.13502035D 00	2.9	0.16045597D 00
3.0	0.18123099D 00	3.1	0.19762642D 00	3.2	0.20996089D 00	3.3	0.21857269D 00
3.4	0.22380660D 00	3.5	0.22600433D 00	3.6	0.22549780D 00	3.7	0.22260459D 00
3.8	0.21762489D 00	3.9	0.21083969D 00	4.0	0.20250990D 00	4.1	0.19287606D 00
4.2	0.18215862D 00	4.3	0.17055842D 00	4.4	0.15825756D 00	4.5	0.14542033D 00
4.6	0.13219422D 00	4.7	0.11871107D 00	4.8	0.10508812D 00	4.9	0.91429152D-01
5.0	0.77825503D-01	5.1	0.64357142D-01	5.2	0.51093614D-01	5.3	0.38094979D-01
5.4	0.25412671D-01	5.5	0.13090312D-01	5.6	0.11644609D-02	5.7	-0.10334684D-01
5.8	-0.21382645D-01	5.9	-0.31960073D-01	6.0	-0.42052202D-01	6.1	-0.51648353D-01
6.2	-0.60741477D-01	6.3	-0.69327742D-01	6.4	-0.77406153D-01	6.5	-0.84978213D-01
6.6	-0.92047605D-01	6.7	-0.98619916D-01	6.8	-0.10470238D 00	6.9	-0.11030364D 00
7.0	-0.11543356D 00	7.1	-0.12010302D 00	7.2	-0.12432374D 00	7.3	-0.12810817D 00
7.4	-0.13146930D 00	7.5	-0.13442058D 00	7.6	-0.13697580D 00	7.7	-0.13914900D 00
7.8	-0.14095436D 00	7.9	-0.14240617D 00	8.0	-0.14351871D 00	8.1	-0.14430624D 00
8.2	-0.14478293D 00	8.3	-0.14496280D 00	8.4	-0.14485970D 00	8.5	-0.14448728D 00
8.6	-0.14385894D 00	8.7	-0.14298783D 00	8.8	-0.14188683D 00	8.9	-0.14056850D 00
9.0	-0.13904510D 00	9.1	-0.13732857D 00	9.2	-0.13543051D 00	9.3	-0.13336219D 00
9.4	-0.13113455D 00	9.5	-0.12875816D 00	9.6	-0.12624326D 00	9.7	-0.12359976D 00
9.8	-0.12083719D 00	9.9	-0.11796478D 00	10.0	-0.11499139D 00		

P= 3.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.42766008D 00	1.2	0.55974312D-01	1.3	-0.17282936D 00
1.4	-0.30079890D 00	1.5	-0.35858015D 00	1.6	-0.36850327D 00	1.7	-0.34680555D 00
1.8	-0.30524377D 00	1.9	-0.25227158D 00	2.0	-0.19390398D 00	2.1	-0.13435513D 00
2.2	-0.76510893D-01	2.3	-0.22279987D-01	2.4	0.27144349D-01	2.5	0.71089109D-01
2.6	0.10925571D 00	2.7	0.14161112D 00	2.8	0.16830671D 00	2.9	0.18961789D 00
3.0	0.20589944D 00	3.1	0.21755265D 00	3.2	0.22500136D 00	3.3	0.22867472D 00
3.4	0.22899495D 00	3.5	0.22636902D 00	3.6	0.22118301D 00	3.7	0.21379866D 00
3.8	0.20455153D 00	3.9	0.19375016D 00	4.0	0.18167619D 00	4.1	0.16858496D 00
4.2	0.15470657D 00	4.3	0.14024716D 00	4.4	0.12539036D 00	4.5	0.11029884D 00
4.6	0.95115858D-01	4.7	0.79966807D-01	4.8	0.64960726D-01	4.9	0.50191740D-01
5.0	0.35740436D-01	5.1	0.21675151D-01	5.2	0.80531758D-02	5.3	-0.50781224D-02
5.4	-0.17680267D-01	5.5	-0.29722716D-01	5.6	-0.41181995D-01	5.7	-0.52040891D-01
5.8	-0.62287728D-01	5.9	-0.71915705D-01	6.0	-0.80922297D-01	6.1	-0.89308707D-01
6.2	-0.97079380D-01	6.3	-0.10424156D 00	6.4	-0.11080489D 00	6.5	-0.11678105D 00
6.6	-0.12218347D 00	6.7	-0.12702697D 00	6.8	-0.13132759D 00	6.9	-0.13510232D 00
7.0	-0.13836889D 00	7.1	-0.14114561D 00	7.2	-0.14345119D 00	7.3	-0.14530465D 00
7.4	-0.14672512D 00	7.5	-0.14773180D 00	7.6	-0.14834386D 00	7.7	-0.14858029D 00
7.8	-0.14845992D 00	7.9	-0.14800130D 00	8.0	-0.14722269D 00	8.1	-0.14614198D 00
8.2	-0.14477665D 00	8.3	-0.14314380D 00	8.4	-0.14126004D 00	8.5	-0.13914155D 00
8.6	-0.13680402D 00	8.7	-0.13426262D 00	8.8	-0.13153206D 00	8.9	-0.12862653D 00
9.0	-0.12555972D 00	9.1	-0.12234478D 00	9.2	-0.11899442D 00	9.3	-0.11552079D 00
9.4	-0.11193558D 00	9.5	-0.10824999D 00	9.6	-0.10447474D 00	9.7	-0.10062008D 00
9.8	-0.96695790D-01	9.9	-0.92711220D-01	10.0	-0.88675272D-01		

P= 3.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.40249468D 00	1.2	0.23638470D-01	1.3	-0.20179605D 00
1.4	-0.32078392D 00	1.5	-0.36717314D 00	1.6	-0.36532043D 00	1.7	-0.33269124D 00
1.8	-0.28173248D 00	1.9	-0.22122955D 00	2.0	-0.15729505D 00	2.1	-0.94092271D-01
2.2	-0.34366454D-01	2.3	0.20163996D-01	2.4	0.68518971D-01	2.5	0.11024365D 00
2.6	0.14525131D 00	2.7	0.17370752D 00	2.8	0.19594505D 00	2.9	0.21240172D 00
3.0	0.22357520D 00	3.1	0.22999063D 00	3.2	0.23217764D 00	3.3	0.23065443D 00
3.4	0.22591704D 00	3.5	0.21843249D 00	3.6	0.20863475D 00	3.7	0.19692273D 00
3.8	0.18365975D 00	3.9	0.16917401D 00	4.0	0.15375986D 00	4.1	0.13767941D 00
4.2	0.12116453D 00	4.3	0.10441894D 00	4.4	0.87620396D-01	4.5	0.70922867D-01
4.6	0.54458638D-01	4.7	0.38340345D-01	4.8	0.22662897D-01	4.9	0.75052750D-02
5.0	-0.70677892D-02	5.1	-0.21004262D-01	5.2	-0.34263382D-01	5.3	-0.46814359D-01
5.4	-0.58635181D-01	5.5	-0.69711536D-01	5.6	-0.80035837D-01	5.7	-0.89606335D-01
5.8	-0.98426331D-01	5.9	-0.10650346D 00	6.0	-0.11384904D 00	6.1	-0.12047754D 00
6.2	-0.12640601D 00	6.3	-0.13165370D 00	6.4	-0.13624160D 00	6.5	-0.14019212D 00
6.6	-0.14352875D 00	6.7	-0.14627580D 00	6.8	-0.14845818D 00	6.9	-0.15010114D 00
7.0	-0.15123012D 00	7.1	-0.15187058D 00	7.2	-0.15204787D 00	7.3	-0.15178710D 00
7.4	-0.15111303D 00	7.5	-0.15005003D 00	7.6	-0.14862195D 00	7.7	-0.14685209D 00
7.8	-0.14476316D 00	7.9	-0.14237722D 00	8.0	-0.13971566D 00	8.1	-0.13679918D 00
8.2	-0.13364776D 00	8.3	-0.13028066D 00	8.4	-0.12671642D 00	8.5	-0.12297283D 00
8.6	-0.11906696D 00	8.7	-0.11501516D 00	8.8	-0.11083306D 00	8.9	-0.10653558D 00
9.0	-0.10213695D 00	9.1	-0.97650723D-01	9.2	-0.93089777D-01	9.3	-0.88466350D-01
9.4	-0.83792043D-01	9.5	-0.79077846D-01	9.6	-0.74334151D-01	9.7	-0.69570776D-01
9.8	-0.64796979D-01	9.9	-0.60021480D-01	10.0	-0.55252480D-01		

P= 3.90

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.37720449D 00	1.2	-0.79387622D-02	1.3	-0.22885637D 00
1.4	-0.33785221D 00	1.5	-0.37216764D 00	1.6	-0.35825051D 00	1.7	-0.31478616D 00
1.8	-0.25486423D 00	1.9	-0.18753660D 00	2.0	-0.11894315D 00	2.1	-0.53128332D-01
2.2	0.73646898D-02	2.3	0.61060891D-01	2.4	0.10723736D 00	2.5	0.14569275D 00
2.6	0.17657936D 00	2.7	0.20028125D 00	2.8	0.21732622D 00	2.9	0.22832271D 00
3.0	0.23391511D 00	3.1	0.23475258D 00	3.2	0.23146782D 00	3.3	0.22466310D 00
3.4	0.21490155D 00	3.5	0.20270234D 00	3.6	0.18853853D 00	3.7	0.17283681D 00
3.8	0.15597869D 00	3.9	0.13830235D 00	4.0	0.12010528D 00	4.1	0.10164703D 00
4.2	0.83152259D-01	4.3	0.64813701D-01	4.4	0.46795147D-01	4.5	0.29234273D-01
4.6	0.12245337D-01	4.7	-0.40783043D-02	4.8	-0.19661858D-01	4.9	-0.34446939D-01
5.0	-0.48389606D-01	5.1	-0.61458560D-01	5.2	-0.73633528D-01	5.3	-0.84903791D-01
5.4	-0.95266867D-01	5.5	-0.10472733D 00	5.6	-0.11329576D 00	5.7	-0.12098780D 00
5.8	-0.12782330D 00	5.9	-0.13382563D 00	6.0	-0.13902096D 00	6.1	-0.14343776D 00
6.2	-0.14710622D 00	6.3	-0.15005786D 00	6.4	-0.15232514D 00	6.5	-0.15394110D 00
6.6	-0.15493910D 00	6.7	-0.15535255D 00	6.8	-0.15521472D 00	6.9	-0.15455852D 00
7.0	-0.15341640D 00	7.1	-0.15182018D 00	7.2	-0.14980095D 00	7.3	-0.14738902D 00
7.4	-0.14461379D 00	7.5	-0.14150374D 00	7.6	-0.13808639D 00	7.7	-0.13438823D 00
7.8	-0.13043473D 00	7.9	-0.12625035D 00	8.0	-0.12185846D 00	8.1	-0.11728144D 00
8.2	-0.11254060D 00	8.3	-0.10765625D 00	8.4	-0.10264769D 00	8.5	-0.97533220D-01
8.6	-0.92330197D-01	8.7	-0.87055025D-01	8.8	-0.81723195D-01	8.9	-0.76349310D-01
9.0	-0.70947112D-01	9.1	-0.65529511D-01	9.2	-0.60108616D-01	9.3	-0.54695760D-01
9.4	-0.49301534D-01	9.5	-0.43935815D-01	9.6	-0.38607794D-01	9.7	-0.33326004D-01
9.8	-0.28098353D-01	9.9	-0.22932147D-01	10.0	-0.17834119D-01		

P= 4.00

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.35182495D 00	1.2	-0.38677412D-01	1.3	-0.25392509D 00
1.4	-0.35196457D 00	1.5	-0.37361547D 00	1.6	-0.34746739D 00	1.7	-0.29340137D 00
1.8	-0.22508764D 00	1.9	-0.15176703D 00	2.0	-0.79526985D-01	2.1	-0.12218778D-01
2.2	0.47881744D-01	2.3	0.99597591D-01	2.4	0.14250694D 00	2.5	0.17669567D 00
2.6	0.20257972D 00	2.7	0.22077835D 00	2.8	0.23202434D 00	2.9	0.23710118D 00
3.0	0.23679987D 00	3.1	0.23188996D 00	3.2	0.22310085D 00	3.3	0.21111046D 00
3.4	0.19653920D 00	3.5	0.17994760D 00	3.6	0.16183645D 00	3.7	0.14264873D 00
3.8	0.12277257D 00	3.9	0.10254489D 00	4.0	0.82255414D-01	4.1	0.62150778D-01
4.2	0.42438624D-01	4.3	0.23291563D-01	4.4	0.48509423D-02	4.5	-0.12769633D-01
4.6	-0.29481027D-01	4.7	-0.45215574D-01	4.8	-0.59924341D-01	4.9	-0.73574650D-01
5.0	-0.86147844D-01	5.1	-0.97637283D-01	5.2	-0.10804655D 00	5.3	-0.11738787D 00
5.4	-0.12568067D 00	5.5	-0.13295038D 00	5.6	-0.13922728D 00	5.7	-0.14454557D 00
5.8	-0.14894253D 00	5.9	-0.15245775D 00	6.0	-0.15513254D 00	6.1	-0.15700934D 00
6.2	-0.15813128D 00	6.3	-0.15854174D 00	6.4	-0.15828404D 00	6.5	-0.15740110D 00
6.6	-0.15593526D 00	6.7	-0.15392803D 00	6.8	-0.15141993D 00	6.9	-0.14845036D 00
7.0	-0.14505750D 00	7.1	-0.14127822D 00	7.2	-0.13714800D 00	7.3	-0.13270093D 00
7.4	-0.12796963D 00	7.5	-0.12298526D 00	7.6	-0.11777754D 00	7.7	-0.11237471D 00
7.8	-0.10680356D 00	7.9	-0.10108946D 00	8.0	-0.95256404D-01	8.1	-0.89326991D-01
8.2	-0.83322504D-01	8.3	-0.77262929D-01	8.4	-0.71167001D-01	8.5	-0.65052245D-01
8.6	-0.58935019D-01	8.7	-0.52830558D-01	8.8	-0.46753021D-01	8.9	-0.40715531D-01
9.0	-0.34730226D-01	9.1	-0.28808296D-01	9.2	-0.22960033D-01	9.3	-0.17194869D-01
9.4	-0.11521421D-01	9.5	-0.59475297D-02	9.6	-0.48029960D-03	9.7	0.48738610D-02
9.8	0.10109204D-01	9.9	0.15220604D-01	10.0	0.20203527D-01		

P= 4.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.32639150D 00	1.2	-0.68500614D-01	1.3	-0.27692695D 00
1.4	-0.36310034D 00	1.5	-0.37159476D 00	1.6	-0.33317606D 00	1.7	-0.26887934D 00
1.8	-0.19287774D 00	1.9	-0.11451106D 00	2.0	-0.39725723D-01	2.1	0.27900180D-01
2.2	0.86426201D-01	2.3	0.13502853D 00	2.4	0.17362903D 00	2.5	0.20263194D 00
2.6	0.22273796D 00	2.7	0.23481413D 00	2.8	0.23980380D 00	2.9	0.23866546D 00
3.0	0.23233227D 00	3.1	0.22168636D 00	3.2	0.20754354D 00	3.3	0.19064547D 00
3.4	0.17165683D 00	3.5	0.15116592D 00	3.6	0.12968757D 00	3.7	0.10766744D 00
3.8	0.85487145D-01	3.9	0.63469732D-01	4.0	0.41885289D-01	4.1	0.20956426D-01
4.2	0.86353191D-03	4.3	-0.18250285D-01	4.4	-0.36274615D-01	4.5	-0.53127547D-01
4.6	-0.68751844D-01	4.7	-0.83111491D-01	4.8	-0.96188602D-01	4.9	-0.10798066D 00
5.0	-0.11849805D 00	5.1	-0.12776193D 00	5.2	-0.13580226D 00	5.3	-0.14265621D 00
5.4	-0.14836663D 00	5.5	-0.15298081D 00	5.6	-0.15654939D 00	5.7	-0.15912541D 00
5.8	-0.16076348D 00	5.9	-0.16151910D 00	6.0	-0.16144808D 00	6.1	-0.16060604D 00
6.2	-0.15904798D 00	6.3	-0.15682795D 00	6.4	-0.15399878D 00	6.5	-0.15061184D 00
6.6	-0.14671686D 00	6.7	-0.14236179D 00	6.8	-0.13759272D 00	6.9	-0.13245376D 00
7.0	-0.12698703D 00	7.1	-0.12123264D 00	7.2	-0.11522863D 00	7.3	-0.10901103D 00
7.4	-0.10261388D 00	7.5	-0.96069225D-01	7.6	-0.89407178D-01	7.7	-0.82655973D-01
7.8	-0.75842014D-01	7.9	-0.68989928D-01	8.0	-0.62122633D-01	8.1	-0.55261399D-01
8.2	-0.48425915D-01	8.3	-0.41634355D-01	8.4	-0.34903443D-01	8.5	-0.28248524D-01
8.6	-0.21683627D-01	8.7	-0.15221529D-01	8.8	-0.88738229D-02	8.9	-0.26509777D-02
9.0	0.34376006D-02	9.1	0.93835093D-02	9.2	0.15179292D-01	9.3	0.20818383D-01
9.4	0.26295053D-01	9.5	0.31604360D-01	9.6	0.36742099D-01	9.7	0.41704753D-01
9.8	0.46489451D-01	9.9	0.51093923D-01	10.0	0.55516459D-01		

P= 4.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.30093951D 00	1.2	-0.97334837D-01	1.3	-0.29779692D 00
1.4	-0.37125737D 00	1.5	-0.36620913D 00	1.6	-0.31561036D 00	1.7	-0.24158980D 00
1.8	-0.15872966D 00	1.9	-0.76366456D-01	2.0	-0.20866781D-03	2.1	0.66523510D-01
2.2	0.12229476D 00	2.3	0.16668889D 00	2.4	0.20001153D 00	2.5	0.22301267D 00
2.6	0.23669435D 00	2.7	0.24217906D 00	2.8	0.24062122D 00	2.9	0.23314927D 00
3.0	0.22082928D 00	3.1	0.20464372D 00	3.2	0.18548060D 00	3.3	0.16412981D 00
3.4	0.14128419D 00	3.5	0.11754368D 00	3.6	0.93421249D-01	3.7	0.69349983D-01
3.8	0.45690480D-01	3.9	0.22738378D-01	4.0	0.73162826D-03	4.1	-0.20142588D-01
4.2	-0.39741430D-01	4.3	-0.57960483D-01	4.4	-0.74728388D-01	4.5	-0.90002003D-01
4.6	-0.10376207D 00	4.7	-0.11600939D 00	4.8	-0.12676141D 00	4.9	-0.13604924D 00
5.0	-0.14391509D 00	5.1	-0.15040993D 00	5.2	-0.15559160D 00	5.3	-0.15952309D 00
5.4	-0.16227106D 00	5.5	-0.16390468D 00	5.6	-0.16449449D 00	5.7	-0.16411160D 00
5.8	-0.16282691D 00	5.9	-0.16071050D 00	6.0	-0.15783114D 00	6.1	-0.15425590D 00
6.2	-0.15004980D 00	6.3	-0.14527555D 00	6.4	-0.13999341D 00	6.5	-0.13426101D 00
6.6	-0.12813326D 00	6.7	-0.12166231D 00	6.8	-0.11489751D 00	6.9	-0.10788543D 00
7.0	-0.10066986D 00	7.1	-0.93291883D-01	7.2	-0.85789899D-01	7.3	-0.78199720D-01
7.4	-0.70554636D-01	7.5	-0.62885503D-01	7.6	-0.55220834D-01	7.7	-0.47586892D-01
7.8	-0.40007792D-01	7.9	-0.32505593D-01	8.0	-0.25100400D-01	8.1	-0.17810462D-01
8.2	-0.10652267D-01	8.3	-0.36406406D-02	8.4	0.32111618D-02	8.5	0.98913656D-02
8.6	0.16389587D-01	8.7	0.22696749D-01	8.8	0.28804998D-01	8.9	0.34707624D-01
9.0	0.40398987D-01	9.1	0.45874443D-01	9.2	0.51130274D-01	9.3	0.56163622D-01
9.4	0.60972427D-01	9.5	0.65555368D-01	9.6	0.69911802D-01	9.7	0.74041715D-01
9.8	0.77945668D-01	9.9	0.81624753D-01	10.0	0.85080540D-01		

P= 4.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.27550426D 00	1.2	-0.12511011D 00	1.3	-0.31648044D 00
1.4	-0.37645187D 00	1.5	-0.35758655D 00	1.6	-0.29503041D 00	1.7	-0.21192532D 00
1.8	-0.12315223D 00	1.9	-0.37930133D-01	2.0	0.38374834D-01	2.1	0.10298860D 00
2.2	0.15485107D 00	2.3	0.19400600D 00	2.4	0.22117913D 00	2.5	0.23748871D 00
2.6	0.24424916D 00	2.7	0.24284030D 00	2.8	0.23462244D 00	2.9	0.22088346D 00
3.0	0.20280810D 00	3.1	0.18146240D 00	3.2	0.15778830D 00	3.3	0.13260504D 00
3.4	0.10661472D 00	3.5	0.80410407D-01	3.6	0.54485596D-01	3.7	0.29244176D-01
3.8	0.50103944D-02	3.9	-0.17961549D-01	4.0	-0.39478470D-01	4.1	-0.59399865D-01
4.2	-0.77630319D-01	4.3	-0.94112670D-01	4.4	-0.10882191D 00	4.5	-0.12175977D 00
4.6	-0.13295001D 00	4.7	-0.14243420D 00	4.8	-0.15026821D 00	4.9	-0.15651900D 00
5.0	-0.16126202D 00	5.1	-0.16457886D 00	5.2	-0.16655538D 00	5.3	-0.16728002D 00
5.4	-0.16684245D 00	5.5	-0.16533241D 00	5.6	-0.16283882D 00	5.7	-0.15944895D 00
5.8	-0.15524787D 00	5.9	-0.15031792D 00	6.0	-0.14473836D 00	6.1	-0.13858509D 00
6.2	-0.13193045D 00	6.3	-0.12484307D 00	6.4	-0.11738782D 00	6.5	-0.10962578D 00
6.6	-0.10161424D 00	6.7	-0.93406750D-01	6.8	-0.85053193D-01	6.9	-0.76599873D-01
7.0	-0.68089623D-01	7.1	-0.59561925D-01	7.2	-0.51053041D-01	7.3	-0.42596146D-01
7.4	-0.34221472D-01	7.5	-0.25956448D-01	7.6	-0.17825845D-01	7.7	-0.98519194D-02
7.8	-0.20545523D-02	7.9	0.55486098D-02	8.0	0.12942020D-01	8.1	0.20112100D-01
8.2	0.27047110D-01	8.3	0.33737030D-01	8.4	0.40173439D-01	8.5	0.46349402D-01
8.6	0.52259361D-01	8.7	0.57899036D-01	8.8	0.63265321D-01	8.9	0.68356196D-01
9.0	0.73170637D-01	9.1	0.77708533D-01	9.2	0.81970607D-01	9.3	0.85958341D-01
9.4	0.89673909D-01	9.5	0.93120111D-01	9.6	0.96300308D-01	9.7	0.99218370D-01
9.8	0.10187862D 00	9.9	0.10428578D 00	10.0	0.10644492D 00		

P= 4.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.25012085D 00	1.2	-0.15176020D 00	1.3	-0.33293363D 00
1.4	-0.37871815D 00	1.5	-0.34587808D 00	1.6	-0.27171984D 00	1.7	-0.18029675D 00
1.8	-0.86661468D-01	1.9	0.21013932D-03	2.0	0.75405022D-01	2.1	0.13668564D 00
2.2	0.18353606D 00	2.3	0.21650885D 00	2.4	0.23678106D 00	2.5	0.24585562D 00
2.6	0.24536397D 00	2.7	0.23693869D 00	2.8	0.22213453D 00	2.9	0.20238258D 00
3.0	0.17896683D 00	3.1	0.15301560D 00	3.2	0.12550273D 00	3.3	0.97254760D-01
3.4	0.68961632D-01	3.5	0.41189236D-01	3.6	0.14392617D-01	3.7	-0.11070891D-01
3.8	-0.34929094D-01	3.9	-0.56983187D-01	4.0	-0.77096998D-01	4.1	-0.95187264D-01
4.2	-0.11121496D 00	4.3	-0.12517763D 00	4.4	-0.13710265D 00	4.5	-0.14704140D 00
4.6	-0.15506418D 00	4.7	-0.16125587D 00	4.8	-0.16571225D 00	4.9	-0.16853684D 00
5.0	-0.16983825D 00	5.1	-0.16972799D 00	5.2	-0.16831863D 00	5.3	-0.16572230D 00
5.4	-0.16204948D 00	5.5	-0.15740802D 00	5.6	-0.15190235D 00	5.7	-0.14563296D 00
5.8	-0.13869587D 00	5.9	-0.13118240D 00	6.0	-0.12317891D 00	6.1	-0.11476669D 00
6.2	-0.10602197D 00	6.3	-0.97015837D-01	6.4	-0.87814397D-01	6.5	-0.78478820D-01
6.6	-0.69065494D-01	6.7	-0.59626185D-01	6.8	-0.50208205D-01	6.9	-0.40854612D-01
7.0	-0.31604399D-01	7.1	-0.22492705D-01	7.2	-0.13551015D-01	7.3	-0.48073715D-02
7.4	0.37134236D-02	7.5	0.11989609D-01	7.6	0.20002265D-01	7.7	0.27735126D-01
7.8	0.35174394D-01	7.9	0.42308564D-01	8.0	0.49128250D-01	8.1	0.55626027D-01
8.2	0.61796273D-01	8.3	0.67635027D-01	8.4	0.73139842D-01	8.5	0.78309663D-01
8.6	0.83144697D-01	8.7	0.87646297D-01	8.8	0.91816854D-01	8.9	0.95659695D-01
9.0	0.99178984D-01	9.1	0.10237964D 00	9.2	0.10526723D 00	9.3	0.10784794D 00
9.4	0.11012844D 00	9.5	0.11211587D 00	9.6	0.11381774D 00	9.7	0.11524191D 00
9.8	0.11639648D 00	9.9	0.11728982D 00	10.0	0.11793045D 00		

P= 4.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.22482417D 00	1.2	-0.17722285D 00	1.3	-0.34712337D 00
1.4	-0.37810821D 00	1.5	-0.33125642D 00	1.6	-0.24598283D 00	1.7	-0.14712843D 00
1.8	-0.49774000D-01	1.9	0.37483786D-01	2.0	0.11030078D 00	2.1	0.16706694D 00
2.2	0.20787674D 00	2.3	0.23383537D 00	2.4	0.24659591D 00	2.5	0.24805533D 00
2.6	0.24015934D 00	2.7	0.22478160D 00	2.8	0.20365333D 00	2.9	0.17832668D 00
3.0	0.15016030D 00	3.1	0.12031926D 00	3.2	0.89783726D-01	3.3	0.59362793D-01
3.4	0.29710874D-01	3.5	0.13450436D-02	3.6	-0.25337757D-01	3.7	-0.50044310D-01
3.8	-0.72569887D-01	3.9	-0.92784389D-01	4.0	-0.11061993D 00	4.1	-0.12605981D 00
4.2	-0.13912896D 00	4.3	-0.14988547D 00	4.4	-0.15841348D 00	4.5	-0.16481698D 00
4.6	-0.16921461D 00	4.7	-0.17173526D 00	4.8	-0.17251437D 00	4.9	-0.17169094D 00
5.0	-0.16940500D 00	5.1	-0.16579563D 00	5.2	-0.16099927D 00	5.3	-0.15514852D 00
5.4	-0.14837111D 00	5.5	-0.14078919D 00	5.6	-0.13251879D 00	5.7	-0.12366946D 00
5.8	-0.11434405D 00	5.9	-0.10463864D 00	6.0	-0.94642517D-01	6.1	-0.84438255D-01
6.2	-0.74101861D-01	6.3	-0.63702954D-01	6.4	-0.53304991D-01	6.5	-0.42965519D-01
6.6	-0.32736444D-01	6.7	-0.22664313D-01	6.8	-0.12790604D-01	6.9	-0.31520239D-02
7.0	0.62192054D-02	7.1	0.15295053D-01	7.2	0.24051397D-01	7.3	0.32467751D-01
7.4	0.40526995D-01	7.5	0.48215119D-01	7.6	0.55520983D-01	7.7	0.62436076D-01
7.8	0.68954296D-01	7.9	0.75071745D-01	8.0	0.80786520D-01	8.1	0.86098536D-01
8.2	0.91009342D-01	8.3	0.95521962D-01	8.4	0.99640738D-01	8.5	0.10337119D 00
8.6	0.10671987D 00	8.7	0.10969426D 00	8.8	0.11230264D 00	8.9	0.11455398D 00
9.0	0.11645785D 00	9.1	0.11802433D 00	9.2	0.11926392D 00	9.3	0.12018747D 00
9.4	0.12080610D 00	9.5	0.12113114D 00	9.6	0.12117408D 00	9.7	0.12094652D 00
9.8	0.12046009D 00	9.9	0.11972646D 00	10.0	0.11875723D 00		

P= 4.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.19964882D 00	1.2	-0.20143992D 00	1.3	-0.35902742D 00
1.4	-0.37469126D 00	1.5	-0.31391434D 00	1.6	-0.21814097D 00	1.7	-0.11285334D 00
1.8	-0.13000625D-01	1.9	0.73345143D-01	2.0	0.14252796D 00	2.1	0.19365514D 00
2.2	0.22749331D 00	2.3	0.24573749D 00	2.4	0.25053385D 00	2.5	0.24417448D 00
2.6	0.22890859D 00	2.7	0.20683170D 00	2.8	0.17982673D 00	2.9	0.14953883D 00
3.0	0.11737189D 00	3.1	0.84498167D-01	3.2	0.51875620D-01	3.3	0.20269027D-01
3.4	-0.97274418D-02	3.5	-0.37667789D-01	3.6	-0.63232853D-01	3.7	-0.86210582D-01
3.8	-0.10647819D 00	3.9	-0.12398631D 00	4.0	-0.13874503D 00	4.1	-0.15081180D 00
4.2	-0.16028106D 00	4.3	-0.16727533D 00	4.4	-0.17193778D 00	4.5	-0.17442591D 00
4.6	-0.17490639D 00	4.7	-0.17355075D 00	4.8	-0.17053192D 00	4.9	-0.16602142D 00
5.0	-0.16018716D 00	5.1	-0.15319174D 00	5.2	-0.14519111D 00	5.3	-0.13633364D 00
5.4	-0.12675944D 00	5.5	-0.11659999D 00	5.6	-0.10597781D 00	5.7	-0.95006506D-01
5.8	-0.83790734D-01	5.9	-0.72426414D-01	6.0	-0.61000948D-01	6.1	-0.49593525D-01
6.2	-0.38275461D-01	6.3	-0.27110573D-01	6.4	-0.16155578D-01	6.5	-0.54604980D-02
6.6	0.49309266D-02	6.7	0.14980826D-01	6.8	0.24656794D-01	6.9	0.33931491D-01
7.0	0.42782260D-01	7.1	0.51190756D-01	7.2	0.59142585D-01	7.3	0.66626973D-01
7.4	0.73636437D-01	7.5	0.80166484D-01	7.6	0.86215323D-01	7.7	0.91783595D-01
7.8	0.96874123D-01	7.9	0.10149167D 00	8.0	0.10564274D 00	8.1	0.10933533D 00
8.2	0.11257880D 00	8.3	0.11538364D 00	8.4	0.11776137D 00	8.5	0.11972433D 00
8.6	0.12128559D 00	8.7	0.12245880D 00	8.8	0.12325810D 00	8.9	0.12369797D 00
9.0	0.12379320D 00	9.1	0.12355876D 00	9.2	0.12300974D 00	9.3	0.12216127D 00
9.4	0.12102846D 00	9.5	0.11962635D 00	9.6	0.11796988D 00	9.7	0.11607378D 00
9.8	0.11395261D 00	9.9	0.11162068D 00	10.0	0.10909203D 00		

P= 4.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.17462909D 00	1.2	-0.22435757D 00	1.3	-0.36863444D 00
1.4	-0.36855314D 00	1.5	-0.29406286D 00	1.6	-0.18852999D 00	1.7	-0.77908211D-01
1.8	0.23160032D-01	1.9	0.10728072D 00	2.0	0.17160691D 00	2.1	0.21605002D 00
2.2	0.24210439D 00	2.3	0.25208396D 00	2.4	0.24863593D 00	2.5	0.23443957D 00
2.6	0.21202815D 00	2.7	0.18369203D 00	2.8	0.15143422D 00	2.9	0.11695919D 00
3.0	0.81682073D-01	3.1	0.46749429D-01	3.2	0.13066086D-01	3.3	-0.18675767D-01
3.4	-0.47967486D-01	3.5	-0.74456272D-01	3.6	-0.97919667D-01	3.7	-0.11824259D 00
3.8	-0.13539706D 00	3.9	-0.14942449D 00	4.0	-0.16042048D 00	4.1	-0.16852189D 00
4.2	-0.17389588D 00	4.3	-0.17673082D 00	4.4	-0.17722874D 00	4.5	-0.17559919D 00
4.6	-0.17205427D 00	4.7	-0.16680462D 00	4.8	-0.16005637D 00	4.9	-0.15200868D 00
5.0	-0.14285202D 00	5.1	-0.13276682D 00	5.2	-0.12192264D 00	5.3	-0.11047760D 00
5.4	-0.98578167D-01	5.5	-0.86359043D-01	5.6	-0.73943351D-01	5.7	-0.61442883D-01
5.8	-0.48958470D-01	5.9	-0.36580434D-01	6.0	-0.24389089D-01	6.1	-0.12455283D-01
6.2	-0.84095842D-03	6.3	0.10400272D-01	6.4	0.21222555D-01	6.5	0.31587225D-01
6.6	0.41462254D-01	6.7	0.50821712D-01	6.8	0.59645249D-01	6.9	0.67917605D-01
7.0	0.75628134D-01	7.1	0.82770365D-01	7.2	0.89341581D-01	7.3	0.95342427D-01
7.4	0.10077655D 00	7.5	0.10565023D 00	7.6	0.10997211D 00	7.7	0.11375286D 00
7.8	0.11700493D 00	7.9	0.11974229D 00	8.0	0.12198020D 00	8.1	0.12373500D 00
8.2	0.12502395D 00	8.3	0.12586500D 00	8.4	0.12627667D 00	8.5	0.12627790D 00
8.6	0.12588792D 00	8.7	0.12512614D 00	8.8	0.12401203D 00	8.9	0.12256504D 00
9.0	0.12080451D 00	9.1	0.11874960D 00	9.2	0.11641925D 00	9.3	0.11383208D 00
9.4	0.11100636D 00	9.5	0.10795998D 00	9.6	0.10471041D 00	9.7	0.10127465D 00
9.8	0.97669235D-01	9.9	0.93910189D-01	10.0	0.90013022D-01		

P= 4.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.14979887D 00	1.2	-0.24592637D 00	1.3	-0.37594396D 00
1.4	-0.35979555D 00	1.5	-0.27192941D 00	1.6	-0.15749636D 00	1.7	-0.42728636D-01
1.8	0.58227612D-01	1.9	0.13881593D 00	2.0	0.19711897D 00	2.1	0.23393400D 00
2.2	0.25153045D 00	2.3	0.25286065D 00	2.4	0.24107066D 00	2.5	0.21920904D 00
2.6	0.19006469D 00	2.7	0.15608784D 00	2.8	0.11936340D 00	2.9	0.81616244D-01
3.0	0.44234837D-01	3.1	0.83044717D-02	3.2	-0.25355598D-01	3.3	-0.56154607D-01
3.4	-0.83694431D-01	3.5	-0.10773659D 00	3.6	-0.12817270D 00	3.7	-0.14499845D 00
3.8	-0.15829114D 00	3.9	-0.16819047D 00	4.0	-0.17488241D 00	4.1	-0.17858564D 00
4.2	-0.17954047D 00	4.3	-0.17799974D 00	4.4	-0.17422157D 00	4.5	-0.16846352D 00
4.6	-0.16097808D 00	4.7	-0.15200926D 00	4.8	-0.14179000D 00	4.9	-0.13054035D 00
5.0	-0.11846627D 00	5.1	-0.10575885D 00	5.2	-0.92594041D-01	5.3	-0.79132549D-01
5.4	-0.65520103D-01	5.5	-0.51887836D-01	5.6	-0.38352828D-01	5.7	-0.25018753D-01
5.8	-0.11976592D-01	5.9	0.69460492D-03	6.0	0.12926938D-01	6.1	0.24662864D-01
6.2	0.35854410D-01	6.3	0.46462399D-01	6.4	0.56455707D-01	6.5	0.65810537D-01
6.6	0.74509734D-01	6.7	0.82542131D-01	6.8	0.89901937D-01	6.9	0.96588155D-01
7.0	0.10260404D 00	7.1	0.10795661D 00	7.2	0.11265617D 00	7.3	0.11671586D 00
7.4	0.12015132D 00	7.5	0.12298026D 00	7.6	0.12522217D 00	7.7	0.12689600D 00
7.8	0.12802989D 00	7.9	0.12864093D 00	8.0	0.12875491D 00	8.1	0.12839614D 00
8.2	0.12758928D 00	8.3	0.12635913D 00	8.4	0.12473055D 00	8.5	0.12272827D 00
8.6	0.12037683D 00	8.7	0.11770045D 00	8.8	0.11472297D 00	8.9	0.11146774D 00
9.0	0.10795760D 00	9.1	0.10421479D 00	9.2	0.10026094D 00	9.3	0.96116993D-01
9.4	0.91803216D-01	9.5	0.87339150D-01	9.6	0.82743607D-01	9.7	0.78034651D-01
9.8	0.73229593D-01	9.9	0.68344990D-01	10.0	0.63396642D-01		

P= 4.90

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.12519161D 00	1.2	-0.26610151D 00	1.3	-0.38096626D 00
1.4	-0.34853531D 00	1.5	-0.24775583D 00	1.6	-0.12539385D 00	1.7	-0.77442319D-02
1.8	0.91745818D-01	1.9	0.16752118D 00	2.0	0.21871206D 00	2.1	0.24707605D 00
2.2	0.25569527D 00	2.3	0.24816868D 00	2.4	0.22812791D 00	2.5	0.19896252D 00
2.6	0.16367931D 00	2.7	0.12484564D 00	2.8	0.84584006D-01	2.9	0.44596114D-01
3.0	0.62027828D-02	3.1	-0.29609329D-01	3.2	-0.62139582D-01	3.3	-0.90927058D-01
3.4	-0.11570791D 00	3.5	-0.13637722D 00	3.6	-0.15295560D 00	3.7	-0.16556055D 00
3.8	-0.17438214D 00	3.9	-0.17966271D 00	4.0	-0.18168016D 00	4.1	-0.18073427D 00
4.2	-0.17713580D 00	4.3	-0.17119774D 00	4.4	-0.16322863D 00	4.5	-0.15352738D 00
4.6	-0.14237948D 00	4.7	-0.13005430D 00	4.8	-0.11680328D 00	4.9	-0.10285884D 00
5.0	-0.88433862D-01	5.1	-0.73721657D-01	5.2	-0.58896225D-01	5.3	-0.44112830D-01
5.4	-0.29508751D-01	5.5	-0.15204183D-01	5.6	-0.13032204D-02	5.7	0.12105089D-01
5.8	0.24945670D-01	5.9	0.37156343D-01	6.0	0.48686749D-01	6.1	0.59497316D-01
6.2	0.69558251D-01	6.3	0.78848581D-01	6.4	0.87355246D-01	6.5	0.95072238D-01
6.6	0.10199980D 00	6.7	0.10814368D 00	6.8	0.11351444D 00	6.9	0.11812681D 00
7.0	0.12199908D 00	7.1	0.12515261D 00	7.2	0.12761130D 00	7.3	0.12940115D 00
7.4	0.13054986D 00	7.5	0.13108647D 00	7.6	0.13104101D 00	7.7	0.13044424D 00
7.8	0.12932736D 00	7.9	0.12772179D 00	8.0	0.12565897D 00	8.1	0.12317017D 00
8.2	0.12028633D 00	8.3	0.11703793D 00	8.4	0.11345489D 00	8.5	0.10956644D 00
8.6	0.10540105D 00	8.7	0.10098636D 00	8.8	0.96349115D-01	8.9	0.91515137D-01
9.0	0.86509271D-01	9.1	0.81355366D-01	9.2	0.76076256D-01	9.3	0.70693749D-01
9.4	0.65228624D-01	9.5	0.59700629D-01	9.6	0.54128488D-01	9.7	0.48529912D-01
9.8	0.42921613D-01	9.9	0.37319323D-01	10.0	0.31737807D-01		

P= 5.00

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.10084028D 00	1.2	-0.28484285D 00	1.3	-0.38372231D 00
1.4	-0.33490341D 00	1.5	-0.22179623D 00	1.6	-0.92580064D-01	1.7	0.26626056D-01
1.8	0.12328784D 00	1.9	0.19301722D 00	2.0	0.23610498D 00	2.1	0.25533419D 00
2.2	0.25462542D 00	2.3	0.23822022D 00	2.4	0.21021043D 00	2.5	0.17428750D 00
2.6	0.13362923D 00	2.7	0.90870064D-01	2.8	0.48120173D-01	2.9	0.70108315D-02
3.0	-0.31247941D-01	3.1	-0.65806199D-01	3.2	-0.96114487D-01	3.3	-0.12186871D 00
3.4	-0.14296082D 00	3.5	-0.15943585D 00	3.6	-0.17145519D 00	3.7	-0.17926563D 00
3.8	-0.18317385D 00	3.9	-0.18352550D 00	4.0	-0.18068840D 00	4.1	-0.17503922D 00
4.2	-0.16695311D 00	4.3	-0.15679581D 00	4.4	-0.14491778D 00	4.5	-0.13165006D 00
4.6	-0.11730137D 00	4.7	-0.10215647D 00	4.8	-0.86475223D-01	4.9	-0.70492420D-01
5.0	-0.54418129D-01	5.1	-0.38438396D-01	5.2	-0.22715266D-01	5.3	-0.73929837D-02
5.4	0.74106534D-02	5.5	0.21592877D-01	5.6	0.35069486D-01	5.7	0.47772387D-01
5.8	0.59648157D-01	5.9	0.70656642D-01	6.0	0.80769625D-01	6.1	0.89969551D-01
6.2	0.98248324D-01	6.3	0.10560619D 00	6.4	0.11205068D 00	6.5	0.11759566D 00
6.6	0.12226043D 00	6.7	0.12606891D 00	6.8	0.12904890D 00	6.9	0.13123141D 00
7.0	0.13265003D 00	7.1	0.13334040D 00	7.2	0.13333973D 00	7.3	0.13268632D 00
7.4	0.13141922D 00	7.5	0.12957785D 00	7.6	0.12720170D 00	7.7	0.12433009D 00
7.8	0.12100193D 00	7.9	0.11725550D 00	8.0	0.11312833D 00	8.1	0.10865699D 00
8.2	0.10387705D 00	8.3	0.98822922D-01	8.4	0.93527795D-01	8.5	0.88023594D-01
8.6	0.82340912D-01	8.7	0.76508988D-01	8.8	0.70555680D-01	8.9	0.64507458D-01
9.0	0.58389399D-01	9.1	0.52225195D-01	9.2	0.46037168D-01	9.3	0.39846286D-01
9.4	0.33672188D-01	9.5	0.27533213D-01	9.6	0.21446432D-01	9.7	0.15427685D-01
9.8	0.94916128D-02	9.9	0.36517024D-02	10.0	-0.20796754D-02		

P= 5.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.76777300D-01	1.2	-0.30211504D 00	1.3	-0.38424351D 00
1.4	-0.31904408D 00	1.5	-0.19431482D 00	1.6	-0.59412905D-01	1.7	0.59978489D-01
1.8	0.15246132D 00	1.9	0.21497978D 00	2.0	0.24909074D 00	2.1	0.25865631D 00
2.2	0.24844797D 00	2.3	0.22333214D 00	2.4	0.18782296D 00	2.5	0.14586376D 00
2.6	0.10074757D 00	2.7	0.55119129D-01	2.8	0.11021790D-01	2.9	-0.30033587D-01
3.0	-0.66991917D-01	3.1	-0.99177593D-01	3.2	-0.12622329D 00	3.3	-0.14800613D 00
3.4	-0.16459219D 00	3.5	-0.17618914D 00	3.6	-0.18310681D 00	3.7	-0.18572470D 00
3.8	-0.18446577D 00	3.9	-0.17977556D 00	4.0	-0.17210590D 00	4.1	-0.16190245D 00
4.2	-0.14959536D 00	4.3	-0.13559262D 00	4.4	-0.12027541D 00	4.5	-0.10399523D 00
4.6	-0.87072276D-01	4.7	-0.69794935D-01	4.8	-0.52420018D-01	4.9	-0.35173610D-01
5.0	-0.18252362D-01	5.1	-0.18250688D-02	5.2	0.13965533D-01	5.3	0.29000864D-01
5.4	0.43184546D-01	5.5	0.56440434D-01	5.6	0.68710694D-01	5.7	0.79953938D-01
5.8	0.90143440D-01	5.9	0.99265451D-01	6.0	0.10731762D 00	6.1	0.11430752D 00
6.2	0.12025128D 00	6.3	0.12517235D 00	6.4	0.12910036D 00	6.5	0.13207004D 00
6.6	0.13412035D 00	6.7	0.13529357D 00	6.8	0.13563459D 00	6.9	0.13519020D 00
7.0	0.13400853D 00	7.1	0.13213850D 00	7.2	0.12962934D 00	7.3	0.12653024D 00
7.4	0.12288996D 00	7.5	0.11875654D 00	7.6	0.11417706D 00	7.7	0.10919743D 00
7.8	0.10386218D 00	7.9	0.98214358D-01	8.0	0.92295383D-01	8.1	0.86144975D-01
8.2	0.79801083D-01	8.3	0.73299833D-01	8.4	0.66675508D-01	8.5	0.59960527D-01
8.6	0.53185454D-01	8.7	0.46378999D-01	8.8	0.39568051D-01	8.9	0.32777700D-01
9.0	0.26031280D-01	9.1	0.19350408D-01	9.2	0.12755036D-01	9.3	0.62635019D-02
9.4	-0.10741508D-03	9.5	-0.63424363D-02	9.6	-0.12427724D-01	9.7	-0.18350821D-01
9.8	-0.24100588D-01	9.9	-0.29667145D-01	10.0	-0.35041804D-01		

P= 5.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.53034497D-01	1.2	-0.31788763D 00	1.3	-0.38257149D 00
1.4	-0.30111364D 00	1.5	-0.16558368D 00	1.6	-0.26247155D-01	1.7	0.91928849D-01
1.8	0.17891287D 00	1.9	0.23314326D 00	2.0	0.25753850D 00	2.1	0.25707955D 00
2.2	0.23738624D 00	2.3	0.20391767D 00	2.4	0.16155943D 00	2.5	0.11444599D 00
2.6	0.65921577D-01	2.7	0.18578592D-01	2.8	-0.25664385D-01	2.9	-0.65468340D-01
3.0	-0.99976950D-01	3.1	-0.12872485D 00	3.2	-0.15155493D 00	3.3	-0.16854632D 00
3.4	-0.17995323D 00	3.5	-0.18615412D 00	3.6	-0.18761026D 00	3.7	-0.18483253D 00
3.8	-0.17835526D 00	3.9	-0.16871615D 00	4.0	-0.15644112D 00	4.1	-0.14203321D 00
4.2	-0.12596492D 00	4.3	-0.10867307D 00	4.4	-0.90555885D-01	4.5	-0.71971598D-01
4.6	-0.53238403D-01	4.7	-0.34635287D-01	4.8	-0.16403560D-01	4.9	0.12511336D-02
5.0	0.18156489D-01	5.1	0.34171030D-01	5.2	0.49181507D-01	5.3	0.63100279D-01
5.4	0.75862748D-01	5.5	0.87424869D-01	5.6	0.97760784D-01	5.7	0.10686059D 00
5.8	0.11472824D 00	5.9	0.12137963D 00	6.0	0.12684081D 00	6.1	0.13114638D 00
6.2	0.13433797D 00	6.3	0.13646299D 00	6.4	0.13757337D 00	6.5	0.13772452D 00
6.6	0.13697441D 00	6.7	0.13538271D 00	6.8	0.13301008D 00	6.9	0.12991753D 00
7.0	0.12616588D 00	7.1	0.12181529D 00	7.2	0.11692484D 00	7.3	0.11155224D 00
7.4	0.10575350D 00	7.5	0.99582761D-01	7.6	0.93092064D-01	7.7	0.86331242D-01
7.8	0.79347801D-01	7.9	0.72186849D-01	8.0	0.64891056D-01	8.1	0.57500621D-01
8.2	0.50053281D-01	8.3	0.42584319D-01	8.4	0.35126599D-01	8.5	0.27710607D-01
8.6	0.20364506D-01	8.7	0.13114201D-01	8.8	0.59834045D-02	8.9	-0.10062851D-02
9.0	-0.78353041D-02	9.1	-0.14486041D-01	9.2	-0.20942750D-01	9.3	-0.27191467D-01
9.4	-0.33219919D-01	9.5	-0.39017439D-01	9.6	-0.44574881D-01	9.7	-0.49884528D-01
9.8	-0.54940016D-01	9.9	-0.59736246D-01	10.0	-0.64269306D-01		

P= 5.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.29643063D-01	1.2	-0.33213510D 00	1.3	-0.37875783D 00
1.4	-0.28127948D 00	1.5	-0.13588044D 00	1.6	0.65689359D-02	1.7	0.12211648D 00
1.8	0.20233197D 00	1.9	0.24730362D 00	2.0	0.26139436D 00	2.1	0.25072803D 00
2.2	0.22175386D 00	2.3	0.18047624D 00	2.4	0.13208841D 00	2.5	0.80844959D-01
2.6	0.30069863D-01	2.7	-0.17764016D-01	2.8	-0.60923034D-01	2.9	-0.98291464D-01
3.0	-0.12925387D 00	3.1	-0.15358794D 00	3.2	-0.17137082D 00	3.3	-0.18289963D 00
3.4	-0.18862563D 00	3.5	-0.18910088D 00	3.6	-0.18493581D 00	3.7	-0.17676622D 00
3.8	-0.16522813D 00	3.9	-0.15093923D 00	4.0	-0.13448555D 00	4.1	-0.11641246D 00
4.2	-0.97219045D-01	4.3	-0.77355053D-01	4.4	-0.57219938D-01	4.5	-0.37163367D-01
4.6	-0.17486848D-01	4.7	0.15538575D-02	4.8	0.19745806D-01	4.9	0.36915615D-01
5.0	0.52926051D-01	5.1	0.67672567D-01	5.2	0.81079888D-01	5.3	0.93098706D-01
5.4	0.10370253D 00	5.5	0.11288471D 00	5.6	0.12065572D 00	5.7	0.12704054D 00
5.8	0.13207642D 00	5.9	0.13581067D 00	6.0	0.13829884D 00	6.1	0.13960296D 00
6.2	0.13979003D 00	6.3	0.13893070D 00	6.4	0.13709808D 00	6.5	0.13436668D 00
6.6	0.13081158D 00	6.7	0.12650761D 00	6.8	0.12152873D 00	6.9	0.11594749D 00
7.0	0.10983452D 00	7.1	0.10325824D 00	7.2	0.96284491D-01	7.3	0.88976319D-01
7.4	0.81393813D-01	7.5	0.73593958D-01	7.6	0.65630556D-01	7.7	0.57554180D-01
7.8	0.49412155D-01	7.9	0.41248573D-01	8.0	0.33104331D-01	8.1	0.25017181D-01
8.2	0.17021811D-01	8.3	0.91499242D-02	8.4	0.14303375D-02	8.5	-0.61109113D-02
8.6	-0.13450453D-01	8.7	-0.20567472D-01	8.8	-0.27443582D-01	8.9	-0.34062709D-01
9.0	-0.40410967D-01	9.1	-0.46476535D-01	9.2	-0.52249537D-01	9.3	-0.57721922D-01
9.4	-0.62887349D-01	9.5	-0.67741068D-01	9.6	-0.72279816D-01	9.7	-0.76501702D-01
9.8	-0.80406111D-01	9.9	-0.83993598D-01	10.0	-0.87265796D-01		

P= 5.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	0.66334957D-02	1.2	-0.34483695D 00	1.3	-0.37286370D 00
1.4	-0.25971875D 00	1.5	-0.10548598D 00	1.6	0.38696937D-01	1.7	0.15020803D 00
1.8	0.22245431D 00	1.9	0.25732035D 00	2.0	0.26068098D 00	2.1	0.23980923D 00
2.2	0.20194726D 00	2.3	0.15358182D 00	2.4	0.10013722D 00	2.5	0.45907688D-01
2.6	-0.58806457D-02	2.7	-0.52944946D-01	2.8	-0.93797266D-01	2.9	-0.12759509D 00
3.0	-0.15400278D 00	3.1	-0.17306948D 00	3.2	-0.18512532D 00	3.3	-0.19069564D 00
3.4	-0.19043171D 00	3.5	-0.18505606D 00	3.6	-0.17532029D 00	3.7	-0.16197333D 00
3.8	-0.14573833D 00	3.9	-0.12729641D 00	4.0	-0.10727603D 00	4.1	-0.86246564D-01
4.2	-0.64715331D-01	4.3	-0.43127011D-01	4.4	-0.21864991D-01	4.5	-0.12539850D-02
4.6	0.18436430D-01	4.7	0.36987719D-01	4.8	0.54227978D-01	4.9	0.70027397D-01
5.0	0.84293747D-01	5.1	0.96967996D-01	5.2	0.10802012D 00	5.3	0.11744515D 00
5.4	0.12525954D 00	5.5	0.13149778D 00	5.6	0.13620932D 00	5.7	0.13945582D 00
5.8	0.14130864D 00	5.9	0.14184668D 00	6.0	0.14115437D 00	6.1	0.13931997D 00
6.2	0.13643413D 00	6.3	0.13258854D 00	6.4	0.12787488D 00	6.5	0.12238384D 00
6.6	0.11620440D 00	6.7	0.10942315D 00	6.8	0.10212376D 00	6.9	0.94386575D-01
7.0	0.86288297D-01	7.1	0.77901732D-01	7.2	0.69295627D-01	7.3	0.60534557D-01
7.4	0.51678874D-01	7.5	0.42784698D-01	7.6	0.33903945D-01	7.7	0.25084387D-01
7.8	0.16369746D-01	7.9	0.77997977D-02	8.0	-0.58949484D-03	8.1	-0.87658425D-02
8.2	-0.16700476D-01	8.3	-0.24367987D-01	8.4	-0.31746160D-01	8.5	-0.38815806D-01
8.6	-0.45560592D-01	8.7	-0.51966870D-01	8.8	-0.58023510D-01	8.9	-0.63721735D-01
9.0	-0.69054956D-01	9.1	-0.74018617D-01	9.2	-0.78610040D-01	9.3	-0.82828280D-01
9.4	-0.86673980D-01	9.5	-0.90149235D-01	9.6	-0.93257464D-01	9.7	-0.96003287D-01
9.8	-0.98392402D-01	9.9	-0.10043148D 00	10.0	-0.10212805D 00		

P= 5.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.15964441D-01	1.2	-0.35597772D 00	1.3	-0.36495950D 00
1.4	-0.23661718D 00	1.5	-0.74682112D-01	1.6	0.69811407D-01	1.7	0.17590082D 00
1.8	0.23906448D 00	1.9	0.26311740D 00	2.0	0.25549588D 00	2.1	0.22460888D 00
2.2	0.17843671D 00	2.3	0.12386981D 00	2.4	0.66474923D-01	2.5	0.10497150D-01
2.6	-0.41017675D-01	2.7	-0.86048777D-01	2.8	-0.12341360D 00	2.9	-0.15258961D 00
3.0	-0.17355514D 00	3.1	-0.18665339D 00	3.2	-0.19247984D 00	3.3	-0.19179159D 00
3.4	-0.18543623D 00	3.5	-0.17429728D 00	3.6	-0.15925357D 00	3.7	-0.14115003D 00
3.8	-0.12077757D 00	3.9	-0.98860196D-01	4.0	-0.76047658D-01	4.1	-0.52912370D-01
4.2	-0.29949468D-01	4.3	-0.75791386D-02	4.4	0.13849476D-01	4.5	0.34053330D-01
4.6	0.52809920D-01	4.7	0.69951428D-01	4.8	0.85358806D-01	4.9	0.98955975D-01
5.0	0.11070426D 00	5.1	0.12059715D 00	5.2	0.12865540D 00	5.3	0.13492259D 00
5.4	0.13946102D 00	5.5	0.14234808D 00	5.6	0.14367296D 00	5.7	0.14353377D 00
5.8	0.14203500D 00	5.9	0.13928534D 00	6.0	0.13539575D 00	6.1	0.13047784D 00
6.2	0.12464251D 00	6.3	0.11799881D 00	6.4	0.11065297D 00	6.5	0.10270765D 00
6.6	0.94261324D-01	6.7	0.85407818D-01	6.8	0.76235963D-01	6.9	0.66829346D-01
7.0	0.57266160D-01	7.1	0.47619126D-01	7.2	0.37955489D-01	7.3	0.28337056D-01
7.4	0.18820292D-01	7.5	0.94564446D-02	7.6	0.29170465D-03	7.7	-0.86326113D-02
7.8	-0.17279861D-01	7.9	-0.25617860D-01	8.0	-0.33618660D-01	8.1	-0.41258314D-01
8.2	-0.48516649D-01	8.3	-0.55377025D-01	8.4	-0.61826108D-01	8.5	-0.67853638D-01
8.6	-0.73452204D-01	8.7	-0.78617029D-01	8.8	-0.83345757D-01	8.9	-0.87638247D-01
9.0	-0.91496382D-01	9.1	-0.94923879D-01	9.2	-0.97926111D-01	9.3	-0.10050994D 00
9.4	-0.10268356D 00	9.5	-0.10445635D 00	9.6	-0.10583869D 00	9.7	-0.10684191D 00
9.8	-0.10747808D 00	9.9	-0.10775997D 00	10.0	-0.10770087D 00		

P= 5.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.38121767D-01	1.2	-0.36554701D 00	1.3	-0.35512445D 00
1.4	-0.21216774D 00	1.5	-0.43749253D-01	1.6	0.99602895D-01	1.7	0.19892585D 00
1.8	0.25199805D 00	1.9	0.26468333D 00	2.0	0.24600864D 00	2.1	0.20548466D 00
2.2	0.15175604D 00	2.3	0.92022972D-01	2.4	0.31894841D-01	2.5	-0.24528120D-01
2.6	-0.74465810D-01	2.7	-0.11623132D 00	2.8	-0.14900430D 00	2.9	-0.17262408D 00
3.0	-0.18741094D 00	3.1	-0.19401769D 00	3.2	-0.19331004D 00	3.3	-0.18627306D 00
3.4	-0.17393983D 00	3.5	-0.15733876D 00	3.6	-0.13745605D 00	3.7	-0.11521026D 00
3.8	-0.91436372D-01	3.9	-0.66877072D-01	4.0	-0.42179531D-01	4.1	-0.17896162D-01
4.2	0.55117852D-02	4.3	0.27668723D-01	4.4	0.48278013D-01	4.5	0.67114581D-01
4.6	0.84017277D-01	4.7	0.98881257D-01	4.8	0.11165061D 00	4.9	0.12231137D 00
5.0	0.13088498D 00	5.1	0.13742236D 00	5.2	0.14199841D 00	5.3	0.14470718D 00
5.4	0.14565750D 00	5.5	0.14496919D 00	5.6	0.14276974D 00	5.7	0.13919140D 00
5.8	0.13436873D 00	5.9	0.12843656D 00	6.0	0.12152818D 00	6.1	0.11377399D 00
6.2	0.10530024D 00	6.3	0.96228183D-01	6.4	0.86673332D-01	6.5	0.76744913D-01
6.6	0.66545506D-01	6.7	0.56170800D-01	6.8	0.45709462D-01	6.9	0.35243114D-01
7.0	0.24846383D-01	7.1	0.14587016D-01	7.2	0.45260589D-02	7.3	-0.52819311D-02
7.4	-0.14788633D-01	7.5	-0.23951683D-01	7.6	-0.32734385D-01	7.7	-0.41105396D-01
7.8	-0.49038412D-01	7.9	-0.56511849D-01	8.0	-0.63508526D-01	8.1	-0.70015347D-01
8.2	-0.76022989D-01	8.3	-0.81525605D-01	8.4	-0.86520531D-01	8.5	-0.91008001D-01
8.6	-0.94990878D-01	8.7	-0.98474400D-01	8.8	-0.10146593D 00	8.9	-0.10397473D 00
9.0	-0.10601173D 00	9.1	-0.10758935D 00	9.2	-0.10872126D 00	9.3	-0.10942226D 00
9.4	-0.10970805D 00	9.5	-0.10959514D 00	9.6	-0.10910063D 00	9.7	-0.10824216D 00
9.8	-0.10703772D 00	9.9	-0.10550557D 00	10.0	-0.10366413D 00		

P= 5.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.59810328D-01	1.2	-0.37353946D 00	1.3	-0.34344608D 00
1.4	-0.18656932D 00	1.5	-0.12964165D-01	1.6	0.12778076D 00	1.7	0.21905038D 00
1.8	0.26114285D 00	1.9	0.26207038D 00	2.0	0.23245700D 00	2.1	0.18285865D 00
2.2	0.12249142D 00	2.3	0.58756606D-01	2.4	-0.28033462D-02	2.5	-0.58333975D-01
2.6	-0.10540747D 00	2.7	-0.14274035D 00	2.8	-0.16992649D 00	2.9	-0.18720244D 00
3.0	-0.19525071D 00	3.1	-0.19504105D 00	3.2	-0.18770613D 00	3.3	-0.17444714D 00
3.4	-0.15646451D 00	3.5	-0.13490883D 00	3.6	-0.11084808D 00	3.7	-0.85247317D-01
3.8	-0.58958022D-01	3.9	-0.32714501D-01	4.0	-0.71354384D-02	4.1	0.17270976D-01
4.2	0.40099563D-01	4.3	0.61039239D-01	4.4	0.79863370D-01	4.5	0.96419918D-01
4.6	0.11062173D 00	4.7	0.12243723D 00	4.8	0.13188173D 00	4.9	0.13900938D 00
5.0	0.14390584D 00	5.1	0.14668178D 00	5.2	0.14746702D 00	5.3	0.14640547D 00
5.4	0.14365070D 00	5.5	0.13936208D 00	5.6	0.13370165D 00	5.7	0.12683131D 00
5.8	0.11891064D 00	5.9	0.11009506D 00	6.0	0.10053432D 00	6.1	0.90371440D-01
6.2	0.79741767D-01	6.3	0.68772408D-01	6.4	0.57581801D-01	6.5	0.46279471D-01
6.6	0.34965950D-01	6.7	0.23732803D-01	6.8	0.12662769D-01	6.9	0.18299706D-02
7.0	-0.86997997D-02	7.1	-0.18868748D-01	7.2	-0.28626707D-01	7.3	-0.37930740D-01
7.4	-0.46744720D-01	7.5	-0.55038905D-01	7.6	-0.62789504D-01	7.7	-0.69978244D-01
7.8	-0.76591950D-01	7.9	-0.82622120D-01	8.0	-0.88064526D-01	8.1	-0.92918824D-01
8.2	-0.97188172D-01	8.3	-0.10087888D 00	8.4	-0.10400007D 00	8.5	-0.10656335D 00
8.6	-0.10858251D 00	8.7	-0.11007325D 00	8.8	-0.11105289D 00	8.9	-0.11154017D 00
9.0	-0.11155497D 00	9.1	-0.11111812D 00	9.2	-0.11025121D 00	9.3	-0.10897641D 00
9.4	-0.10731629D 00	9.5	-0.10529370D 00	9.6	-0.10293161D 00	9.7	-0.10025300D 00
9.8	-0.97280744D-01	9.9	-0.94037510D-01	10.0	-0.90545686D-01		

P= 5.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.81002839D-01	1.2	-0.37995480D 00	1.3	-0.33001978D 00
1.4	-0.16002537D 00	1.5	0.17402279D-01	1.6	0.15407578D 00	1.7	0.23608007D 00
1.8	0.26643971D 00	1.9	0.25539272D 00	2.0	0.21514189D 00	2.1	0.15720881D 00
2.2	0.91269203D-01	2.3	0.24803263D-01	2.4	-0.36831274D-01	2.5	-0.90129746D-01
2.6	-0.13310212D 00	2.7	-0.16493359D 00	2.8	-0.18567761D 00	2.9	-0.19599502D 00
3.0	-0.19694194D 00	3.1	-0.18980323D 00	3.2	-0.17596628D 00	3.3	-0.15682841D 00
3.4	-0.13373182D 00	3.5	-0.10792027D 00	3.6	-0.80512636D-01	3.7	-0.52489217D-01
3.8	-0.24687387D-01	3.9	0.21960392D-02	4.0	0.27597860D-01	4.1	0.51078580D-01
4.2	0.72310777D-01	4.3	0.91066656D-01	4.4	0.10720545D 00	4.5	0.12066113D 00
4.6	0.13143079D 00	4.7	0.13956379D 00	4.8	0.14515194D 00	4.9	0.14832057D 00
5.0	0.14922075D 00	5.1	0.14802230D 00	5.2	0.14490793D 00	5.3	0.14006804D 00
5.4	0.13369647D 00	5.5	0.12598688D 00	5.6	0.11712978D 00	5.7	0.10731019D 00
5.8	0.96705715D-01	5.9	0.85485112D-01	6.0	0.73807226D-01	6.1	0.61820244D-01
6.2	0.49661225D-01	6.3	0.37455862D-01	6.4	0.25318435D-01	6.5	0.13351930D-01
6.6	0.16482732D-02	6.7	-0.97113199D-02	6.8	-0.20655925D-01	6.9	-0.31124426D-01
7.0	-0.41064986D-01	7.1	-0.50434495D-01	7.2	-0.59197997D-01	7.3	-0.67328105D-01
7.4	-0.74804430D-01	7.5	-0.81613000D-01	7.6	-0.87745706D-01	7.7	-0.93199759D-01
7.8	-0.97977167D-01	7.9	-0.10208423D 00	8.0	-0.10553109D 00	8.1	-0.10833124D 00
8.2	-0.11050114D 00	8.3	-0.11205979D 00	8.4	-0.11302840D 00	8.5	-0.11343000D 00
8.6	-0.11328917D 00	8.7	-0.11263170D 00	8.8	-0.11148436D 00	8.9	-0.10987464D 00
9.0	-0.10783052D 00	9.1	-0.10538029D 00	9.2	-0.10255232D 00	9.3	-0.99374962D-01
9.4	-0.95876362D-01	9.5	-0.92084334D-01	9.6	-0.88026253D-01	9.7	-0.83728952D-01
9.8	-0.79218638D-01	9.9	-0.74520812D-01	10.0	-0.69660211D-01		

P= 5.90

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.10167292D 00	1.2	-0.38479774D 00	1.3	-0.31494824D 00
1.4	-0.13274250D 00	1.5	0.47087243D-01	1.6	0.17824250D 00	1.7	0.24986063D 00
1.8	0.26788237D 00	1.9	0.24482378D 00	2.0	0.19442152D 00	2.1	0.12905960D 00
2.2	0.58743297D-01	2.3	-0.91026622D-02	2.4	-0.69429090D-01	2.5	-0.11918603D 00
2.6	-0.15690336D 00	2.7	-0.18229368D 00	2.8	-0.19590678D 00	2.9	-0.19884526D 00
3.0	-0.19253994D 00	3.1	-0.17857913D 00	3.2	-0.15858349D 00	3.3	-0.13411816D 00
3.4	-0.10663448D 00	3.5	-0.77434477D-01	3.6	-0.47652585D-01	3.7	-0.18249774D-01
3.8	0.99833713D-02	3.9	0.36417629D-01	4.0	0.60572308D-01	4.1	0.82100277D-01
4.2	0.10077205D 00	4.3	0.11645983D 00	4.4	0.12912203D 00	4.5	0.13878875D 00
4.6	0.14554845D 00	4.7	0.14953584D 00	4.8	0.15092114D 00	4.9	0.14990072D 00
5.0	0.14668888D 00	5.1	0.14151095D 00	5.2	0.13459743D 00	5.3	0.12617908D 00
5.4	0.11648299D 00	5.5	0.10572940D 00	5.6	0.94129174D-01	5.7	0.81881945D-01
5.8	0.69174722D-01	5.9	0.56180967D-01	6.0	0.43060031D-01	6.1	0.29956900D-01
6.2	0.17002201D-01	6.3	0.43124119D-02	6.4	-0.80097646D-02	6.5	-0.19874894D-01
6.6	-0.31206208D-01	6.7	-0.41938928D-01	6.8	-0.52019540D-01	6.9	-0.61405039D-01
7.0	-0.70062158D-01	7.1	-0.77966594D-01	7.2	-0.85102243D-01	7.3	-0.91460450D-01
7.4	-0.97039280D-01	7.5	-0.10184283D 00	7.6	-0.10588054D 00	7.7	-0.10916660D 00
7.8	-0.11171930D 00	7.9	-0.11356052D 00	8.0	-0.11471520D 00	8.1	-0.11521082D 00
8.2	-0.11507699D 00	8.3	-0.11434503D 00	8.4	-0.11304760D 00	8.5	-0.11121833D 00
8.6	-0.10889153D 00	8.7	-0.10610193D 00	8.8	-0.10288439D 00	8.9	-0.99273666D-01
9.0	-0.95304272D-01	9.1	-0.91010247D-01	9.2	-0.86425020D-01	9.3	-0.81581281D-01
9.4	-0.76510856D-01	9.5	-0.71244617D-01	9.6	-0.65812397D-01	9.7	-0.60242924D-01
9.8	-0.54563766D-01	9.9	-0.48801288D-01	10.0	-0.42980627D-01		

P= 6.00

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.12179515D 00	1.2	-0.38807796D 00	1.3	-0.29834088D 00
1.4	-0.10492907D 00	1.5	0.75838000D-01	1.6	0.20006135D 00	1.7	0.26027906D 00
1.8	0.26551677D 00	1.9	0.23059281D 00	2.0	0.17070467D 00	2.1	0.98971917D-01
2.2	0.25582092D-01	2.3	-0.42240139D-01	2.4	-0.99881759D-01	2.5	-0.14485078D 00
2.6	-0.17627337D 00	2.7	-0.19443959D 00	2.8	-0.20042201D 00	2.9	-0.19577126D 00
3.0	-0.18228307D 00	3.1	-0.16182709D 00	3.2	-0.13622647D 00	3.3	-0.10717787D 00
3.4	-0.76202586D-01	3.5	-0.44621059D-01	3.6	-0.13544288D-01	3.7	0.16122802D-01
3.8	0.43670045D-01	3.9	0.68565745D-01	4.0	0.90437634D-01	4.1	0.10905270D 00
4.2	0.12429706D 00	4.3	0.13615655D 00	4.4	0.14469872D 00	4.5	0.15005632D 00
4.6	0.15241256D 00	4.7	0.15198812D 00	4.8	0.14902994D 00	4.9	0.14380154D 00
5.0	0.13657501D 00	5.1	0.12762424D 00	5.2	0.11721949D 00	5.3	0.10562304D 00
5.4	0.93085803D-01	5.5	0.79844758D-01	5.6	0.66121125D-01	5.7	0.52119132D-01
5.8	0.38025293D-01	5.9	0.24008111D-01	6.0	0.10218121D-01	6.1	-0.32117913D-02
6.2	-0.16165846D-01	6.3	-0.28544666D-01	6.4	-0.40264445D-01	6.5	-0.51256024D-01
6.6	-0.61463917D-01	6.7	-0.70845296D-01	6.8	-0.79368969D-01	6.9	-0.87014360D-01
7.0	-0.93770505D-01	7.1	-0.99635086D-01	7.2	-0.10461349D 00	7.3	-0.10871791D 00
7.4	-0.11196652D 00	7.5	-0.11438267D 00	7.6	-0.11599410D 00	7.7	-0.11683235D 00
7.8	-0.11693200D 00	7.9	-0.11633017D 00	8.0	-0.11506596D 00	8.1	-0.11317992D 00
8.2	-0.11071366D 00	8.3	-0.10770945D 00	8.4	-0.10420980D 00	8.5	-0.10025721D 00
8.6	-0.95893874D-01	8.7	-0.91161401D-01	8.8	-0.86100635D-01	8.9	-0.80751455D-01
9.0	-0.75152620D-01	9.1	-0.69341635D-01	9.2	-0.63354640D-01	9.3	-0.57226322D-01
9.4	-0.50989846D-01	9.5	-0.44676797D-01	9.6	-0.38317146D-01	9.7	-0.31939223D-01
9.8	-0.25569706D-01	9.9	-0.19233621D-01	10.0	-0.12954348D-01		

P= 6.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.14134509D 00	1.2	-0.38981007D 00	1.3	-0.28031325D 00
1.4	-0.76793841D-01	1.5	0.10341390D 00	1.6	0.21934050D 00	1.7	0.25726432D 00
1.8	0.25943969D 00	1.9	0.21298059D 00	2.0	0.14444321D 00	2.1	0.67532529D-01
2.2	-0.75446391D-02	2.3	-0.73916225D-01	2.4	-0.12753416D 00	2.5	-0.16656342D 00
2.6	-0.19079452D 00	2.7	-0.20113440D 00	2.8	-0.19919280D 00	2.9	-0.18696249D 00
3.0	-0.16658263D 00	3.1	-0.14017155D 00	3.2	-0.10971506D 00	3.3	-0.76997629D-01
3.4	-0.43565669D-01	3.5	-0.10714115D-01	3.6	0.20510772D-01	3.7	0.49297779D-01
3.8	0.75050441D-01	3.9	0.97363092D-01	4.0	0.11599548D 00	4.1	0.13084747D 00
4.2	0.14193483D 00	4.3	0.14936685D 00	4.4	0.15332601D 00	4.5	0.15404996D 00
4.6	0.15181587D 00	4.7	0.14692694D 00	4.8	0.13970111D 00	4.9	0.13046159D 00
5.0	0.11952926D 00	5.1	0.10721658D 00	5.2	0.93822777D-01	5.3	0.79630361D-01
5.4	0.64902516D-01	5.5	0.49881408D-01	5.6	0.34787183D-01	5.7	0.19817547D-01
5.8	0.51478254D-02	5.9	-0.90686072D-02	6.0	-0.22699596D-01	6.1	-0.35633254D-01
6.2	-0.47776848D-01	6.3	-0.59055569D-01	6.4	-0.69411232D-01	6.5	-0.78800936D-01
6.6	-0.87195725D-01	6.7	-0.94579249D-01	6.8	-0.10094647D 00	6.9	-0.10630240D 00
7.0	-0.11066091D 00	7.1	-0.11404359D 00	7.2	-0.11647867D 00	7.3	-0.11800004D 00
7.4	-0.11864633D 00	7.5	-0.11846001D 00	7.6	-0.11748668D 00	7.7	-0.11577429D 00
7.8	-0.11337255D 00	7.9	-0.11033229D 00	8.0	-0.10670503D 00	8.1	-0.10254240D 00
8.2	-0.97895858D-01	8.3	-0.92816228D-01	8.4	-0.87353452D-01	8.5	-0.81556305D-01
8.6	-0.75472167D-01	8.7	-0.69146835D-01	8.8	-0.62624370D-01	8.9	-0.55946964D-01
9.0	-0.49154847D-01	9.1	-0.42286212D-01	9.2	-0.35377158D-01	9.3	-0.28461667D-01
9.4	-0.21571578D-01	9.5	-0.14736600D-01	9.6	-0.79843141D-02	9.7	-0.13402095D-02
9.8	0.51722862D-02	9.9	0.11531760D-01	10.0	0.17718762D-01		

P= 6.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.16029932D 00	1.2	-0.39001347D 00	1.3	-0.26098639D 00
1.4	-0.48544533D-01	1.5	0.12958821D 00	1.6	0.23591734D 00	1.7	0.27078756D 00
1.8	0.24979667D 00	1.9	0.19231450D 00	2.0	0.11612407D 00	2.1	0.35343184D-01
2.2	-0.39978700D-01	2.3	-0.10348000D 00	2.4	-0.15180466D 00	2.5	-0.18386650D 00
2.6	-0.20017787D 00	2.7	-0.20228920D 00	2.8	-0.19234846D 00	2.9	-0.17277162D 00
3.0	-0.14600779D 00	3.1	-0.11438100D 00	3.2	-0.79991360D-01	3.3	-0.44660930D-01
3.4	-0.99118383D-02	3.5	0.23033047D-01	3.6	0.53234535D-01	3.7	0.80012315D-01
3.8	0.10291518D 00	3.9	0.12168928D 00	4.0	0.13624646D 00	4.1	0.14663407D 00
4.2	0.15300722D 00	4.3	0.15560369D 00	4.4	0.15472194D 00	4.5	0.15070216D 00
4.6	0.14391010D 00	4.7	0.13472377D 00	4.8	0.12352241D 00	4.9	0.11067781D 00
5.0	0.96547507D-01	5.1	0.81469614D-01	5.2	0.65759145D-01	5.3	0.49705510D-01
5.4	0.33571020D-01	5.5	0.17590224D-01	5.6	0.19698991D-02	5.7	-0.13110421D-01
5.8	-0.27497515D-01	5.9	-0.41063238D-01	6.0	-0.53703053D-01	6.1	-0.65334421D-01
6.2	-0.75895097D-01	6.3	-0.85341376D-01	6.4	-0.93646345D-01	6.5	-0.10079814D 00
6.6	-0.10679830D 00	6.7	-0.11166008D 00	6.8	-0.11540702D 00	6.9	-0.11807141D 00
7.0	-0.11969299D 00	7.1	-0.12031770D 00	7.2	-0.11999649D 00	7.3	-0.11878433D 00
7.4	-0.11673919D 00	7.5	-0.11392125D 00	7.6	-0.11039204D 00	7.7	-0.10621384D 00
7.8	-0.10144902D 00	7.9	-0.96159527D-01	8.0	-0.90406406D-01	8.1	-0.84249428D-01
8.2	-0.77746736D-01	8.3	-0.70954563D-01	8.4	-0.63927001D-01	8.5	-0.56715814D-01
8.6	-0.49370283D-01	8.7	-0.41937103D-01	8.8	-0.34460298D-01	8.9	-0.26981174D-01
9.0	-0.19538295D-01	9.1	-0.12167480D-01	9.2	-0.49018209D-02	9.3	0.22282798D-02
9.4	0.91950628D-02	9.5	0.15973348D-01	9.6	0.22540463D-01	9.7	0.28876161D-01
9.8	0.34962535D-01	9.9	0.40783922D-01	10.0	0.46326807D-01		

P= 6.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.17863549D 00	1.2	-0.38871232D 00	1.3	-0.24048622D 00
1.4	-0.20386500D-01	1.5	0.15414983D 00	1.6	0.24965973D 00	1.7	0.27086189D 00
1.8	0.23677937D 00	1.9	0.16896296D 00	2.0	0.86260692D-01	2.1	0.30096343D-02
2.2	-0.71086027D-01	2.3	-0.13033544D 00	2.4	-0.17219686D 00	2.5	-0.19641495D 00
2.6	-0.20426833D 00	2.7	-0.19796313D 00	2.8	-0.18017206D 00	2.9	-0.15370178D 00
3.0	-0.12126603D 00	3.1	-0.85341659D-01	3.2	-0.48087188D-01	3.3	-0.11306730D-01
3.4	0.23554416D-01	3.5	0.55393202D-01	3.6	0.83419299D-01	3.7	0.10711852D 00
3.8	0.12621334D 00	3.9	0.14062347D 00	4.0	0.15042826D 00	4.1	0.15583218D 00
4.2	0.15713397D 00	4.3	0.15469970D 00	4.4	0.14893972D 00	4.5	0.14028934D 00
4.6	0.12919298D 00	4.7	0.11609139D 00	4.8	0.10141171D 00	4.9	0.85559822D-01
5.0	0.68914865D-01	5.1	0.51825403D-01	5.2	0.34607089D-01	5.3	0.17541491D-01
5.4	0.87588451D-03	5.5	-0.15176211D-01	5.6	-0.30433907D-01	5.7	-0.44747388D-01
5.8	-0.57996036D-01	5.9	-0.70086350D-01	6.0	-0.80949731D-01	6.1	-0.90540217D-01
6.2	-0.98832203D-01	6.3	-0.10581821D 00	6.4	-0.11150670D 00	6.5	-0.11592004D 00
6.6	-0.11909250D 00	6.7	-0.12106847D 00	6.8	-0.12190069D 00	6.9	-0.12164876D 00
7.0	-0.12037764D 00	7.1	-0.11815639D 00	7.2	-0.11505700D 00	7.3	-0.11115333D 00
7.4	-0.10652019D 00	7.5	-0.10123253D 00	7.6	-0.95364727D-01	7.7	-0.88989984D-01
7.8	-0.82179787D-01	7.9	-0.75003482D-01	8.0	-0.67527899D-01	8.1	-0.59817066D-01
8.2	-0.51931962D-01	8.3	-0.43930347D-01	8.4	-0.35866624D-01	8.5	-0.27791758D-01
8.6	-0.19753221D-01	8.7	-0.11794983D-01	8.8	-0.39575261D-02	8.9	0.37221184D-02
9.0	0.11210305D-01	9.1	0.18476696D-01	9.2	0.25494162D-01	9.3	0.32238679D-01
9.4	0.38689205D-01	9.5	0.44827559D-01	9.6	0.50638282D-01	9.7	0.56108509D-01
9.8	0.61227820D-01	9.9	0.65988109D-01	10.0	0.70383437D-01		

P= 6.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.19633232D 00	1.2	-0.38593537D 00	1.3	-0.21894283D 00
1.4	0.74786333D-02	1.5	0.17690487D 00	1.6	0.26046695D 00	1.7	0.26754155D 00
1.8	0.22062237D 00	1.9	0.14332936D 00	2.0	0.55384250D-01	2.1	-0.28869284D-01
2.2	-0.10026861D 00	2.3	-0.15395297D 00	2.4	-0.18830928D 00	2.5	-0.20398243D 00
2.6	-0.20304647D 00	2.7	-0.18835951D 00	2.8	-0.16309022D 00	2.9	-0.13038977D 00
3.0	-0.93179727D-01	3.1	-0.54027995D-01	3.2	-0.15089249D-01	3.3	0.21909674D-01
3.4	0.55654373D-01	3.5	0.85208879D-01	3.6	0.10997136D 00	3.7	0.12962544D 00
3.8	0.14409118D 00	3.9	0.15347835D 00	4.0	0.15804351D 00	4.1	0.15815195D 00
4.2	0.15424452D 00	4.3	0.14680963D 00	4.4	0.13635996D 00	4.5	0.12341360D 00
4.6	0.10847911D 00	4.7	0.92044036D-01	4.8	0.74566478D-01	4.9	0.56469100D-01
5.0	0.38135301D-01	5.1	0.19907105D-01	5.2	0.20844462D-02	5.3	-0.15074440D-01
5.4	-0.31351779D-01	5.5	-0.46568374D-01	5.6	-0.60581272D-01	5.7	-0.73281119D-01
5.8	-0.84589322D-01	5.9	-0.94455131D-01	6.0	-0.10285270D 00	6.1	-0.10977821D 00
6.2	-0.11524707D 00	6.3	-0.11929127D 00	6.4	-0.12195685D 00	6.5	-0.12330163D 00
6.6	-0.12339296D 00	6.7	-0.12230583D 00	6.8	-0.12012104D 00	6.9	-0.11692361D 00
7.0	-0.11280133D 00	7.1	-0.10784348D 00	7.2	-0.10213976D 00	7.3	-0.95779268D-01
7.4	-0.88849696D-01	7.5	-0.81436627D-01	7.6	-0.73622935D-01	7.7	-0.65488304D-01
7.8	-0.57108832D-01	7.9	-0.48556729D-01	8.0	-0.39900084D-01	8.1	-0.31202706D-01
8.2	-0.22524016D-01	8.3	-0.13919004D-01	8.4	-0.54382165D-02	8.5	0.28722050D-02
8.6	0.10970458D-01	8.7	0.18818981D-01	8.8	0.26384338D-01	8.9	0.33637077D-01
9.0	0.40551574D-01	9.1	0.47105872D-01	9.2	0.53281498D-01	9.3	0.59063286D-01
9.4	0.64439191D-01	9.5	0.69400095D-01	9.6	0.73939623D-01	9.7	0.78053952D-01
9.8	0.81741629D-01	9.9	0.85003383D-01	10.0	0.87841952D-01		

P= 6.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.21336966D 00	1.2	-0.38171589D 00	1.3	-0.19648980D 00
1.4	0.34854270D-01	1.5	0.19767803D 00	1.6	0.26827021D 00	1.7	0.26092079D 00
1.8	0.20159937D 00	1.9	0.11584550D 00	2.0	0.24034614D-01	2.1	-0.59712289D-01
2.2	-0.12697555D 00	2.3	-0.17387954D 00	2.4	-0.19984295D 00	2.5	-0.20646490D 00
2.6	-0.19662696D 00	2.7	-0.17381835D 00	2.8	-0.14165923D 00	2.9	-0.10358571D 00
3.0	-0.62659748D-01	3.1	-0.21470899D-01	3.2	0.17896994D-01	3.3	0.53855774D-01
3.4	0.85275709D-01	3.5	0.11143290D 00	3.6	0.13194979D 00	3.7	0.14673448D 00
3.8	0.15592264D 00	3.9	0.15982423D 00	4.0	0.15887622D 00	4.1	0.15360185D 00
4.2	0.14457627D 00	4.3	0.13239840D 00	4.4	0.11766829D 00	4.5	0.10096964D 00
4.6	0.82856439D-01	4.7	0.63843565D-01	4.8	0.44400312D-01	4.9	0.24946588D-01
5.0	0.58511508D-02	5.1	-0.12568492D-01	5.2	-0.30044921D-01	5.3	-0.46358682D-01
5.4	-0.61335461D-01	5.5	-0.74842800D-01	5.6	-0.86786515D-01	5.7	-0.97106960D-01
5.8	-0.10577527D 00	5.9	-0.11278963D 00	6.0	-0.11817174D 00	6.1	-0.12196334D 00
6.2	-0.12422302D 00	6.3	-0.12502327D 00	6.4	-0.12444772D 00	6.5	-0.12258862D 00
6.6	-0.11954464D 00	6.7	-0.11541884D 00	6.8	-0.11031690D 00	6.9	-0.10434556D 00
7.0	-0.97611259D-01	7.1	-0.90219008D-01	7.2	-0.82271378D-01	7.3	-0.73867696D-01
7.4	-0.65103374D-01	7.5	-0.56069368D-01	7.6	-0.46851763D-01	7.7	-0.37531464D-01
7.8	-0.28183974D-01	7.9	-0.18879266D-01	8.0	-0.96817172D-02	8.1	-0.65011145D-03
8.2	0.81623072D-02	8.3	0.16707735D-01	8.4	0.24943674D-01	8.5	0.32832757D-01
8.6	0.40342566D-01	8.7	0.47445406D-01	8.8	0.54118088D-01	8.9	0.60341682D-01
9.0	0.66101274D-01	9.1	0.71385715D-01	9.2	0.76187369D-01	9.3	0.80501859D-01
9.4	0.84327823D-01	9.5	0.87666664D-01	9.6	0.90522319D-01	9.7	0.92901023D-01
9.8	0.94811088D-01	9.9	0.96262689D-01	10.0	0.97267659D-01		

P= 6.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.22972852D 00	1.2	-0.37609146D 00	1.3	-0.17326353D 00
1.4	0.61550101D-01	1.5	0.21631388D 00	1.6	0.27303300D 00	1.7	0.25113211D 00
1.8	0.18001905D 00	1.9	0.86964737D-01	2.0	-0.72486561D-02	2.1	-0.88965842D-01
2.2	-0.15071309D 00	2.3	-0.18974687D 00	2.4	-0.20660642D 00	2.5	-0.20388126D 00
2.6	-0.18525370D 00	2.7	-0.15480535D 00	2.8	-0.11654774D 00	2.9	-0.74129672D-01
3.0	-0.30676685D-01	3.1	0.11275428D-01	3.2	0.49783525D-01	3.3	0.83459638D-01
3.4	0.11141024D 00	3.5	0.13316438D 00	3.6	0.14859903D 00	3.7	0.15786696D 00
3.8	0.16133044D 00	3.9	0.15950249D 00	4.0	0.15299645D 00	4.1	0.14248375D 00
4.2	0.12865958D 00	4.3	0.11221569D 00	4.4	0.93819479D-01	4.5	0.74098638D-01
4.6	0.53630378D-01	4.7	0.32934530D-01	4.8	0.12469772D-01	4.9	-0.73676410D-02
5.0	-0.26243201D-01	5.1	-0.43881994D-01	5.2	-0.60065479D-01	5.3	-0.74627507D-01
5.4	-0.87449878D-01	5.5	-0.98457628D-01	5.6	-0.10761423D 00	5.7	-0.11491684D 00
5.8	-0.12039170D 00	5.9	-0.12408972D 00	6.0	-0.12608237D 00	6.1	-0.12645788D 00
6.2	-0.12531771D 00	6.3	-0.12277341D 00	6.4	-0.11894374D 00	6.5	-0.11395218D 00
6.6	-0.10792473D 00	6.7	-0.10098794D 00	6.8	-0.93267281D-01	6.9	-0.84885767D-01
7.0	-0.75962752D-01	7.1	-0.66612993D-01	7.2	-0.56945877D-01	7.3	-0.47064825D-01
7.4	-0.37066849D-01	7.5	-0.27042239D-01	7.6	-0.17074367D-01	7.7	-0.72395985D-02
7.8	0.23927146D-02	7.9	0.11760165D-01	8.0	0.20807157D-01	8.1	0.29484723D-01
8.2	0.37750298D-01	8.3	0.45567457D-01	8.4	0.52905631D-01	8.5	0.59739799D-01
8.6	0.66050168D-01	8.7	0.71821845D-01	8.8	0.77044503D-01	8.9	0.81712047D-01
9.0	0.85822282D-01	9.1	0.89376590D-01	9.2	0.92379607D-01	9.3	0.94838917D-01
9.4	0.96764752D-01	9.5	0.98169708D-01	9.6	0.99068465D-01	9.7	0.99477536D-01
9.8	0.99415012D-01	9.9	0.98900336D-01	10.0	0.97954081D-01		

P= 6.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.24539107D 00	1.2	-0.36910388D 00	1.3	-0.14940247D 00
1.4	0.87383315D-01	1.5	0.23267787D 00	1.6	0.27475097D 00	1.7	0.23834424D 00
1.8	0.15622035D 00	1.9	0.57154894D-01	2.0	-0.37935206D-01	2.1	-0.11611371D 00
2.2	-0.17105335D 00	2.3	-0.20127798D 00	2.4	-0.20851857D 00	2.5	-0.19637109D 00
2.6	-0.16929182D 00	2.7	-0.13189782D 00	2.8	-0.88516690D-01	2.9	-0.42926119D-01
3.0	0.17692229D-02	3.1	0.43166824D-01	3.2	0.79534886D-01	3.3	0.10974512D 00
3.4	0.13318749D 00	3.5	0.14967894D 00	3.6	0.15937416D 00	3.7	0.16268299D 00
3.8	0.16019723D 00	3.9	0.15262791D 00	4.0	0.14075308D 00	4.1	0.12537562D 00
4.2	0.10729020D 00	4.3	0.87258253D-01	4.4	0.65989859D-01	4.5	0.44131491D-01
4.6	0.22258489D-01	4.7	0.87139203D-03	4.8	-0.19604703D-01	4.9	-0.38818477D-01
5.0	-0.56489013D-01	5.1	-0.72401210D-01	5.2	-0.86400397D-01	5.3	-0.98386484D-01
5.4	-0.10830790D 00	5.5	-0.11615552D 00	5.6	-0.12195678D 00	5.7	-0.12576998D 00
5.8	-0.12767902D 00	5.9	-0.12778845D 00	6.0	-0.12621899D 00	6.1	-0.12310347D 00
6.2	-0.11858320D 00	6.3	-0.11280478D 00	6.4	-0.10591732D 00	6.5	-0.98070007D-01
6.6	-0.89410080D-01	6.7	-0.80081119D-01	6.8	-0.70221629D-01	6.9	-0.59963908D-01
7.0	-0.49433156D-01	7.1	-0.38746804D-01	7.2	-0.28014041D-01	7.3	-0.17335501D-01
7.4	-0.68031046D-02	7.5	0.34999795D-02	7.6	0.13499264D-01	7.7	0.23128783D-01
7.8	0.32330861D-01	7.9	0.41055818D-01	8.0	0.49261630D-01	8.1	0.56913551D-01
8.2	0.63983711D-01	8.3	0.70450694D-01	8.4	0.76299101D-01	8.5	0.81519114D-01
8.6	0.86106055D-01	8.7	0.90059955D-01	8.8	0.93385123D-01	8.9	0.96089735D-01
9.0	0.98185433D-01	9.1	0.99686938D-01	9.2	0.10061168D 00	9.3	0.10097946D 00
9.4	0.10081209D 00	9.5	0.10013313D 00	9.6	0.98967550D-01	9.7	0.97341474D-01
9.8	0.95281936D-01	9.9	0.92816641D-01	10.0	0.89973746D-01		

P= 6.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.26034068D 00	1.2	-0.36079899D 00	1.3	-0.12504649D 00
1.4	0.11217974D 00	1.5	0.24665727D 00	1.6	0.27345156D 00	1.7	0.22275961D 00
1.8	0.13056758D 00	1.9	0.26891047D-01	2.0	-0.67512528D-01	2.1	-0.14068573D 00
2.2	-0.18764178D 00	2.3	-0.20829162D 00	2.4	-0.20560872D 00	2.5	-0.18418962D 00
2.6	-0.14921680D 00	2.7	-0.10576785D 00	2.8	-0.58396906D-01	2.9	-0.10916871D-01
3.0	0.33678652D-01	3.1	0.73201749D-01	3.2	0.10620110D 00	3.3	0.13186338D 00
3.4	0.14990305D 00	3.5	0.16045225D 00	3.6	0.16395803D 00	3.7	0.16109087D 00
3.8	0.15266631D 00	3.9	0.13957995D 00	4.0	0.12275524D 00	4.1	0.10310293D 00
4.2	0.81490725D-01	4.3	0.58721787D-01	4.4	0.35520451D-01	4.5	0.12523964D-01
4.6	-0.97210600D-02	4.7	-0.30758508D-01	4.8	-0.50219880D-01	4.9	-0.67819402D-01
5.0	-0.83347779D-01	5.1	-0.96665097D-01	5.2	-0.10769328D 00	5.3	-0.11640840D 00
5.4	-0.12283316D 00	5.5	-0.12702957D 00	5.6	-0.12909214D 00	5.7	-0.12914147D 00
5.8	-0.12731849D 00	5.9	-0.12377918D 00	6.0	-0.11868995D 00	6.1	-0.11222349D 00
6.2	-0.10455528D 00	6.3	-0.95860460D-01	6.4	-0.86311359D-01	6.5	-0.76075291D-01
6.6	-0.65312864D-01	6.7	-0.54176594D-01	6.8	-0.42809859D-01	6.9	-0.31346132D-01
7.0	-0.19908461D-01	7.1	-0.86091621D-02	7.2	0.24502975D-02	7.3	0.13179267D-01
7.4	0.23497745D-01	7.5	0.33336105D-01	7.6	0.42634731D-01	7.7	0.51343594D-01
7.8	0.59421759D-01	7.9	0.66836869D-01	8.0	0.73564590D-01	8.1	0.79588049D-01
8.2	0.84897257D-01	8.3	0.89488535D-01	8.4	0.93363951D-01	8.5	0.96530760D-01
8.6	0.99008866D-01	8.7	0.10079030D 00	8.8	0.10191874D 00	8.9	0.10240901D 00
9.0	0.10228663D 00	9.1	0.10157944D 00	9.2	0.10031715D 00	9.3	0.98531012D-01
9.4	0.96253455D-01	9.5	0.93517781D-01	9.6	0.90357879D-01	9.7	0.86807954D-01
9.8	0.82902298D-01	9.9	0.78675066D-01	10.0	0.74160089D-01		

P= 6.90

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.27456195D 00	1.2	-0.35122645D 00	1.3	-0.10033610D 00
1.4	0.13577494D 00	1.5	0.25816178D 00	1.6	0.26919333D 00	1.7	0.20461154D 00
1.8	0.10344511D 00	1.9	-0.33517179D-02	2.0	-0.95494186D-01	2.1	-0.16226564D 00
2.2	-0.20020296D 00	2.3	-0.21070472D 00	2.4	-0.19801449D 00	2.5	-0.16770001D 00
2.6	-0.12560102D 00	2.7	-0.77163236D-01	2.8	-0.27065180D-01	2.9	0.20946665D-01
3.0	0.64083299D-01	3.1	0.10045208D 00	3.2	0.12894737D 00	3.3	0.14911966D 00
3.4	0.16104074D 00	3.5	0.16517597D 00	3.6	0.16226971D 00	3.7	0.15324704D 00
3.8	0.13913221D 00	3.9	0.12098356D 00	4.0	0.99843195D-01	4.1	0.76699819D-01
4.2	0.52462666D-01	4.3	0.27944721D-01	4.4	0.38533915D-02	4.5	-0.19212936D-01
4.6	-0.40763833D-01	4.7	-0.60412451D-01	4.8	-0.77868796D-01	4.9	-0.92931458D-01
5.0	-0.10547844D 00	5.1	-0.11545760D 00	5.2	-0.12287700D 00	5.3	-0.12779566D 00
5.4	-0.13031462D 00	5.5	-0.13056875D 00	5.6	-0.12871915D 00	5.7	-0.12494637D 00
5.8	-0.11944435D 00	5.9	-0.11241509D 00	6.0	-0.10406407D 00	6.1	-0.94596365D-01
6.2	-0.84213334D-01	6.3	-0.73109941D-01	6.4	-0.61472589D-01	6.5	-0.49477426D-01
6.6	-0.37289072D-01	6.7	-0.25059715D-01	6.8	-0.12928530D-01	6.9	-0.10213700D-02
7.0	0.10549305D-01	7.1	0.21684303D-01	7.2	0.32297415D-01	7.3	0.42314996D-01
7.4	0.51675442D-01	7.5	0.60328585D-01	7.6	0.68235027D-01	7.7	0.75365442D-01
7.8	0.81699837D-01	7.9	0.87226808D-01	8.0	0.91942798D-01	8.1	0.95851358D-01
8.2	0.98962421D-01	8.3	0.10129160D 00	8.4	0.10285953D 00	8.5	0.10369118D 00
8.6	0.10381531D 00	8.7	0.10326381D 00	8.8	0.10207126D 00	8.9	0.10027432D 00
9.0	0.97911350D-01	9.1	0.95021935D-01	9.2	0.91646505D-01	9.3	0.87825976D-01
9.4	0.83601426D-01	9.5	0.79013800D-01	9.6	0.74103653D-01	9.7	0.68910916D-01
9.8	0.63474694D-01	9.9	0.57833089D-01	10.0	0.52023045D-01		

P= 7.00

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.28804073D 00	1.2	-0.34043958D 00	1.3	-0.75411739D-01
1.4	0.15801517D 00	1.5	0.26712408D 00	1.6	0.26206489D 00	1.7	0.18416100D 00
1.8	0.75251961D-01	1.9	-0.33105450D-01	2.0	-0.12142752D 00	2.1	-0.18049788D 00
2.2	-0.20854495D 00	2.3	-0.20853277D 00	2.4	-0.18597718D 00	2.5	-0.14736324D 00
2.6	-0.99097976D-01	2.7	-0.46886726D-01	2.8	0.45806420D-02	2.9	0.51731144D-01
3.0	0.92074642D-01	3.1	0.12409117D 00	3.2	0.14707962D 00	3.3	0.16099453D 00
3.4	0.16628793D 00	3.5	0.16376578D 00	3.6	0.15446415D 00	3.7	0.13954648D 00
3.8	0.12022151D 00	3.9	0.97680331D-01	4.0	0.73050118D-01	4.1	0.47362138D-01
4.2	0.21531516D-01	4.3	-0.36535697D-02	4.4	-0.27535317D-01	4.5	-0.49584181D-01
4.6	-0.69392708D-01	4.7	-0.86666590D-01	4.8	-0.10121395D 00	4.9	-0.11293369D 00
5.0	-0.12180342D 00	5.1	-0.12786766D 00	5.2	-0.13122632D 00	5.3	-0.13202405D 00
5.4	-0.13044036D 00	5.5	-0.12668069D 00	5.6	-0.12096848D 00	5.7	-0.11353820D 00
5.8	-0.10462940D 00	5.9	-0.94481524D-01	6.0	-0.83329749D-01	6.1	-0.71401449D-01
6.2	-0.58913443D-01	6.3	-0.46069846D-01	6.4	-0.33060481D-01	6.5	-0.20059774D-01
6.6	-0.72260714D-02	6.7	0.52986842D-02	6.8	0.17388978D-01	6.9	0.28935492D-01
7.0	0.39844678D-01	7.1	0.50038153D-01	7.2	0.59451981D-01	7.3	0.68035855D-01
7.4	0.75752200D-01	7.5	0.82575251D-01	7.6	0.88490092D-01	7.7	0.93491688D-01
7.8	0.97583933D-01	7.9	0.10077871D 00	8.0	0.10309496D 00	8.1	0.10455786D 00
8.2	0.10519789D 00	8.3	0.10505015D 00	8.4	0.10415353D 00	8.5	0.10255005D 00
8.6	0.10028422D 00	8.7	0.97402405D-01	8.8	0.93952343D-01	8.9	0.89982588D-01
9.0	0.85542088D-01	9.1	0.80679778D-01	9.2	0.75444215D-01	9.3	0.69883259D-01
9.4	0.64043798D-01	9.5	0.57971498D-01	9.6	0.51710596D-01	9.7	0.45303727D-01
9.8	0.38791773D-01	9.9	0.32213741D-01	10.0	0.25606676D-01		

P= 7.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.30076410D 00	1.2	-0.32849517D 00	1.3	-0.50413110D-01
1.4	0.17875836D 00	1.5	0.27350007D 00	1.6	0.25218369D 00	1.7	0.16169318D 00
1.8	0.46396200D-01	1.9	-0.61916282D-01	2.0	-0.14490070D 00	2.1	-0.19509314D 00
2.2	-0.21256185D 00	2.3	-0.20188812D 00	2.4	-0.16983491D 00	2.5	-0.12372580D 00
2.6	-0.70424861D-01	2.7	-0.15774048D-01	2.8	0.35646798D-01	2.9	0.80548033D-01
3.0	0.11683053D 00	3.1	0.14341842D 00	3.2	0.16006515D 00	3.3	0.16715901D 00
3.4	0.16554359D 00	3.5	0.15636128D 00	3.6	0.14092303D 00	3.7	0.12060423D 00
3.8	0.96764887D-01	3.9	0.70691379D-01	4.0	0.43556431D-01	4.1	0.16394164D-01
4.2	-0.99128582D-02	4.3	-0.34637277D-01	4.4	-0.57203654D-01	4.5	-0.77180213D-01
4.6	-0.94267234D-01	4.7	-0.10828340D 00	4.8	-0.11915114D 00	4.9	-0.12688162D 00
5.0	-0.13156021D 00	5.1	-0.13333252D 00	5.2	-0.13239151D 00	5.3	-0.12896576D 00
5.4	-0.12330899D 00	5.5	-0.11569087D 00	5.6	-0.10638901D 00	5.7	-0.95682293D-01
5.8	-0.83845218D-01	5.9	-0.71143323D-01	6.0	-0.57829560D-01	6.1	-0.44141493D-01
6.2	-0.30299248D-01	6.3	-0.16504105D-01	6.4	-0.29376519D-02	6.5	0.10238599D-01
6.6	0.22883188D-01	6.7	0.34874371D-01	6.8	0.46109507D-01	6.9	0.56504257D-01
7.0	0.65991602D-01	7.1	0.74520769D-01	7.2	0.82056054D-01	7.3	0.88575618D-01
7.4	0.94070246D-01	7.5	0.98542109D-01	7.6	0.10200355D 00	7.7	0.10447589D 00
7.8	0.10598830D 00	7.9	0.10657669D 00	8.0	0.10628266D 00	8.1	0.10515260D 00
8.2	0.10323670D 00	8.3	0.10058817D 00	8.4	0.97262436D-01	8.5	0.93316465D-01
8.6	0.88808113D-01	8.7	0.83795557D-01	8.8	0.78336783D-01	8.9	0.72489131D-01
9.0	0.66308904D-01	9.1	0.59851017D-01	9.2	0.53168704D-01	9.3	0.46313267D-01
9.4	0.39333868D-01	9.5	0.32277360D-01	9.6	0.25188151D-01	9.7	0.18108104D-01
9.8	0.11076465D-01	9.9	0.41298181D-02	10.0	-0.26979389D-02		

P= 7.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.31272045D 00	1.2	-0.31545322D 00	1.3	-0.25478437D-01
1.4	0.19787492D 00	1.5	0.27726902D 00	1.6	0.23969437D 00	1.7	0.13751378D 00
1.8	0.17289313D-01	1.9	-0.89351093D-01	2.0	-0.16554901D 00	2.1	-0.20583275D 00
2.2	-0.21223470D 00	2.3	-0.19097629D 00	2.4	-0.15001376D 00	2.5	-0.97405556D-01
2.6	-0.40343636D-01	2.7	0.15328597D-01	2.8	0.65268305D-01	2.9	0.10657873D 00
3.0	0.13763879D 00	3.1	0.15787949D 00	3.2	0.16754779D 00	3.3	0.16748299D 00
3.4	0.15891889D 00	3.5	0.14331779D 00	3.6	0.12223740D 00	3.7	0.97228689D-01
3.8	0.69761458D-01	3.9	0.41173422D-01	4.0	0.12638666D-01	4.1	-0.14848486D-01
4.2	-0.40477772D-01	4.3	-0.63618511D-01	4.4	-0.83808947D-01	4.5	-0.10074149D 00
4.6	-0.11424554D 00	4.7	-0.12426921D 00	4.8	-0.13086082D 00	4.9	-0.13415107D 00
5.0	-0.13433614D 00	5.1	-0.13166225D 00	5.2	-0.12641172D 00	5.3	-0.11889069D 00
5.4	-0.10941846D 00	5.5	-0.98318487D-01	5.6	-0.85910800D-01	5.7	-0.72505879D-01
5.8	-0.58399770D-01	5.9	-0.43870337D-01	6.0	-0.29174524D-01	6.1	-0.14546475D-01
6.2	-0.19640877D-03	6.3	0.13689881D-01	6.4	0.26950998D-01	6.5	0.39449488D-01
6.6	0.51071043D-01	6.7	0.61723430D-01	6.8	0.71335227D-01	6.9	0.79854412D-01
7.0	0.87246850D-01	7.1	0.93494736D-01	7.2	0.98595011D-01	7.3	0.10255779D 00
7.4	0.10540481D 00	7.5	0.10716796D 00	7.6	0.10788784D 00	7.7	0.10761238D 00
7.8	0.10639564D 00	7.9	0.10429653D 00	8.0	0.10137781D 00	8.1	0.97705005D-01
8.2	0.93345554D-01	8.3	0.88367946D-01	8.4	0.82840994D-01	8.5	0.76833177D-01
8.6	0.70412065D-01	8.7	0.63643812D-01	8.8	0.56592727D-01	8.9	0.49320905D-01
9.0	0.41887915D-01	9.1	0.34350559D-01	9.2	0.26762662D-01	9.3	0.19174925D-01
9.4	0.11634811D-01	9.5	0.41864722D-02	9.6	-0.31292893D-02	9.7	-0.10275030D-01
9.8	-0.17216647D-01	9.9	-0.23923338D-01	10.0	-0.30367540D-01		

P= 7.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.32389944D 00	1.2	-0.30137675D 00	1.3	-0.74379439D-03
1.4	0.21524847D 00	1.5	0.27843338D 00	1.6	0.22476698D 00	1.7	0.11194508D 00
1.8	-0.11659407D-01	1.9	-0.11500377D 00	2.0	-0.18306029D 00	2.1	-0.21257166D 00
2.2	-0.20763072D 00	2.3	-0.17609044D 00	2.4	-0.12701704D 00	2.5	-0.69076099D-01
2.6	-0.96413194D-02	2.7	0.45587038D-01	2.8	0.92632530D-01	2.9	0.12909730D 00
3.0	0.15391729D 00	3.1	0.16708178D 00	3.2	0.16935706D 00	3.3	0.16203690D 00
3.4	0.14673038D 00	3.5	0.12519058D 00	3.6	0.99182768D-01	3.7	0.70387957D-01
3.8	0.40337059D-01	3.9	0.10370048D-01	4.0	-0.18384956D-01	4.1	-0.45017361D-01
4.2	-0.68828574D-01	4.3	-0.89318648D-01	4.4	-0.10616783D 00	4.5	-0.11921520D 00
4.6	-0.12843602D 00	4.7	-0.13391906D 00	4.8	-0.13584484D 00	4.9	-0.13446517D 00
5.0	-0.13008473D 00	5.1	-0.12304451D 00	5.2	-0.11370750D 00	5.3	-0.10244632D 00
5.4	-0.89633071D-01	5.5	-0.75630817D-01	5.6	-0.60786968D-01	5.7	-0.45428109D-01
5.8	-0.29856211D-01	5.9	-0.14346025D-01	6.0	0.85651317D-03	6.1	0.15535023D-01
6.2	0.29502712D-01	6.3	0.42601833D-01	6.4	0.54702680D-01	6.5	0.65702232D-01
6.6	0.75522531D-01	6.7	0.84108877D-01	6.8	0.91427895D-01	6.9	0.97465551D-01
7.0	0.10222513D 00	7.1	0.10572527D 00	7.2	0.10799796D 00	7.3	0.10908676D 00
7.4	0.10904493D 00	7.5	0.10793383D 00	7.6	0.10582129D 00	7.7	0.10278022D 00
7.8	0.98887259D-01	7.9	0.94221558D-01	8.0	0.88863723D-01	8.1	0.82894828D-01
8.2	0.76395563D-01	8.3	0.69445482D-01	8.4	0.62122349D-01	8.5	0.54501587D-01
8.6	0.46655808D-01	8.7	0.38654424D-01	8.8	0.30563336D-01	8.9	0.22444690D-01
9.0	0.14356695D-01	9.1	0.63534928D-02	9.2	-0.15149147D-02	9.3	-0.92027016D-02
9.4	-0.16668211D-01	9.5	-0.23873914D-01	9.6	-0.30786343D-01	9.7	-0.37375992D-01
9.8	-0.43617205D-01	9.9	-0.49488039D-01	10.0	-0.54970118D-01		

P= 7.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.33429203D 00	1.2	-0.28633158D 00	1.3	0.23657562D-01
1.4	0.23077651D 00	1.5	0.27701856D 00	1.6	0.20759488D 00	1.7	0.85321883D-01
1.8	-0.40048388D-01	1.9	-0.13850100D 00	2.0	-0.19717943D 00	2.1	-0.21524014D 00
2.2	-0.19890082D 00	2.3	-0.15760402D 00	2.4	-0.10141297D 00	2.5	-0.39449964D-01
2.6	0.20890131D-01	2.7	0.74201025D-01	2.8	0.11700103D 00	2.9	0.14749034D 00
3.0	0.16522975D 00	3.1	0.17080454D 00	3.2	0.16551127D 00	3.3	0.15108665D 00
3.4	0.12948610D 00	3.5	0.10271203D 00	3.6	0.72687685D-01	3.7	0.41170493D-01
3.8	0.96980676D-02	3.9	-0.20439903D-01	4.0	-0.48209944D-01	4.1	-0.72828595D-01
4.2	-0.93746159D-01	4.3	-0.11062399D 00	4.4	-0.12330815D 00	4.5	-0.13180157D 00
4.6	-0.13623633D 00	4.7	-0.13684712D 00	4.8	-0.13394668D 00	4.9	-0.12790365D 00
5.0	-0.11912307D 00	5.1	-0.10802954D 00	5.2	-0.95053082D-01	5.3	-0.80617467D-01
5.4	-0.65130885D-01	5.5	-0.48978704D-01	5.6	-0.32518084D-01	5.7	-0.16074216D-01
5.8	0.62044282D-04	5.9	0.15635374D-01	6.0	0.30426208D-01	6.1	0.44250174D-01
6.2	0.56956899D-01	6.3	0.68428342D-01	6.4	0.78576758D-01	6.5	0.87342421D-01
6.6	0.94691189D-01	6.7	0.10061198D 00	6.8	0.10511422D 00	6.9	0.10822535D 00
7.0	0.10998838D 00	7.1	0.11045953D 00	7.2	0.10970604D 00	7.3	0.10780407D 00
7.4	0.10483680D 00	7.5	0.10089264D 00	7.6	0.96063625D-01	7.7	0.90443996D-01
7.8	0.84128872D-01	7.9	0.77213131D-01	8.0	0.69790405D-01	8.1	0.61952219D-01
8.2	0.53787266D-01	8.3	0.45380782D-01	8.4	0.36814054D-01	8.5	0.28164009D-01
8.6	0.19502905D-01	8.7	0.10898106D-01	8.8	0.24119224D-02	8.9	-0.58984731D-02
9.0	-0.13981077D-01	9.1	-0.21789032D-01	9.2	-0.29280564D-01	9.3	-0.36418874D-01
9.4	-0.43172006D-01	9.5	-0.49512681D-01	9.6	-0.55418114D-01	9.7	-0.60869811D-01
9.8	-0.65853349D-01	9.9	-0.70358156D-01	10.0	-0.74377268D-01		

P= 7.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.34389052D 00	1.2	-0.27038604D 00	1.3	0.47595852D-01
1.4	0.24437096D 00	1.5	0.27307238D 00	1.6	0.18839247D 00	1.7	0.57987362D-01
1.8	-0.67489118D-01	1.9	-0.15950749D 00	2.0	-0.20771192D 00	2.1	-0.21384404D 00
2.2	-0.18627556D 00	2.3	-0.13596186D 00	2.4	-0.73821094D-01	2.5	-0.92611726D-02
2.6	0.50473789D-01	2.7	0.10042495D 00	2.8	0.13772903D 00	2.9	0.16127319D 00
3.0	0.17129699D 00	3.1	0.16900359D 00	3.2	0.15621436D 00	3.3	0.13508195D 00
3.4	0.10786527D 00	3.5	0.76762809D-01	3.6	0.43797064D-01	3.7	0.10741704D-01
3.8	-0.20917321D-01	3.9	-0.49994693D-01	4.0	-0.75599653D-01	4.1	-0.97116826D-01
4.2	-0.11417868D 00	4.3	-0.12663334D 00	4.4	-0.13451055D 00	4.5	-0.13798783D 00
4.6	-0.13735830D 00	4.7	-0.13300104D 00	4.8	-0.12535462D 00	4.9	-0.11489394D 00
5.0	-0.10211057D 00	5.1	-0.87496415D-01	5.2	-0.71530434D-01	5.3	-0.54668304D-01
5.4	-0.37334589D-01	5.5	-0.19917162D-01	5.6	-0.27635675D-02	5.7	0.13820997D-01
5.8	0.29574339D-01	5.9	0.44276827D-01	6.0	0.57750081D-01	6.1	0.69854983D-01
6.2	0.80489197D-01	6.3	0.89584336D-01	6.4	0.97102913D-01	6.5	0.10303517D 00
6.6	0.10739588D 00	6.7	0.11022120D 00	6.8	0.11156557D 00	6.9	0.11149884D 00
7.0	0.11010344D 00	7.1	0.10747187D 00	7.2	0.10370428D 00	7.3	0.98906287D-01
7.4	0.93187091D-01	7.5	0.86657660D-01	7.6	0.79429213D-01	7.7	0.71611867D-01
7.8	0.63313475D-01	7.9	0.54638634D-01	8.0	0.45687857D-01	8.1	0.36556902D-01
8.2	0.27336223D-01	8.3	0.18110564D-01	8.4	0.89586501D-02	8.5	-0.47009982D-04
8.6	-0.88402404D-02	8.7	-0.17361230D-01	8.8	-0.25556501D-01	8.9	-0.33378813D-01
9.0	-0.40787023D-01	9.1	-0.47745898D-01	9.2	-0.54225897D-01	9.3	-0.60202924D-01
9.4	-0.65658056D-01	9.5	-0.70577257D-01	9.6	-0.74951081D-01	9.7	-0.78774362D-01
9.8	-0.82045910D-01	9.9	-0.84768195D-01	10.0	-0.86947040D-01		

P= 7.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.35268848D 00	1.2	-0.25361076D 00	1.3	0.70945399D-01
1.4	0.25595860D 00	1.5	0.26666441D 00	1.6	0.16739270D 00	1.7	0.30288829D-01
1.8	-0.93611273D-01	1.9	-0.17773058D 00	2.0	-0.21452629D 00	2.1	-0.20846372D 00
2.2	-0.17005962D 00	2.3	-0.11166989D 00	2.4	-0.44897675D-01	2.5	0.20752533D-01
2.6	0.78367112D-01	2.7	0.12358688D 00	2.8	0.15428214D 00	2.9	0.17010239D 00
3.0	0.17200336D 00	3.1	0.16181031D 00	3.2	0.14184680D 00	3.3	0.11463869D 00
3.4	0.82692173D-01	3.5	0.48338011D-01	3.6	0.13631643D-01	3.7	-0.19701852D-01
3.8	-0.50288439D-01	3.9	-0.77100669D-01	4.0	-0.99435885D-01	4.1	-0.11688336D 00
4.2	-0.12928532D 00	4.3	-0.13669551D 00	4.4	-0.13933803D 00	4.5	-0.13756833D 00
4.6	-0.13183742D 00	4.7	-0.12266015D 00	4.8	-0.11058780D 00	4.9	-0.96184928D-01
5.0	-0.80010453D-01	5.1	-0.62602609D-01	5.2	-0.44467397D-01	5.3	-0.26070157D-01
5.4	-0.78298487D-02	5.5	0.98843852D-02	5.6	0.26754737D-01	5.7	0.42514695D-01
5.8	0.56947767D-01	5.9	0.69885240D-01	6.0	0.81203233D-01	6.1	0.90819248D-01
6.2	0.98688394D-01	6.3	0.10479943D 00	6.4	0.10917079D 00	6.5	0.11184655D 00
6.6	0.11289268D 00	6.7	0.11239328D 00	6.8	0.11044719D 00	6.9	0.10716474D 00
7.0	0.10266484D 00	7.1	0.97072275D-01	7.2	0.90515346D-01	7.3	0.83123719D-01
7.4	0.75026579D-01	7.5	0.66351021D-01	7.6	0.57220680D-01	7.7	0.47754587D-01
7.8	0.38066229D-01	7.9	0.28262798D-01	8.0	0.18444611D-01	8.1	0.87046883D-02
8.2	-0.87153304D-03	8.3	-0.10206365D-01	8.4	-0.19229922D-01	8.5	-0.27880087D-01
8.6	-0.36102396D-01	8.7	-0.43849849D-01	8.8	-0.51082676D-01	8.9	-0.57768047D-01
9.0	-0.63879750D-01	9.1	-0.69397837D-01	9.2	-0.74308254D-01	9.3	-0.78602451D-01
9.4	-0.82276985D-01	9.5	-0.85333121D-01	9.6	-0.87776430D-01	9.7	-0.89616398D-01
9.8	-0.90866031D-01	9.9	-0.91541482D-01	10.0	-0.91661685D-01		

P= 7.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.36068085D 00	1.2	-0.23607840D 00	1.3	0.93585232D-01
1.4	0.26548142D 00	1.5	0.25788509D 00	1.6	0.14484447D 00	1.7	0.25735235D-02
1.8	-0.11806752D 00	1.9	-0.19292414D 00	2.0	-0.21755550D 00	2.1	-0.19925164D 00
2.2	-0.15062483D 00	2.3	-0.85283688D-01	2.4	-0.15320514D-01	2.5	0.49867320D-01
2.6	0.10388019D 00	2.7	0.14310538D 00	2.8	0.16624973D 00	2.9	0.17378378D 00
3.0	0.16739807D 00	3.1	0.14952527D 00	3.2	0.12295084D 00	3.3	0.90515880D-01
3.4	0.54905384D-01	3.5	0.18509663D-01	3.6	-0.16654830D-01	3.7	-0.48978866D-01
3.8	-0.77260164D-01	3.9	-0.10067983D 00	4.0	-0.11876392D 00	4.1	-0.13133665D 00
4.2	-0.13847028D 00	4.3	-0.14043512D 00	4.4	-0.13765220D 00	4.5	-0.13065011D 00
4.6	-0.12002690D 00	4.7	-0.10641743D 00	4.8	-0.90466093D-01	4.9	-0.72804879D-01
5.0	-0.54036122D-01	5.1	-0.34719644D-01	5.2	-0.15363665D-01	5.3	0.35810342D-02
5.4	0.21724218D-01	5.5	0.38737069D-01	5.6	0.54351271D-01	5.7	0.68356686D-01
5.8	0.80598003D-01	5.9	0.90970625D-01	6.0	0.99416079D-01	6.1	0.10591713D 00
6.2	0.11049279D 00	6.3	0.11319338D 00	6.4	0.11409567D 00	6.5	0.11329830D 00
6.6	0.11091744D 00	6.7	0.10708279D 00	6.8	0.10193389D 00	6.9	0.95616827D-01
7.0	0.88281281D-01	7.1	0.80077910D-01	7.2	0.71156105D-01	7.3	0.61662054D-01
7.4	0.51737117D-01	7.5	0.41516483D-01	7.6	0.31128094D-01	7.7	0.20691801D-01
7.8	0.10318747D-01	7.9	0.11092600D-03	8.0	-0.98390707D-02	8.1	-0.19448173D-01
8.2	-0.28642810D-01	8.3	-0.37358790D-01	8.4	-0.45541070D-01	8.5	-0.53143467D-01
8.6	-0.60128288D-01	8.7	-0.66465924D-01	8.8	-0.72134395D-01	8.9	-0.77118874D-01
9.0	-0.81411186D-01	9.1	-0.85009301D-01	9.2	-0.87916817D-01	9.3	-0.90142450D-01
9.4	-0.91699531D-01	9.5	-0.92605505D-01	9.6	-0.92881459D-01	9.7	-0.92551654D-01
9.8	-0.91643087D-01	9.9	-0.90185068D-01	10.0	-0.88208820D-01		

P= 7.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.36786386D 00	1.2	-0.21786340D 00	1.3	0.11539966D 00
1.4	0.27289683D 00	1.5	0.24684473D 00	1.6	0.12100981D 00	1.7	-0.24815561D-01
1.8	-0.14053795D 00	1.9	-0.20489172D 00	2.0	-0.21679729D 00	2.1	-0.18642870D 00
2.2	-0.12840204D 00	2.3	-0.57396167D-01	2.4	0.14226572D-01	2.5	0.77390346D-01
2.6	0.12639244D 00	2.7	0.15850379D 00	2.8	0.17335477D 00	2.9	0.17227622D 00
3.0	0.15769160D 00	3.1	0.13260656D 00	3.2	0.10021069D 00	3.3	0.63588074D-01
3.4	0.25523382D-01	3.5	-0.11613111D-01	3.6	-0.45919119D-01	3.7	-0.75968499D-01
3.8	-0.10078775D 00	3.9	-0.11981213D 00	4.0	-0.13283020D 00	4.1	-0.13992342D 00
4.2	-0.14140569D 00	4.3	-0.13776590D 00	4.4	-0.12961575D 00	4.5	-0.11764385D 00
4.6	-0.10257673D 00	4.7	-0.85146559D-01	4.8	-0.66065466D-01	4.9	-0.46005743D-01
5.0	-0.25585409D-01	5.1	-0.53583786D-02	5.2	0.14191456D-01	5.3	0.32652891D-01
5.4	0.49687458D-01	5.5	0.65027408D-01	5.6	0.78472136D-01	5.7	0.89883462D-01
5.8	0.99180155D-01	5.9	0.10633197D 00	6.0	0.11135352D 00	6.1	0.11429802D 00
6.2	0.11525133D 00	6.3	0.11432610D 00	6.4	0.11165639D 00	6.5	0.10739255D 00
6.6	0.10169671D 00	6.7	0.94738570D-01	6.8	0.86691776D-01	6.9	0.77730676D-01
7.0	0.68027557D-01	7.1	0.57750294D-01	7.2	0.47060389D-01	7.3	0.36111386D-01
7.4	0.25047606D-01	7.5	0.14003198D-01	7.6	0.31014572D-02	7.7	-0.75456201D-02
7.8	-0.17837566D-01	7.9	-0.27685487D-01	8.0	-0.37011959D-01	8.1	-0.45750782D-01
8.2	-0.53846632D-01	8.3	-0.61254617D-01	8.4	-0.67939770D-01	8.5	-0.73876476D-01
8.6	-0.79047871D-01	8.7	-0.83445205D-01	8.8	-0.87067195D-01	8.9	-0.89919368D-01
9.0	-0.92013407D-01	9.1	-0.93366509D-01	9.2	-0.94000756D-01	9.3	-0.93942504D-01
9.4	-0.93221801D-01	9.5	-0.91871829D-01	9.6	-0.89928373D-01	9.7	-0.87429327D-01
9.8	-0.84414227D-01	9.9	-0.80923817D-01	10.0	-0.76999652D-01		

P= 7.90

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.37423509D 00	1.2	-0.19904171D 00	1.3	0.13627882D 00
1.4	0.27817779D 00	1.5	0.23367230D 00	1.6	0.96161023D-01	1.7	-0.51543751D-01
1.8	-0.16073416D 00	1.9	-0.21348894D 00	2.0	-0.21231340D 00	2.1	-0.17027934D 00
2.2	-0.10387192D 00	2.3	-0.28624567D-01	2.4	0.43069209D-01	2.5	0.10267591D 00
2.6	0.14536741D 00	2.7	0.16942153D 00	2.8	0.17545985D 00	2.9	0.16569090D 00
3.0	0.14324731D 00	3.1	0.11165333D 00	3.2	0.74428299D-01	3.3	0.34814210D-01
3.4	-0.43922149D-02	3.5	-0.40924746D-01	3.6	-0.73071069D-01	3.7	-0.99652734D-01
3.8	-0.11997676D 00	3.9	-0.13377084D 00	4.0	-0.14111115D 00	4.1	-0.14234925D 00
4.2	-0.13804251D 00	4.3	-0.12889081D 00	4.4	-0.11568115D 00	4.5	-0.99240766D-01
4.6	-0.80398752D-01	4.7	-0.59955796D-01	4.8	-0.38661281D-01	4.9	-0.17196942D-01
5.0	0.38333841 3D-02	5.1	0.23912025D-01	5.2	0.42605004D-01	5.3	0.59565632D-01
5.4	0.74528878D-01	5.5	0.87306715D-01	5.6	0.97781787D-01	5.7	0.10590030D 00
5.8	0.11166447D 00	5.9	0.11512490D 00	6.0	0.11637298D 00	6.1	0.11553366D 00
6.2	0.11275857D 00	6.3	0.10821965D 00	6.4	0.10210339D 00	6.5	0.94605569D-01
6.6	0.85926675D-01	6.7	0.76267876D-01	6.8	0.65827572D-01	6.9	0.54798494D-01
7.0	0.43365310D-01	7.1	0.31702704D-01	7.2	0.19973879D-01	7.3	0.83294560D-02
7.4	-0.30932955D-02	7.5	-0.14170913D-01	7.6	-0.24793941D-01	7.7	-0.34866896D-01
7.8	-0.44308050D-01	7.9	-0.53049048D-01	8.0	-0.61034393D-01	8.1	-0.68220836D-01
8.2	-0.74576675D-01	8.3	-0.80081002D-01	8.4	-0.84722905D-01	8.5	-0.88500651D-01
8.6	-0.91420851D-01	8.7	-0.93497630D-01	8.8	-0.94751810D-01	8.9	-0.95210108D-01
9.0	-0.94904364D-01	9.1	-0.93870797D-01	9.2	-0.92149304D-01	9.3	-0.89782789D-01
9.4	-0.86816539D-01	9.5	-0.83297645D-01	9.6	-0.79274456D-01	9.7	-0.74796093D-01
9.8	-0.69911989D-01	9.9	-0.64671484D-01	10.0	-0.59123455D-01		

P= 8.00

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.37979342D 00	1.2	-0.17969052D 00	1.3	0.15611919D 00
1.4	0.28131285D 00	1.5	0.21851410D 00	1.6	0.70577751D-01	1.7	-0.77288625D-01
1.8	-0.17840272D 00	1.9	-0.21862505D 00	2.0	-0.20422773D 00	2.1	-0.15114559D 00
2.2	-0.77554853D-01	2.3	0.40285899D-03	2.4	0.70557526D-01	2.5	0.12514028D 00
2.6	0.16036522D 00	2.7	0.17562225D 00	2.8	0.17256926D 00	2.9	0.15428615D 00
3.0	0.12456842D 00	3.1	0.87385134D-01	3.2	0.46495594D-01	3.3	0.52040192D-02
3.4	-0.33774456D-01	3.5	-0.68363266D-01	3.6	-0.97113686D-01	3.7	-0.11915425D 00
3.8	-0.13411653D 00	3.9	-0.14204962D 00	4.0	-0.14333214D 00	4.1	-0.13858792D 00
4.2	-0.12860917D 00	4.3	-0.11428929D 00	4.4	-0.96566265D-01	4.5	-0.76376698D-01
4.6	-0.54619863D-01	4.7	-0.32130952D-01	4.8	-0.96623864D-02	4.9	0.12127976D-01
5.0	0.32682898D-01	5.1	0.51547177D-01	5.2	0.68365433D-01	5.3	0.82877142D-01
5.4	0.94909703D-01	5.5	0.10437018D 00	5.6	0.11123625D 00	5.7	0.11554683D 00
5.8	0.11739268D 00	5.9	0.11690728D 00	6.0	0.11425818D 00	6.1	0.10963893D 00
6.2	0.10326176D 00	6.3	0.95350916D-01	6.4	0.86136830D-01	6.5	0.75851050D-01
6.6	0.64721884D-01	6.7	0.52970765D-01	6.8	0.40809274D-01	6.9	0.28436767D-01
7.0	0.16038561D-01	7.1	0.37846126D-02	7.2	-0.81713650D-02	7.3	-0.19692397D-01
7.4	-0.30658413D-01	7.5	-0.40966128D-01	7.6	-0.50528685D-01	7.7	-0.59275110D-01
7.8	-0.67149600D-01	7.9	-0.74110699D-01	8.0	-0.80130372D-01	8.1	-0.85193024D-01
8.2	-0.89294474D-01	8.3	-0.92440904D-01	8.4	-0.94647810D-01	8.5	-0.95938962D-01
8.6	-0.96345388D-01	8.7	-0.95904386D-01	8.8	-0.94658585D-01	8.9	-0.92655046D-01
9.0	-0.89944415D-01	9.1	-0.86580137D-01	9.2	-0.82617717D-01	9.3	-0.78114047D-01
9.4	-0.73126784D-01	9.5	-0.67713790D-01	9.6	-0.61932626D-01	9.7	-0.55840100D-01
9.8	-0.49491868D-01	9.9	-0.42942086D-01	10.0	-0.36243105D-01		

P= 8.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.38453908D 00	1.2	-0.15988797D 00	1.3	0.17482404D 00
1.4	0.28230603D 00	1.5	0.20153228D 00	1.6	0.44543950D-01	1.7	-0.10174376D 00
1.8	-0.19332831D 00	1.9	-0.22026365D 00	2.0	-0.19272358D 00	2.1	-0.12942016D 00
2.2	-0.50000240D-01	2.3	0.29060252D-01	2.4	0.96080493D-01	2.5	0.14427488D 00
2.6	0.17105248D 00	2.7	0.17699852D 00	2.8	0.16482715D 00	2.9	0.13845814D 00
3.0	0.10228110D 00	3.1	0.60617709D-01	3.2	0.17364155D-01	3.3	-0.24216901D-01
3.4	-0.61588597D-01	3.5	-0.92948257D-01	3.6	-0.11717925D 00	3.7	-0.13376844D 00
3.8	-0.14270605D 00	3.9	-0.14438041D 00	4.0	-0.13947583D 00	4.1	-0.12887917D 00
4.2	-0.11359809D 00	4.3	-0.94692311D-01	4.4	-0.73218108D-01	4.5	-0.50185259D-01
4.6	-0.26525428D-01	4.7	-0.30705072D-02	4.8	0.19460570D-01	4.9	0.40468031D-01
5.0	0.59471879D-01	5.1	0.76108204D-01	5.2	0.90122296D-01	5.3	0.10135953D 00
5.4	0.10975483D 00	5.5	0.11532133D 00	5.6	0.11813888D 00	5.7	0.11834257D 00
5.8	0.11611191D 00	5.9	0.11166052D 00	6.0	0.10522678D 00	6.1	0.97065389D-01
6.2	0.87439857D-01	6.3	0.76616010D-01	6.4	0.64856443D-01	6.5	0.52415889D-01
6.6	0.39537441D-01	6.7	0.26449562D-01	6.8	0.13363822D-01	6.9	0.47326233D-03
7.0	-0.12048664D-01	7.1	-0.24048662D-01	7.2	-0.35393739D-01	7.3	-0.45970995D-01
7.4	-0.55687114D-01	7.5	-0.64467613D-01	7.6	-0.72255905D-01	7.7	-0.79012206D-01
7.8	-0.84712349D-01	7.9	-0.89346515D-01	8.0	-0.92917929D-01	8.1	-0.95441539D-01
8.2	-0.96942695D-01	8.3	-0.97455864D-01	8.4	-0.97023370D-01	8.5	-0.95694191D-01
8.6	-0.93522819D-01	8.7	-0.90568173D-01	8.8	-0.86892602D-01	8.9	-0.82560941D-01
9.0	-0.77639661D-01	9.1	-0.72196081D-01	9.2	-0.66297666D-01	9.3	-0.60011390D-01
9.4	-0.53403175D-01	9.5	-0.46537403D-01	9.6	-0.39476479D-01	9.7	-0.32280476D-01
9.8	-0.25006819D-01	9.9	-0.17710038D-01	10.0	-0.10441562D-01		

P= 8.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.38847359D 00	1.2	-0.13971292D 00	1.3	0.19230390D 00
1.4	0.28117664D 00	1.5	0.18290323D 00	1.6	0.18344893D-01	1.7	-0.12462223D 00
1.8	-0.20533616D 00	1.9	-0.21842265D 00	2.0	-0.17803981D 00	2.1	-0.10553862D 00
2.2	-0.21775293D-01	2.3	0.56737350D-01	2.4	0.11907922D 00	2.5	0.15965752D 00
2.6	0.17720945D 00	2.7	0.17357317D 00	2.8	0.15251173D 00	2.9	0.11872761D 00
3.0	0.77113869D-01	3.1	0.32236028D-01	3.2	-0.11986734D-01	3.3	-0.52442119D-01
3.4	-0.86868059D-01	3.5	-0.11381542D 00	3.6	-0.13256020D 00	3.7	-0.14298844D 00
3.8	-0.14547123D 00	3.9	-0.14074139D 00	4.0	-0.12977972D 00	4.1	-0.11371515D 00
4.2	-0.93740826D-01	4.3	-0.71046531D-01	4.4	-0.46766649D-01	4.5	-0.21942312D-01
4.6	0.25040943D-02	4.7	0.25783975D-01	4.8	0.47249017D-01	4.9	0.66390924D-01
5.0	0.82835998D-01	5.1	0.96335984D-01	5.2	0.10675637D 00	5.3	0.11406316D 00
5.4	0.11830888D 00	5.5	0.11961849D 00	5.6	0.11817570D 00	5.7	0.11420997D 00
5.8	0.10798450D 00	5.9	0.99785375D-01	6.0	0.89911907D-01	6.1	0.78668217D-01
6.2	0.66356065D-01	6.3	0.53268848D-01	6.4	0.39686729D-01	6.5	0.25872778D-01
6.6	0.12070061D-01	6.7	-0.15004449D-02	6.8	-0.14641193D-01	6.9	-0.27178772D-01
7.0	-0.38964070D-01	7.1	-0.49871971D-01	7.2	-0.59800679D-01	7.3	-0.68670744D-01
7.4	-0.76423837D-01	7.5	-0.83021359D-01	7.6	-0.88442919D-01	7.7	-0.92684731D-01
7.8	-0.95757968D-01	7.9	-0.97687107D-01	8.0	-0.98508294D-01	8.1	-0.98267744D-01
8.2	-0.97020201D-01	8.3	-0.94827476D-01	8.4	-0.91757058D-01	8.5	-0.87880823D-01
8.6	-0.83273836D-01	8.7	-0.78013251D-01	8.8	-0.72177311D-01	8.9	-0.65844445D-01
9.0	-0.59092468D-01	9.1	-0.51997866D-01	9.2	-0.44635180D-01	9.3	-0.37076471D-01
9.4	-0.29390865D-01	9.5	-0.21644180D-01	9.6	-0.13898619D-01	9.7	-0.62125308D-02
9.8	0.13597708D-02	9.9	0.87681323D-02	10.0	0.15966636D-01		

P= 8.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.39159978D 00	1.2	-0.11924465D 00	1.3	0.20847691D 00
1.4	0.27795894D 00	1.5	0.16281592D 00	1.6	-0.77358208D-02	1.7	-0.14565986D 00
1.8	-0.21429398D 00	1.9	-0.21317331D 00	2.0	-0.16046621D 00	2.1	-0.79970966D-01
2.2	0.65463488D-02	2.3	0.82852210D-01	2.4	0.13905892D 00	2.5	0.17096150D 00
2.6	0.17873419D 00	2.7	0.16549707D 00	2.8	0.13602599D 00	2.9	0.95723270D-01
3.0	0.49874298D-01	3.1	0.31654986D-02	3.2	-0.40582535D-01	3.3	-0.78518339D-01
3.4	-0.10874757D 00	3.5	-0.13024656D 00	3.6	-0.14273361D 00	3.7	-0.14652224D 00
3.8	-0.14237296D 00	3.9	-0.13135483D 00	4.0	-0.11472312D 00	4.1	-0.93816299D-01
4.2	-0.69973149D-01	4.3	-0.44469187D-01	4.4	-0.18470756D-01	4.5	0.69955216D-02
4.6	0.31060323D-01	4.7	0.53018691D-01	4.8	0.72328843D-01	4.9	0.88604777D-01
5.0	0.10160445D 00	5.1	0.11121507D 00	5.2	0.11743663D 00	5.3	0.12036482D 00
5.4	0.12017401D 00	5.5	0.11710078D 00	5.6	0.11142864D 00	5.7	0.10347393D 00
5.8	0.93573368D-01	5.9	0.82073003D-01	6.0	0.69318856D-01	6.1	0.55649035D-01
6.2	0.41387319D-01	6.3	0.26838082D-01	6.4	0.12282445D-01	6.5	-0.20244796D-02
6.6	-0.15855372D-01	6.7	-0.29011655D-01	6.8	-0.41323732D-01	6.9	-0.52650629D-01
7.0	-0.62879143D-01	7.1	-0.71922614D-01	7.2	-0.79719376D-01	7.3	-0.86231004D-01
7.4	-0.91440395D-01	7.5	-0.95349754D-01	7.6	-0.97978540D-01	7.7	-0.99361407D-01
7.8	-0.99546170D-01	7.9	-0.98591848D-01	8.0	-0.96566769D-01	8.1	-0.93546793D-01
8.2	-0.89613641D-01	8.3	-0.84853338D-01	8.4	-0.79354798D-01	8.5	-0.73208523D-01
8.6	-0.66505442D-01	8.7	-0.59335872D-01	8.8	-0.51788604D-01	8.9	-0.43950108D-01
9.0	-0.35903854D-01	9.1	-0.27729734D-01	9.2	-0.19503591D-01	9.3	-0.11296836D-01
9.4	-0.31761556D-02	9.5	0.47967065D-02	9.6	0.12565095D-01	9.7	0.20077525D-01
9.8	0.27287704D-01	9.9	0.34154508D-01	10.0	0.40641910D-01		

P= 8.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.39392177D 00	1.2	-0.98562561D-01	1.3	0.22326922D 00
1.4	0.27270182D 00	1.5	0.14147008D 00	1.6	-0.33419217D-01	1.7	-0.16461818D 00
1.8	-0.22011329D 00	1.9	-0.20463856D 00	2.0	-0.14033804D 00	2.1	-0.53212614D-01
2.2	0.34396023D-01	2.3	0.10686319D 00	2.4	0.15559936D 00	2.5	0.17796226D 00
2.6	0.17564373D 00	2.7	0.15304365D 00	2.8	0.11588495D 00	2.9	0.70162121D-01
3.0	0.21423481D-01	3.1	-0.25657770D-01	3.2	-0.67485385D-01	3.3	-0.10157718D 00
3.4	-0.12649239D 00	3.5	-0.14169384D 00	3.6	-0.14737862D 00	3.7	-0.14430134D 00
3.8	-0.13360585D 00	3.9	-0.11667514D 00	4.0	-0.95004234D-01	4.1	-0.70097762D-01
4.2	-0.43391507D-01	4.3	-0.16195931D-01	4.4	0.10341090D-01	4.5	0.35255398D-01
4.6	0.57774648D-01	4.7	0.77316242D-01	4.8	0.93477729D-01	4.9	0.10602191D 00
5.0	0.11485851D 00	5.1	0.12002409D 00	5.2	0.12166113D 00	5.3	0.11999764D 00
5.4	0.11532754D 00	5.5	0.10799257D 00	5.6	0.98366007D-01	5.7	0.86838203D-01
5.8	0.73804177D-01	5.9	0.59653148D-01	6.0	0.44759948D-01	6.1	0.29478208D-01
6.2	0.14135150D-01	6.3	-0.97217282D-03	6.4	-0.15579356D-01	6.5	-0.29456013D-01
6.6	-0.42406209D-01	6.7	-0.54268105D-01	6.8	-0.64912953D-01	6.9	-0.74243594D-01
7.0	-0.82192555D-01	7.1	-0.88719854D-01	7.2	-0.93810623D-01	7.3	-0.97472597D-01
7.4	-0.99733560D-01	7.5	-0.10063879D 00	7.6	-0.10024857D 00	7.7	-0.98635733D-01
7.8	-0.95883405D-01	7.9	-0.92082799D-01	8.0	-0.87331211D-01	8.1	-0.81730155D-01
8.2	-0.75383672D-01	8.3	-0.68396802D-01	8.4	-0.60874227D-01	8.5	-0.52919071D-01
8.6	-0.44631862D-01	8.7	-0.36109643D-01	8.8	-0.27445224D-01	8.9	-0.18726568D-01
9.0	-0.10036297D-01	9.1	-0.14513156D-02	9.2	0.69574641D-02	9.3	0.15125304D-01
9.4	0.22993739D-01	9.5	0.30510595D-01	9.6	0.37629954D-01	9.7	0.44312045D-01
9.8	0.50523102D-01	9.9	0.56235169D-01	10.0	0.61425875D-01		

P= 8.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.39544498D 00	1.2	-0.77745966D-01	1.3	0.23661528D 00
1.4	0.26546820D 00	1.5	0.11907430D 00	1.6	-0.58433808D-01	1.7	-0.18128697D 00
1.8	-0.22275015D 00	1.9	-0.19299052D 00	2.0	-0.11802985D 00	2.1	-0.25775056D-01
2.2	0.61221212D-01	2.3	0.12827993D 00	2.4	0.16836342D 00	2.5	0.18054156D 00
2.6	0.16807230D 00	2.7	0.13660013D 00	2.8	0.92699946D-01	2.9	0.42827546D-01
3.0	-0.73507617D-02	3.1	-0.53316704D-01	3.2	-0.91824445D-01	3.3	-0.12086369D 00
3.4	-0.13952257D 00	3.5	-0.14779774D 00	3.6	-0.14638600D 00	3.7	-0.13648078D 00
3.8	-0.11958778D 00	3.9	-0.97367624D-01	4.0	-0.71508402D-01	4.1	-0.43627864D-01
4.2	-0.15203054D-01	4.3	0.12476025D-01	4.4	0.38332557D-01	4.5	0.61512775D-01
4.6	0.81383228D-01	4.7	0.97518775D-01	4.8	0.10968416D 00	4.9	0.11781153D 00
5.0	0.12197593D 00	5.1	0.12237011D 00	5.2	0.11927997D 00	5.3	0.11306134D 00
5.4	0.10411872D 00	5.5	0.92886260D-01	5.6	0.79811270D-01	5.7	0.65340124D-01
5.8	0.49906661D-01	5.9	0.33922862D-01	6.0	0.17771669D-01	6.1	0.18017109D-02
6.2	-0.13676253D-01	6.3	-0.28391420D-01	6.4	-0.42113763D-01	6.5	-0.54653803D-01
6.6	-0.65861512D-01	6.7	-0.75624548D-01	6.8	-0.83865951D-01	6.9	-0.90541472D-01
7.0	-0.95636628D-01	7.1	-0.99163612D-01	7.2	-0.10115813D 00	7.3	-0.10167625D 00
7.4	-0.10079133D 00	7.5	-0.98590996D-01	7.6	-0.95174389D-01	7.7	-0.90649476D-01
7.8	-0.85130617D-01	7.9	-0.78736327D-01	8.0	-0.71587255D-01	8.1	-0.63804377D-01
8.2	-0.55507400D-01	8.3	-0.46813382D-01	8.4	-0.37835539D-01	8.5	-0.28682251D-01
8.6	-0.19456239D-01	8.7	-0.10253906D-01	8.8	-0.11648360D-02	8.9	0.77285813D-02
9.0	0.16351389D-01	9.1	0.24636197D-01	9.2	0.32523217D-01	9.3	0.39960207D-01
9.4	0.46902344D-01	9.5	0.53312033D-01	9.6	0.59158656D-01	9.7	0.64418286D-01
9.8	0.69073354D-01	9.9	0.73112298D-01	10.0	0.76529177D-01		

P= 8.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.39617607D 00	1.2	-0.56873761D-01	1.3	0.24845806D 00
1.4	0.25633452D 00	1.5	0.95844167D-01	1.6	-0.82518356D-01	1.7	-0.19548653D 00
1.8	-0.22220522D 00	1.9	-0.17844733D 00	2.0	-0.93948782D-01	2.1	0.18236892D-02
2.2	0.86496275D-01	2.3	0.14667320D 00	2.4	0.17710378D 00	2.5	0.17868910D 00
2.6	0.15626649D 00	2.7	0.11665593D 00	2.8	0.67160496D-01	2.9	0.14545671D-01
3.0	-0.35561199D-01	3.1	-0.78942054D-01	3.2	-0.11282372D 00	3.3	-0.13576072D 00
3.4	-0.14743158D 00	3.5	-0.14839784D 00	3.6	-0.13985977D 00	3.7	-0.12343049D 00
3.8	-0.10094064D 00	3.9	-0.74278865D-01	4.0	-0.45268868D-01	4.1	-0.15580552D-01
4.2	0.13328615D-01	4.3	0.40246856D-01	4.4	0.64220885D-01	4.5	0.84552933D-01
4.6	0.10078624D 00	4.7	0.11268263D 00	4.8	0.12019526D 00	4.9	0.12343887D 00
5.0	0.12265958D 00	5.1	0.11820547D 00	5.2	0.11049909D 00	5.3	0.10001247D 00
5.4	0.87245051D-01	5.5	0.72704707D-01	5.6	0.56891882D-01	5.7	0.40286696D-01
5.8	0.23338870D-01	5.9	0.64601928D-02	6.0	-0.99807131D-02	6.1	-0.25661647D-01
6.2	-0.40308373D-01	6.3	-0.53694734D-01	6.4	-0.65641600D-01	6.5	-0.76014886D-01
6.6	-0.84722857D-01	6.7	-0.91712904D-01	6.8	-0.96967971D-01	6.9	-0.10050276D 00
7.0	-0.10235985D 00	7.1	-0.10260580D 00	7.2	-0.10132737D 00	7.3	-0.98627821D-01
7.4	-0.94623523D-01	7.5	-0.89440675D-01	7.6	-0.83212350D-01	7.7	-0.76075790D-01
7.8	-0.68169978D-01	7.9	-0.59633490D-01	8.0	-0.50602612D-01	8.1	-0.41209728D-01
8.2	-0.31581957D-01	8.3	-0.21840022D-01	8.4	-0.12097345D-01	8.5	-0.24593426D-02
8.6	0.69770941D-02	8.7	0.16123948D-01	8.8	0.24902277D-01	8.9	0.33242287D-01
9.0	0.41083274D-01	9.1	0.48373486D-01	9.2	0.55069882D-01	9.3	0.61137823D-01
9.4	0.66550715D-01	9.5	0.71289586D-01	9.6	0.75342640D-01	9.7	0.78704778D-01
9.8	0.81377103D-01	9.9	0.83366408D-01	10.0	0.84684666D-01		

P= 8.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.39612297D 00	1.2	-0.36024187D-01	1.3	0.25874934D 00
1.4	0.24539002D 00	1.5	0.72000240D-01	1.6	-0.10542450D 00	1.7	-0.20706954D 00
1.8	-0.21852328D 00	1.9	-0.16126925D 00	2.0	-0.68527299D-01	2.1	0.29068637D-01
2.2	0.10973254D 00	2.3	0.16168324D 00	2.4	0.18166747D 00	2.5	0.17250152D 00
2.6	0.14057780D 00	2.7	0.93788452D-01	2.8	0.40014127D-01	2.9	-0.13839226D-01
3.0	-0.62348176D-01	3.1	-0.10173935D 00	3.2	-0.12982644D 00	3.3	-0.14580832D 00
3.4	-0.14998861D 00	3.5	-0.14353641D 00	3.6	-0.12811085D 00	3.7	-0.10571857D 00
3.8	-0.78463341D-01	3.9	-0.48400152D-01	4.0	-0.17421892D-01	4.1	0.12816130D-01
4.2	0.40939562D-01	4.3	0.65872513D-01	4.4	0.86835130D-01	4.5	0.10332665D 00
4.6	0.11509859D 00	4.7	0.12212191D 00	4.8	0.12455128D 00	4.9	0.12268881D 00
5.0	0.11694901D 00	5.1	0.10782620D 00	5.2	0.95865148D-01	5.3	0.81635447D-01
5.4	0.65709695D-01	5.5	0.48645562D-01	5.6	0.30971491D-01	5.7	0.13175814D-01
5.8	-0.43010847D-02	5.9	-0.21071894D-01	6.0	-0.36805182D-01	6.1	-0.51226240D-01
6.2	-0.64116316D-01	6.3	-0.75310562D-01	6.4	-0.84694979D-01	6.5	-0.92202638D-01
6.6	-0.97809384D-01	6.7	-0.10152923D 00	6.8	-0.10340959D 00	6.9	-0.10352654D 00
7.0	-0.10198010D 00	7.1	-0.98889747D-01	7.2	-0.94390199D-01	7.3	-0.88627418D-01
7.4	-0.81754994D-01	7.5	-0.73930861D-01	7.6	-0.65314357D-01	7.7	-0.56063654D-01
7.8	-0.46333514D-01	7.9	-0.36273398D-01	8.0	-0.26025878D-01	8.1	-0.15725353D-01
8.2	-0.54970349D-02	8.3	0.45438075D-02	8.4	0.14292383D-01	8.5	0.23654701D-01
8.6	0.32547724D-01	8.7	0.40899346D-01	8.8	0.48648249D-01	8.9	0.55743631D-01
9.0	0.62144844D-01	9.1	0.67820946D-01	9.2	0.72750190D-01	9.3	0.76919465D-01
9.4	0.80323697D-01	9.5	0.82965226D-01	9.6	0.84853173D-01	9.7	0.86002791D-01
9.8	0.86434830D-01	9.9	0.86174901D-01	10.0	0.85252865D-01		

P= 8.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.39529483D 00	1.2	-0.15274557D-01	1.3	0.26744981D 00
1.4	0.23273597D 00	1.5	0.47766076D-01	1.6	-0.12691920D 00	1.7	-0.21592242D 00
1.8	-0.21179213D 00	1.9	-0.14175429D 00	2.0	-0.42215673D-01	2.1	0.55457238D-01
2.2	0.13048753D 00	2.3	0.17302664D 00	2.4	0.18199826D 00	2.5	0.16217888D 00
2.6	0.12145251D 00	2.7	0.68646709D-01	2.8	0.12044853D-01	2.9	-0.41489578D-01
3.0	-0.86905468D-01	3.1	-0.12101322D 00	3.2	-0.14231531D 00	3.3	-0.15071762D 00
3.4	-0.14719419D 00	3.5	-0.13345447D 00	3.6	-0.11164300D 00	3.7	-0.84087071D-01
3.8	-0.53098078D-01	3.9	-0.20825414D-01	4.0	0.10841956D-01	4.1	0.40333302D-01
4.2	0.66420988D-01	4.3	0.88219946D-01	4.4	0.10516864D 00	4.5	0.11699752D 00
4.6	0.12368986D 00	4.7	0.12543903D 00	4.8	0.12260512D 00	4.9	0.11567315D 00
5.0	0.10521453D 00	5.1	0.91852510D-01	5.2	0.76232321D-01	5.3	0.58996085D-01
5.4	0.40762480D-01	5.5	0.22110858D-01	5.6	0.35694758D-02	5.7	-0.14392612D-01
5.8	-0.31370476D-01	5.9	-0.47025519D-01	6.0	-0.61085445D-01	6.1	-0.73342404D-01
6.2	-0.83649740D-01	6.3	-0.91917672D-01	6.4	-0.98108241D-01	6.5	-0.10222977D 00
6.6	-0.10433110D 00	6.7	-0.10449572D 00	6.8	-0.10283606D 00	6.9	-0.99487920D-01
7.0	-0.94605253D-01	7.1	-0.88355280D-01	7.2	-0.80914039D-01	7.3	-0.72462361D-01
7.4	-0.63182300D-01	7.5	-0.53254007D-01	7.6	-0.42853049D-01	7.7	-0.32148149D-01
7.8	-0.21299323D-01	7.9	-0.10456395D-01	8.0	0.24214945D-03	8.1	0.10669990D-01
8.2	0.20713525D-01	8.3	0.30272165D-01	8.4	0.39258415D-01	8.5	0.47597761D-01
8.6	0.55228393D-01	8.7	0.62100796D-01	8.8	0.68177224D-01	8.9	0.73431085D-01
9.0	0.77846256D-01	9.1	0.81416350D-01	9.2	0.84143936D-01	9.3	0.86039753D-01
9.4	0.87121904D-01	9.5	0.87415054D-01	9.6	0.86949644D-01	9.7	0.85761122D-01
9.8	0.83889194D-01	9.9	0.81377116D-01	10.0	0.78271014D-01		

P= 8.90

X	KP(X)
1.0	0.10000000D 01
1.4	0.21848485D 00
1.8	-0.20214091D 00
2.2	0.14837326D 00
2.6	0.99419312D-01
3.0	-0.10850422D 00
3.4	-0.13917907D 00
3.8	-0.25891290D-01
4.2	0.88671672D-01
4.6	0.12621155D 00
5.0	0.88134283D-01
5.4	0.13828012D-01
5.8	-0.56278939D-01
6.2	-0.97740584D-01
6.6	-0.10393589D 00
7.0	-0.80820277D-01
7.4	-0.40297380D-01
7.8	0.50365872D-02
8.2	0.45029487D-01
8.6	0.73244917D-01
9.0	0.86957160D-01
9.4	0.86448687D-01
9.8	0.74048729D-01

X	KP(X)
1.1	-0.39370201D 00
1.5	0.23366228D-01
1.9	-0.12023315D 00
2.3	0.18050150D 00
2.7	0.41933224D-01
3.1	-0.13618826D 00
3.5	-0.11858085D 00
3.9	0.72945411D-02
4.3	0.10631674D 00
4.7	0.12254085D 00
5.1	0.71192505D-01
5.5	-0.53740533D-02
5.9	-0.69865875D-01
6.3	-0.10252531D 00
6.7	-0.10049395D 00
7.1	-0.71809645D-01
7.5	-0.28961114D-01
7.9	0.15853892D-01
8.3	0.53353581D-01
8.7	0.78065973D-01
9.1	0.88099588D-01
9.5	0.84341566D-01
9.9	0.69480799D-01

X	KP(X)
1.2	0.52990018D-02
1.6	-0.14678709D 00
2.0	-0.15474180D-01
2.4	0.17813686D 00
2.8	-0.15949145D-01
3.2	-0.14992792D 00
3.6	-0.91131779D-01
4.0	0.38328202D-01
4.4	0.11842342D 00
4.8	0.11452605D 00
5.2	0.52713132D-01
5.6	-0.23727475D-01
6.0	-0.81377681D-01
6.4	-0.10508866D 00
6.8	-0.95365734D-01
7.2	-0.61916601D-01
7.6	-0.17517308D-01
8.0	0.26194677D-01
8.4	0.60860636D-01
8.8	0.81955943D-01
9.2	0.88371295D-01
9.6	0.81536300D-01
10.0	0.64442039D-01

X	KP(X)
1.3	0.27452923D 00
1.7	-0.22196643D 00
2.1	0.80508450D-01
2.5	0.14801868D 00
2.9	-0.67598903D-01
3.3	-0.15037870D 00
3.7	-0.59421373D-01
4.1	0.65793936D-01
4.5	0.12497873D 00
4.9	0.10282118D 00
5.3	0.33380445D-01
5.7	-0.40810417D-01
6.1	-0.90689745D-01
6.5	-0.10551794D 00
6.9	-0.88741026D-01
7.3	-0.51346112D-01
7.7	-0.61334915D-02
8.1	0.35950752D-01
8.5	0.67502574D-01
8.9	0.84915970D-01
9.3	0.87807312D-01
9.7	0.78086596D-01

P= 9.00

X	KP(X)
1.0	0.10000000D 01
1.4	0.20275945D 00
1.8	-0.18973788D 00
2.2	0.16306325D 00
2.6	0.75075021D-01
3.0	-0.12651430D 00
3.4	-0.12629646D 00
3.8	0.20490387D-02
4.2	0.10674512D 00
4.6	0.12261061D 00
5.0	0.66655756D-01
5.4	-0.13575063D-01
5.8	-0.77581296D-01
6.2	-0.10557235D 00
6.6	-0.96727582D-01
7.0	-0.61641861D-01
7.4	-0.14783993D-01
7.8	0.30703119D-01
8.2	0.65599160D-01
8.6	0.85214228D-01
9.0	0.88802375D-01
9.4	0.78445268D-01
9.8	0.57853482D-01

X	KP(X)
1.1	-0.39135605D 00
1.5	-0.97575050D-03
1.9	-0.97063845D-01
2.3	0.18399083D 00
2.7	0.14384710D-01
3.1	-0.14682595D 00
3.5	-0.99514444D-01
3.9	0.34799592D-01
4.3	0.11939299D 00
4.7	0.11363978D 00
5.1	0.46991140D-01
5.5	-0.32247562D-01
5.9	-0.88261380D-01
6.3	-0.10653170D 00
6.7	-0.89867521D-01
7.1	-0.50466501D-01
7.5	-0.28461435D-02
7.9	0.40679120D-01
8.3	0.72024438D-01
8.7	0.87571326D-01
9.1	0.87394574D-01
9.5	0.74091883D-01
9.9	0.51610098D-01

X	KP(X)
1.2	0.25621798D-01
1.6	-0.16483244D 00
2.0	0.11234758D-01
2.4	0.17021893D 00
2.8	-0.43177976D-01
3.2	-0.15246690D 00
3.6	-0.67397287D-01
4.0	0.63888494D-01
4.4	0.12604441D 00
4.8	0.10078776D 00
5.2	0.26615995D-01
5.6	-0.49356609D-01
6.0	-0.96495805D-01
6.4	-0.10526270D 00
6.8	-0.81576879D-01
7.2	-0.38789846D-01
7.6	0.88139620D-02
8.0	0.49871439D-01
8.4	0.77444050D-01
8.8	0.88933395D-01
9.2	0.85159959D-01
9.6	0.69163141D-01
10.0	0.45065077D-01

X	KP(X)
1.3	0.27996649D 00
1.7	-0.22515821D 00
2.1	0.10377133D 00
2.5	0.13040768D 00
2.9	-0.91415023D-01
3.3	-0.14486237D 00
3.7	-0.32714424D-01
4.1	0.88122753D-01
4.5	0.12695784D 00
4.9	0.84854335D-01
5.3	0.62223602D-02
5.7	-0.64554373D-01
6.1	-0.10225539D 00
6.5	-0.10193037D 00
6.9	-0.72089685D-01
7.3	-0.26827954D-01
7.7	0.20042307D-01
8.1	0.58199248D-01
8.5	0.81841608D-01
8.9	0.89330765D-01
9.3	0.82156255D-01
9.7	0.63727387D-01

P= 9.10

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.38826965D 00	1.2	0.45620827D-01	1.3	0.28374964D 00
1.4	0.18569184D 00	1.5	-0.25039269D-01	1.6	-0.18088113D 00	1.7	-0.22548991D 00
1.8	-0.17478765D 00	1.9	-0.72625916D-01	2.0	0.37454393D-01	2.1	0.12483297D 00
2.2	0.17429837D 00	2.3	0.18346405D 00	2.4	0.15847093D 00	2.5	0.10981160D 00
2.6	0.49068681D-01	2.7	-0.13247944D-01	2.8	-0.68882033D-01	2.9	-0.11226145D 00
3.0	-0.14042236D 00	3.1	-0.15263703D 00	3.2	-0.14990466D 00	3.3	-0.13441571D 00
3.4	-0.10905788D 00	3.5	-0.77000559D-01	3.6	-0.41371779D-01	3.7	-0.50271781D-02
3.8	0.29598065D-01	3.9	0.60567722D-01	4.0	0.86468139D-01	4.1	0.10639121D 00
4.2	0.11988943D 00	4.3	0.12691366D 00	4.4	0.12774233D 00	4.5	0.12290834D 00
4.6	0.11312873D 00	4.7	0.99240017D-01	4.8	0.82141350D-01	4.9	0.62746317D-01
5.0	0.41943778D-01	5.1	0.20567354D-01	5.2	-0.62700019D-03	5.3	-0.20976223D-01
5.4	-0.39919097D-01	5.5	-0.56999938D-01	5.6	-0.71867992D-01	5.7	-0.84273436D-01
5.8	-0.94060832D-01	5.9	-0.10116076D 00	6.0	-0.10558025D 00	6.1	-0.10739257D 00
6.2	-0.10672682D 00	6.3	-0.10375758D 00	6.4	-0.98695056D-01	6.5	-0.91775768D-01
6.6	-0.83254018D-01	6.7	-0.73394181D-01	6.8	-0.62463889D-01	6.9	-0.50728126D-01
7.0	-0.38444207D-01	7.1	-0.25857625D-01	7.2	-0.13198710D-01	7.3	-0.68004189D-03
7.4	0.11505442D-01	7.5	0.23185725D-01	7.6	0.34210431D-01	7.7	0.44451069D-01
7.8	0.53800864D-01	7.9	0.62174243D-01	8.0	0.69506028D-01	8.1	0.75750399D-01
8.2	0.80879687D-01	8.3	0.84883017D-01	8.4	0.87764873D-01	8.5	0.89543597D-01
8.6	0.90249861D-01	8.7	0.89925153D-01	8.8	0.88620269D-01	8.9	0.86393864D-01
9.0	0.83311045D-01	9.1	0.79442052D-01	9.2	0.74860998D-01	9.3	0.69644712D-01
9.4	0.63871663D-01	9.5	0.57620976D-01	9.6	0.50971547D-01	9.7	0.44001247D-01
9.8	0.36786220D-01	9.9	0.29400267D-01	10.0	0.21914317D-01		

P= 9.20

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.38445667D 00	1.2	0.65225007D-01	1.3	0.28587583D 00
1.4	0.16742238D 00	1.5	-0.48608619D-01	1.6	-0.19478217D 00	1.7	-0.22298896D 00
1.8	-0.15752803D 00	1.9	-0.47314362D-01	2.0	0.62741295D-01	2.1	0.14332570D 00
2.2	0.18189125D 00	2.3	0.17897679D 00	2.4	0.14320408D 00	2.5	0.86763103D-01
2.6	0.22084546D-01	2.7	-0.40219865D-01	2.8	-0.92352902D-01	2.9	-0.12955631D 00
3.0	-0.14984629D 00	3.1	-0.15348905D 00	3.2	-0.14238239D 00	3.3	-0.11945178D 00
3.4	-0.88123281D-01	3.5	-0.51902171D-01	3.6	-0.14063474D-01	3.7	0.22553126D-01
3.8	0.55658950D-01	3.9	0.83560378D-01	4.0	0.10514922D 00	4.1	0.11985573D 00
4.2	0.12757815D 00	4.3	0.12860006D 00	4.4	0.12350400D 00	4.5	0.11308778D 00
4.6	0.98287447D-01	4.7	0.80109688D-01	4.8	0.59574839D-01	4.9	0.37670856D-01
5.0	0.15317900D-01	5.1	-0.66572559D-02	5.2	-0.27538102D-01	5.3	-0.46727749D-01
5.4	-0.63752033D-01	5.5	-0.78257584D-01	5.6	-0.90005988D-01	5.7	-0.98865083D-01
5.8	-0.10479827D 00	5.9	-0.10785259D 00	6.0	-0.10814629D 00	6.1	-0.10585621D 00
6.2	-0.10120565D 00	6.3	-0.94452737D-01	6.4	-0.85879756D-01	6.5	-0.75783460D-01
6.6	-0.64466453D-01	6.7	-0.52229727D-01	6.8	-0.39366311D-01	6.9	-0.26156023D-01
7.0	-0.12861245D-01	7.1	0.27632782D-03	7.2	0.13038064D-01	7.3	0.25229980D-01
7.4	0.36683628D-01	7.5	0.47256371D-01	7.6	0.56831143D-01	7.7	0.65315777D-01
7.8	0.72641965D-01	7.9	0.78763949D-01	8.0	0.83656989D-01	8.1	0.87315668D-01
8.2	0.89752100D-01	8.3	0.90994070D-01	8.4	0.91083162D-01	8.5	0.90072891D-01
8.6	0.88026884D-01	8.7	0.85017118D-01	8.8	0.81122243D-01	8.9	0.76426003D-01
9.0	0.71015758D-01	9.1	0.64981124D-01	9.2	0.58412727D-01	9.3	0.51401084D-01
9.4	0.44035592D-01	9.5	0.36403644D-01	9.6	0.28589859D-01	9.7	0.20675416D-01
9.8	0.12737502D-01	9.9	0.48488513D-02	10.0	-0.29226196D-02		

P= 9.30

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.37993203D 00	1.2	0.84365414D-01	1.3	0.28635131D 00
1.4	0.14809867D 00	1.5	-0.71474832D-01	1.6	-0.20640910D 00	1.7	-0.21771735D 00
1.8	-0.13822638D 00	1.9	-0.21533427D-01	2.0	0.86672692D-01	2.1	0.15893331D 00
2.2	0.18572921D 00	2.3	0.17066872D 00	2.4	0.12480633D 00	2.5	0.61848271D-01
2.6	-0.51756530D-02	2.7	-0.65811996D-01	2.8	-0.11295263D 00	2.9	-0.14282839D 00
3.0	-0.15454536D 00	3.1	-0.14940885D 00	3.2	-0.13020357D 00	3.3	-0.10053368D 00
3.4	-0.64276204D-01	3.5	-0.25167239D-01	3.6	0.13482113D-01	3.7	0.48955262D-01
3.8	0.79206138D-01	3.9	0.10286395D 00	4.0	0.11918755D 00	4.1	0.12798759D 00
4.2	0.12953102D 00	4.3	0.12443954D 00	4.4	0.11358999D 00	4.5	0.98022589D-01
4.6	0.78860166D-01	4.7	0.57240396D-01	4.8	0.34261335D-01	4.9	0.10939930D-01
5.0	-0.11817530D-01	5.1	-0.33234382D-01	5.2	-0.52673521D-01	5.3	-0.69639869D-01
5.4	-0.83776917D-01	5.5	-0.94858744D-01	5.6	-0.10277879D 00	5.7	-0.10753646D 00
5.8	-0.10922253D 00	5.9	-0.10800402D 00	6.0	-0.10410926D 00	6.1	-0.97813534D-01
6.2	-0.89425641D-01	6.3	-0.79275650D-01	6.4	-0.67703970D-01	6.5	-0.55051815D-01
6.6	-0.41653071D-01	6.7	-0.27827561D-01	6.8	-0.13875615D-01	6.9	-0.73871519D-04
7.0	0.13327783D-01	7.1	0.26108274D-01	7.2	0.38076496D-01	7.3	0.49071696D-01
7.4	0.58963194D-01	7.5	0.67649552D-01	7.6	0.75057297D-01	7.7	0.81139285D-01
7.8	0.85872801D-01	7.9	0.89257466D-01	8.0	0.91313019D-01	8.1	0.92077036D-01
8.2	0.91602628D-01	8.3	0.89956176D-01	8.4	0.87215113D-01	8.5	0.83465810D-01
8.6	0.78801565D-01	8.7	0.73320722D-01	8.8	0.67124933D-01	8.9	0.60317562D-01
9.0	0.53002246D-01	9.1	0.45281608D-01	9.2	0.37256121D-01	9.3	0.29023116D-01
9.4	0.20675941D-01	9.5	0.12303249D-01	9.6	0.39884173D-02	9.7	-0.41909117D-02
9.8	-0.12163181D-01	9.9	-0.19863166D-01	10.0	-0.27232125D-01		

P= 9.40

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.37471175D 00	1.2	0.10297550D 00	1.3	0.28519124D 00
1.4	0.12787433D 00	1.5	-0.93437468D-01	1.6	-0.21566105D 00	1.7	-0.20977051D 00
1.8	-0.11717559D 00	1.9	0.43097077D-02	2.0	0.10885340D 00	2.1	0.17139637D 00
2.2	0.18577569D 00	2.3	0.15875970D 00	2.4	0.10373281D 00	2.5	0.35691953D-01
2.6	-0.32010704D-01	2.7	-0.89350021D-01	2.8	-0.13013083D 00	2.9	-0.15172968D 00
3.0	-0.15442611D 00	3.1	-0.14058003D 00	3.2	-0.11382231D 00	3.3	-0.78353777D-01
3.4	-0.38394789D-01	3.5	0.22067214D-02	3.6	0.40221818D-01	3.7	0.73165243D-01
3.8	0.99325341D-01	3.9	0.11772569D 00	4.0	0.12804195D 00	4.1	0.13049330D 00
4.2	0.12572359D 00	4.3	0.11468325D 00	4.4	0.98519650D-01	4.5	0.78480421D-01
4.6	0.55832320D-01	4.7	0.31796333D-01	4.8	0.74985513D-02	4.9	-0.16064372D-01
5.0	-0.38046929D-01	5.1	-0.57765537D-01	5.2	-0.74700381D-01	5.3	-0.88490229D-01
5.4	-0.98921985D-01	5.5	-0.10591652D 00	5.6	-0.10951218D 00	5.7	-0.10984699D 00
5.8	-0.10714058D 00	5.9	-0.10167649D 00	6.0	-0.93785343D-01	6.1	-0.83829356D-01
6.2	-0.72188385D-01	6.3	-0.59247661D-01	6.4	-0.45387279D-01	6.5	-0.30973428D-01
6.6	-0.16351289D-01	6.7	-0.18395169D-02	6.8	0.12273828D-01	6.9	0.25734049D-01
7.0	0.38321418D-01	7.1	0.49851784D-01	7.2	0.60176323D-01	7.3	0.69180582D-01
7.4	0.76782945D-01	7.5	0.82932652D-01	7.6	0.87607472D-01	7.7	0.90811139D-01
7.8	0.92570643D-01	7.9	0.92933432D-01	8.0	0.91964611D-01	8.1	0.89744183D-01
8.2	0.86364370D-01	8.3	0.81927062D-01	8.4	0.76541408D-01	8.5	0.70321578D-01
8.6	0.63384711D-01	8.7	0.55849043D-01	8.8	0.47832236D-01	8.9	0.39449903D-01
9.0	0.30814316D-01	9.1	0.22033304D-01	9.2	0.13209328D-01	9.3	0.44387199D-02
9.4	-0.41889170D-02	9.5	-0.12591167D-01	9.6	-0.20693126D-01	9.7	-0.28427598D-01
9.8	-0.35735194D-01	9.9	-0.42564325D-01	10.0	-0.48871108D-01		

P= 9.50

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.36881290D 00	1.2	0.12099131D 00	1.3	0.28241955D 00
1.4	0.10690795D 00	1.5	-0.11430630D 00	1.6	-0.22246355D 00	1.7	-0.19927579D 00
1.8	-0.94689811D-01	1.9	0.29811214D-01	2.0	0.12892219D 00	2.1	0.18051638D 00
2.2	0.18207024D 00	2.3	0.14354431D 00	2.4	0.80494760D-01	2.5	0.89423348D-02
2.6	-0.57737706D-01	2.7	-0.11022189D 00	2.8	-0.14343914D 00	2.9	-0.15604428D 00
3.0	-0.14954363D 00	3.1	-0.12733544D 00	3.2	-0.93826721D-01	3.3	-0.53708699D-01
3.4	-0.11419807D-01	3.5	0.29209497D-01	3.6	0.65153970D-01	3.7	0.94264914D-01
3.8	0.11524870D 00	3.9	0.12758279D 00	4.0	0.13139479D 00	4.1	0.12732475D 00
4.2	0.11638558D 00	4.3	0.99832005D-01	4.4	0.79044481D-01	4.5	0.55431554D-01
4.6	0.30351881D-01	4.7	0.50554854D-02	4.8	-0.19357233D-01	4.9	-0.41960818D-01
5.0	-0.62016831D-01	5.1	-0.78975156D-01	5.2	-0.92466920D-01	5.3	-0.10229122D 00
5.4	-0.10839757D 00	5.5	-0.11086581D 00	5.6	-0.10988462D 00	5.7	-0.10573008D 00
5.8	-0.98744689D-01	5.9	-0.89317884D-01	6.0	-0.77868161D-01	6.1	-0.64827246D-01
6.2	-0.50626393D-01	6.3	-0.35684848D-01	6.4	-0.20400440D-01	6.5	-0.51421849D-02
6.6	0.97552432D-02	6.7	0.23995344D-01	6.8	0.37322123D-01	6.9	0.49521111D-01
7.0	0.60419276D-01	7.1	0.69884007D-01	7.2	0.77821341D-01	7.3	0.84173617D-01
7.4	0.88916671D-01	7.5	0.92056738D-01	7.6	0.93627147D-01	7.7	0.93684925D-01
7.8	0.92307394D-01	7.9	0.89588810D-01	8.0	0.85637135D-01	8.1	0.80570951D-01
8.2	0.74516576D-01	8.3	0.67605392D-01	8.4	0.59971407D-01	8.5	0.51749064D-01
8.6	0.43071285D-01	8.7	0.34067769D-01	8.8	0.24863528D-01	8.9	0.15577646D-01
9.0	0.63222640D-02	9.1	-0.27982340D-02	9.2	-0.11687842D-01	9.3	-0.20259379D-01
9.4	-0.28434775D-01	9.5	-0.36145214D-01	9.6	-0.43331148D-01	9.7	-0.49942200D-01
9.8	-0.55936969D-01	9.9	-0.61282750D-01	10.0	-0.65955185D-01		

P= 9.60

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.36225354D 00	1.2	0.13835167D 00	1.3	0.27806874D 00
1.4	0.85361841D-01	1.5	-0.13390291D 00	1.6	-0.22676907D 00	1.7	-0.18639062D 00
1.8	-0.71099891D-01	1.9	0.54576787D-01	2.0	0.14655759D 00	2.1	0.18615881D 00
2.2	0.17472689D 00	2.3	0.12538475D 00	2.4	0.55647387D-01	2.5	-0.17744888D-01
2.6	-0.81709217D-01	2.7	-0.12789354D 00	2.8	-0.15254274D 00	2.9	-0.15569337D 00
3.0	-0.14009829D 00	3.1	-0.11014510D 00	3.2	-0.70918194D-01	3.3	-0.27471060D-01
3.4	0.15679197D-01	3.5	0.54855115D-01	3.6	0.87355578D-01	3.7	0.11146644D 00
3.8	0.12638378D 00	3.9	0.13208329D 00	4.0	0.12916289D 00	4.1	0.11867859D 00
4.2	0.10198811D 00	4.3	0.80611121D-01	4.4	0.56111141D-01	4.5	0.30001158D-01
4.6	0.36725816D-02	4.7	-0.21654085D-01	4.8	-0.44959439D-01	4.9	-0.65439215D-01
5.0	-0.82505270D-01	5.1	-0.95776478D-01	5.2	-0.10506228D 00	5.3	-0.11034127D 00
5.4	-0.11173685D 00	5.5	-0.10949163D 00	5.6	-0.10394190D 00	5.7	-0.95493091D-01
5.8	-0.84597082D-01	5.9	-0.71731712D-01	6.0	-0.57382849D-01	6.1	-0.42029109D-01
6.2	-0.26129250D-01	6.3	-0.10112121D-01	6.4	0.56309626D-02	6.5	0.20751663D-01
6.6	0.34948081D-01	6.7	0.47966508D-01	6.8	0.59601705D-01	6.9	0.69695962D-01
7.0	0.78137152D-01	7.1	0.84856020D-01	7.2	0.89822870D-01	7.3	0.93043850D-01
7.4	0.94556972D-01	7.5	0.94427994D-01	7.6	0.92746292D-01	7.7	0.89620796D-01
7.8	0.85176066D-01	7.9	0.79548579D-01	8.0	0.72883250D-01	8.1	0.65330235D-01
8.2	0.57042030D-01	8.3	0.48170873D-01	8.4	0.38866458D-01	8.5	0.29273955D-01
8.6	0.19532329D-01	8.7	0.97729422D-02	8.8	0.11843162D-03	8.9	-0.93181697D-02
9.0	-0.18434072D-01	9.1	-0.27137161D-01	9.2	-0.35346233D-01	9.3	-0.42991068D-01
9.4	-0.50012345D-01	9.5	-0.56361430D-01	9.6	-0.62000058D-01	9.7	-0.66899910D-01
9.8	-0.71042124D-01	9.9	-0.74416739D-01	10.0	-0.77022094D-01		

P= 9.70

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.35505269D 00	1.2	0.15499842D 00	1.3	0.27217962D 00
1.4	0.63400875D-01	1.5	-0.15206215D 00	1.6	-0.22855723D 00	1.7	-0.17130028D 00
1.8	-0.46748624D-01	1.9	0.78227617D-01	2.0	0.16148285D 00	2.1	0.18825490D 00
2.2	0.16393115D 00	2.3	0.10470246D 00	2.4	0.29776801D-01	2.5	-0.43722367D-01
2.6	-0.10332934D 00	2.7	-0.14192230D 00	2.8	-0.15722855D 00	2.9	-0.15073604D 00
3.0	-0.12642818D 00	3.1	-0.89599779D-01	3.2	-0.45887087D-01	3.3	-0.55888695D-03
3.4	0.41938422D-01	3.5	0.78217513D-01	3.6	0.10601618D 00	3.7	0.12414143D 00
3.8	0.13233528D 00	3.9	0.13109834D 00	4.0	0.12149862D 00	4.1	0.10498534D 00
4.2	0.83220486D-01	4.3	0.57935445D-01	4.4	0.30815845D-01	4.5	0.34148035D-02
4.6	-0.22907327D-01	4.7	-0.47019755D-01	4.8	-0.68038260D-01	4.9	-0.85326259D-01
5.0	-0.98483648D-01	5.1	-0.10732681D 00	5.2	-0.11186280D 00	5.3	-0.11226011D 00
5.4	-0.10881828D 00	5.5	-0.10193763D 00	5.6	-0.92090645D-01	5.7	-0.79795579D-01
5.8	-0.65592994D-01	5.9	-0.50025467D-01	6.0	-0.33620583D-01	6.1	-0.16877167D-01
6.2	-0.25461784D-03	6.3	0.15834903D-01	6.4	0.31031706D-01	6.5	0.45031527D-01
6.6	0.57586534D-01	6.7	0.68504693D-01	6.8	0.77647796D-01	6.9	0.84928432D-01
7.0	0.90306163D-01	7.1	0.93783132D-01	7.2	0.95399300D-01	7.3	0.95227494D-01
7.4	0.93368413D-01	7.5	0.89945698D-01	7.6	0.85101190D-01	7.7	0.78990422D-01
7.8	0.71778426D-01	7.9	0.63635886D-01	8.0	0.54735655D-01	8.1	0.45249671D-01
8.2	0.35346251D-01	8.3	0.25187769D-01	8.4	0.14928712D-01	8.5	0.47140898D-02
8.6	-0.53218311D-02	8.7	-0.15056472D-01	8.8	-0.24379612D-01	8.9	-0.33193796D-01
9.0	-0.41414516D-01	9.1	-0.48970188D-01	9.2	-0.55801957D-01	9.3	-0.61863347D-01
9.4	-0.67119780D-01	9.5	-0.71548005D-01	9.6	-0.75135427D-01	9.7	-0.77879379D-01
9.8	-0.79786351D-01	9.9	-0.80871177D-01	10.0	-0.81156216D-01		

P= 9.80

X	KP(X)	X	KP(X)	X	KP(X)	X	KP(X)
1.0	0.10000000D 01	1.1	-0.34723034D 00	1.2	0.17087652D 00	1.3	0.26480101D 00
1.4	0.41191261D-01	1.5	-0.16863348D 00	1.6	-0.22783476D 00	1.7	-0.15421531D 00
1.8	-0.21985896D-01	1.9	0.10040606D 00	2.0	0.17347025D 00	2.1	0.18680226D 00
2.2	0.14993560D 00	2.3	0.81968528D-01	2.4	0.34864012D-02	2.5	-0.68366591D-01
2.6	-0.12206832D 00	2.7	-0.15196786D 00	2.8	-0.15740994D 00	2.9	-0.14136616D 00
3.0	-0.10899732D 00	3.1	-0.66391069D-01	3.2	-0.19585774D-01	3.3	0.26096185D-01
3.4	0.66433777D-01	3.5	0.98463682D-01	3.6	0.12046714D 00	3.7	0.13184358D 00
3.8	0.13291877D 00	3.9	0.12472508D 00	4.0	0.10878103D 00	4.1	0.86888404D-01
4.2	0.60957562D-01	4.3	0.32866214D-01	4.4	0.43523882D-02	4.5	-0.23060176D-01
4.6	-0.48107822D-01	4.7	-0.69808421D-01	4.8	-0.87463227D-01	4.9	-0.10064376D 00
5.0	-0.10916798D 00	5.1	-0.11306941D 00	5.2	-0.11256243D 00	5.3	-0.10800614D 00
5.4	-0.99868775D-01	5.5	-0.88694210D-01	5.6	-0.75071386D-01	5.7	-0.59607406D-01
5.8	-0.42904564D-01	5.9	-0.25541408D-01	6.0	-0.80577274D-02	6.1	0.90567466D-02
6.2	0.25370267D-01	6.3	0.40513975D-01	6.4	0.54184191D-01	6.5	0.66142495D-01
6.6	0.76213979D-01	6.7	0.84284062D-01	6.8	0.90294189D-01	6.9	0.94236732D-01
7.0	0.96149364D-01	7.1	0.96109134D-01	7.2	0.94226445D-01	7.3	0.90639099D-01
7.4	0.85506533D-01	7.5	0.79004358D-01	7.6	0.71319271D-01	7.7	0.62644402D-01
7.8	0.53175114D-01	7.9	0.43105298D-01	8.0	0.32624144D-01	8.1	0.21913397D-01
8.2	0.11145073D-01	8.3	0.47962173D-03	8.4	-0.99355100D-02	8.5	-0.19966922D-01
8.6	-0.29495962D-01	8.7	-0.38419122D-01	8.8	-0.46648142D-01	8.9	-0.54109891D-01
9.0	-0.60746022D-01	9.1	-0.66512467D-01	9.2	-0.71378774D-01	9.3	-0.75327335D-01
9.4	-0.78352519D-01	9.5	-0.80459738D-01	9.6	-0.81664466D-01	9.7	-0.81991235D-01
9.8	-0.81472618D-01	9.9	-0.80148213D-01	10.0	-0.78063650D-01		

P= 9.90

X	KP(X)
1.0	0.10000000D 01
1.4	0.18899353D-01
1.8	0.28362216D-02
2.2	0.13305417D 00
2.6	-0.13747536D 00
3.0	-0.88380249D-01
3.4	0.88313106D-01
3.8	0.12816582D 00
4.2	0.36219408D-01
4.6	-0.70728344D-01
5.0	-0.11405697D 00
5.4	-0.85447578D-01
5.8	-0.17909044D-01
6.2	0.49150409D-01
6.6	0.89651089D-01
7.0	0.95330823D-01
7.4	0.71601719D-01
7.8	0.30802308D-01
8.2	-0.13670429D-01
8.6	-0.51069466D-01
9.0	-0.74874729D-01
9.4	-0.82818012D-01
9.8	-0.76025998D-01

X	KP(X)
1.1	-0.33880735D 00
1.5	-0.18348217D 00
1.9	0.12078094D 00
2.3	0.57693189D-01
2.7	-0.15780029D 00
3.1	-0.41288354D-01
3.5	0.11488308D 00
3.9	0.11328042D 00
4.3	0.65617938D-02
4.7	-0.88926043D-01
5.1	-0.11275987D 00
5.5	-0.70567437D-01
5.9	0.22594107D-03
6.3	0.62379625D-01
6.7	0.94307322D-01
7.1	0.91740326D-01
7.5	0.62460213D-01
7.9	0.19544507D-01
8.3	-0.24014799D-01
8.7	-0.58451951D-01
9.1	-0.78352894D-01
9.5	-0.82399544D-01
9.9	-0.72384739D-01

X	KP(X)
1.2	0.18593431D 00
1.6	-0.22463517D 00
2.0	0.18234438D 00
2.4	-0.22616999D-01
2.8	-0.15312788D 00
3.2	0.71000912D-02
3.6	0.13020534D 00
4.0	0.91597711D-01
4.4	-0.22043790D-01
4.8	-0.10229819D 00
5.2	-0.10718595D 00
5.6	-0.53912824D-01
6.0	0.17735379D-01
6.4	0.73630941D-01
6.8	0.96748136D-01
7.2	0.86455486D-01
7.6	0.52468442D-01
8.0	0.82635619D-02
8.4	-0.33766903D-01
8.8	-0.64905474D-01
9.2	-0.80824850D-01
9.6	-0.81093038D-01
10.0	-0.68091744D-01

X	KP(X)
1.3	0.25598941D 00
1.7	-0.13536875D 00
2.1	0.18186430D 00
2.5	-0.91092656D-01
2.9	-0.12790538D 00
3.3	0.51580867D-01
3.7	0.13432412D 00
4.1	0.65214300D-01
4.5	-0.48174689D-01
4.9	-0.11065639D 00
5.3	-0.97875222D-01
5.7	-0.36149341D-01
6.1	0.34168890D-01
6.5	0.82750405D-01
6.9	0.97050243D-01
7.3	0.79672677D-01
7.7	0.41844881D-01
8.1	-0.28623136D-02
8.5	-0.42816059D-01
8.9	-0.70388420D-01
9.3	-0.82304804D-01
9.7	-0.78949478D-01

P=10.00

X	KP(X)
1.0	0.10000000D 01
1.4	-0.33095476D-02
1.8	0.27368368D-01
2.2	0.11365532D 00
2.6	-0.14918928D 00
3.0	-0.65243327D-01
3.4	0.10682556D 00
3.8	0.11832061D 00
4.2	0.10124927D-01
4.6	-0.89704614D-01
5.0	-0.11295512D 00
5.4	-0.66411891D-01
5.8	0.78943186D-02
6.2	0.69621835D-01
6.6	0.97070034D-01
7.0	0.87974106D-01
7.4	0.52709431D-01
7.8	0.63614398D-02
8.2	-0.37181103D-01
8.6	-0.68348365D-01
9.0	-0.82690294D-01
9.4	-0.80206024D-01
9.8	-0.63993852D-01

X	KP(X)
1.1	-0.32980544D 00
1.5	-0.19649037D 00
1.9	0.13905239D 00
2.3	0.32414518D-01
2.7	-0.15930507D 00
3.1	-0.15113597D-01
3.5	0.12691231D 00
3.9	0.97285857D-01
4.3	-0.19775462D-01
4.7	-0.10346980D 00
5.1	-0.10647584D 00
5.5	-0.48630467D-01
5.9	0.25720514D-01
6.3	0.80079465D-01
6.7	0.97968289D-01
7.1	0.81047296D-01
7.5	0.41564884D-01
7.9	-0.52478640D-02
8.3	-0.46393020D-01
8.7	-0.73578181D-01
9.1	-0.83567062D-01
9.5	-0.77272284D-01
9.9	-0.58333930D-01

X	KP(X)
1.2	0.20012358D 00
1.6	-0.21901812D 00
2.0	0.18798469D 00
2.4	-0.47936984D-01
2.8	-0.14454851D 00
3.2	0.33281106D-01
3.6	0.13491040D 00
4.0	0.70717968D-01
4.4	-0.47152890D-01
4.8	-0.11184608D 00
5.2	-0.96084218D-01
5.6	-0.29870134D-01
6.0	0.42190389D-01
6.4	0.88165874D-01
6.8	0.96639541D-01
7.2	0.72697576D-01
7.6	0.29974944D-01
8.0	-0.16478590D-01
8.4	-0.54706959D-01
8.8	-0.77712669D-01
9.2	-0.83412801D-01
9.6	-0.73542178D-01
10.0	-0.52191927D-01

X	KP(X)
1.3	0.24580862D 00
1.7	-0.11501303D 00
2.1	0.17356834D 00
2.5	-0.11136796D 00
2.9	-0.11079236D 00
3.3	0.75031223D-01
3.7	0.13153941D 00
4.1	0.40935256D-01
4.5	-0.70756427D-01
4.9	-0.11490659D 00
5.3	-0.82481155D-01
5.7	-0.10819715D-01
6.1	0.56922294D-01
6.5	0.93825692D-01
6.9	0.93243914D-01
7.3	0.63169017D-01
7.7	0.18168692D-01
8.1	-0.27169801D-01
8.5	-0.62044831D-01
8.9	-0.80746710D-01
9.3	-0.82273937D-01
9.7	-0.69090051D-01

Table 2

The zeros, value of the derivative at the zeros, bend points and value of the function at the bend points, to 8 decimal places.

ZEROS AND DERIVATIVES AT THE ZERO POINTS,BEND POINTS AND FUNCTION
VALUES AT THE BEND POINTS OF THE CONICAL FUNCTION KP(X)

P*	ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.90	0.81185516D 01	-0.33233290D-01	0.0	0.0
1.00	0.60666379D 01	-0.54660979D-01	0.0	0.0
1.10	0.48026857D 01	-0.82381328D-01	0.0	0.0
1.20	0.39684929D 01	-0.11639415D 00	0.0	0.0
1.30	0.33880547D 01	-0.15658010D 00	0.83648462D 01	-0.22648034D 00
1.40	0.29670618D 01	-0.20276474D 00	0.69322529D 01	-0.24217967D 00

P#	ZERO	DERIVATIVE	BEND POINT	FUNCTION
1.50	0.26513731D 01	-0.25475703D 00	0.58869363D 01	-0.25617995D 00
1.60	0.24081172D 01	-0.31237035D 00	0.51005880D 01	-0.26866050D 00
1.70	0.22163913D 01	-0.37543280D 00	0.44938172D 01	-0.27979230D 00
1.80	0.20623778D 01	-0.44379121D 00	0.40154535D 01	-0.28973276D 00
1.90	0.19366371D 01	-0.51731181D 00	0.36313471D 01	-0.29862366D 00
	0.93792963D 01	0.38623858D-01	0.0	0.0
2.00	0.18325356D 01	-0.59587916D 00	0.33180137D 01	-0.30659097D 00
	0.80961006D 01	0.49560327D-01	0.0	0.0
2.10	0.17452963D 01	-0.67939428D 00	0.30588768D 01	-0.31374566D 00
	0.70902858D 01	0.62187172D-01	0.0	0.0

P#	ZERO	DERIVATIVE	BEND POINT	FUNCTION
2.20				
	0.16714047D 01	-0.76777267D 00	0.28419653D 01	-0.32018488D 00
	0.62875583D 01	0.76540975D-01	0.0	0.0
2.30				
	0.16082264D 01	-0.86094228D 00	0.26584580D 01	-0.32599347D 00
	0.56367396D 01	0.92644161D-01	0.0	0.0
2.40				
	0.15537533D 01	-0.95884168D 00	0.25017371D 01	-0.33124535D 00
	0.51017156D 01	0.11050758D 00	0.89456896D 01	0.16904983D 00
2.50				
	0.15064316D 01	-0.10614184D 01	0.23667577D 01	-0.33600489D 00
	0.46564513D 01	0.13013285D 00	0.79964115D 01	0.17560593D 00
2.60				
	0.14650434D 01	-0.11686277D 01	0.22496181D 01	-0.34032816D 00
	0.42818157D 01	0.15151436D 00	0.72098682D 01	0.18176394D 00
2.70				
	0.14286221D 01	-0.12804311D 01	0.21472614D 01	-0.34426403D 00
	0.39635079D 01	0.17464106D 00	0.65510057D 01	0.18754626D 00

P# 2.80 ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.13963924D 01	-0.13967956D 01	0.20572641D 01	-0.34785514D 00
0.36906706D 01	0.19949785D 00	0.59936455D 01	0.19297519D 00
P# 2.90			
0.13677261D 01	-0.15176928D 01	0.19776844D 01	-0.35113877D 00
0.34549445D 01	0.22606681D 00	0.55179429D 01	0.19807259D 00
P# 3.00			
0.13421097D 01	-0.16430982D 01	0.19069506D 01	-0.35414755D 00
0.32498108D 01	0.25432809D 00	0.51086489D 01	0.20285957D 00
0.90588206D 01	-0.51158784D-01	0.0	0.0
P# 3.10			
0.13191200D 01	-0.17729906D 01	0.18437798D 01	-0.35691010D 00
0.30701263D 01	0.28426073D 00	0.47538992D 01	0.20735634D 00
0.82531355D 01	-0.59916706D-01	0.0	0.0
P# 3.20			
0.12984053D 01	-0.19073514D 01	0.17871154D 01	-0.35945159D 00
0.29117898D 01	0.31584320D 00	0.44443573D 01	0.21158211D 00
0.75641050D 01	-0.69528811D-01	0.0	0.0
P# 3.30			
0.12796716D 01	-0.20461647D 01	0.17360810D 01	-0.36179415D 00
0.27714981D 01	0.34905388D 00	0.41725983D 01	0.21555501D 00
0.69703527D 01	-0.80008615D-01	0.0	0.0

P# 3.40			
ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.12626711D 01	-0.21894163D 01	0.16899445D 01	-0.36395735D 00
0.26465672D 01	0.38387137D 00	0.39326603D 01	0.21929208D 00
0.64551353D 01	-0.91366447D-01	0.97788474D 01	-0.13724093D 00
P# 3.50			
0.12471942D 01	-0.23370939D 01	0.16480901D 01	-0.36595848D 00
0.25347976D 01	0.42027479D 00	0.37197126D 01	0.22280929D 00
0.60052002D 01	-0.10360980D 00	0.89950237D 01	-0.14118591D 00
P# 3.60			
0.12330622D 01	-0.24891868D 01	0.16099967D 01	-0.36781285D 00
0.24343726D 01	0.45824390D 00	0.35298092D 01	0.22612156D 00
0.56099563D 01	-0.11674368D 00	0.83127951D 01	-0.14496511D 00
P# 3.70			
0.12201221D 01	-0.26456854D 01	0.15752209D 01	-0.36953410D 00
0.23437813D 01	0.49775927D 00	0.33597010D 01	0.22924278D 00
0.52608629D 01	-0.13077091D 00	0.77154542D 01	-0.14858452D 00
P# 3.80			
0.12082422D 01	-0.28065812D 01	0.15433836D 01	-0.37113432D 00
0.22617582D 01	0.53880232D 00	0.32066940D 01	0.23218590D 00
0.49509754D 01	-0.14569243D 00	0.71895519D 01	-0.15205027D 00

P# 3.90

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.11973088D 01	-0.29718669D 01	0.15141586D 01	-0.37262433D 00
0.21872370D 01	0.58135542D 00	0.30685392D 01	0.23496293D 00
0.46746028D 01	-0.16150760D 00	0.67241727D 01	-0.15536850D 00

P# 4.00

0.11872229D 01	-0.31415358D 01	0.14872643D 01	-0.37401378D 00
0.21193139D 01	0.62540182D 00	0.29433471D 01	0.23758504D 00
0.44270473D 01	-0.17821438D 00	0.63103892D 01	-0.15854533D 00
0.96088835D 01	0.54016339D-01	0.0	0.0

P# 4.10

0.11778984D 01	-0.33155820D 01	0.14624564D 01	-0.37531133D 00
0.20572184D 01	0.67092577D 00	0.28295213D 01	0.24006258D 00
0.42044052D 01	-0.19580964D 00	0.59408478D 01	-0.16158679D 00
0.89432611D 01	0.60982540D-01	0.0	0.0

P# 4.20

0.11692599D 01	-0.34940003D 01	0.14395220D 01	-0.37652472D 00
0.20002906D 01	0.71791237D 00	0.27257050D 01	0.24240514D 00
0.40034111D 01	-0.21428926D 00	0.56094505D 01	-0.16449881D 00
0.83528320D 01	0.68475865D-01	0.0	0.0

P# 4.30

0.11612412D 01	-0.36767860D 01	0.14182751D 01	-0.37766094D 00
0.19479624D 01	0.76634767D 00	0.26307396D 01	0.24462164D 00
0.38213180D 01	-0.23364839D 00	0.53111082D 01	-0.16728715D 00
0.78267631D 01	0.76504177D-01	0.0	0.0

P# 4.40

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.11537840D 01	-0.38639348D 01	0.13985521D 01	-0.37872627D 00
0.18997425D 01	0.81621854D 00	0.25436306D 01	0.24672032D 00
0.36558013D 01	-0.25388153D 00	0.50415486D 01	-0.16995741D 00
0.73560800D 01	0.85074128D-01	0.0	0.0

P# 4.50

0.11468366D 01	-0.40554431D 01	0.13802094D 01	-0.37972637D 00
0.18552042D 01	0.86751269D 00	0.24635202D 01	0.24870882D 00
0.35048840D 01	-0.27498274D 00	0.47971652D 01	-0.17251501D 00
0.69333026D 01	0.94191243D-01	0.95654168D 01	0.12119042D 00

P# 4.60

0.11403532D 01	-0.42513073D 01	0.13631195D 01	-0.38066638D 00
0.18139755D 01	0.92021863D 00	0.23896658D 01	0.25059425D 00
0.33668759D 01	-0.29694568D 00	0.45748974D 01	-0.17496516D 00
0.65521592D 01	0.10386001D 00	0.89782353D 01	0.12380106D 00

P# 4.70

0.11342932D 01	-0.44515244D 01	0.13471700D 01	-0.38155092D 00
0.17757307D 01	0.97432557D 00	0.23214216D 01	0.25238316D 00
0.32403254D 01	-0.31976374D 00	0.43721347D 01	-0.17731287D 00
0.62073625D 01	0.11408397D 00	0.84501074D 01	0.12632739D 00

P# 4.80

0.11286202D 01	-0.46560917D 01	0.13322605D 01	-0.38238421D 00
0.17401836D 01	0.10298235D 01	0.22582235D 01	0.25408166D 00
0.31239803D 01	-0.34343016D 00	0.41866404D 01	-0.17956294D 00
0.58944318D 01	0.12486580D 00	0.79734305D 01	0.12877175D 00

P* 4.90

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.11233017D 01	-0.48650066D 01	0.13183016D 01	-0.38317008D 00
0.17070821D 01	0.10867029D 01	0.21995776D 01	0.25569541D 00
0.30167554D 01	-0.36793803D 00	0.40164889D 01	-0.18171*00D 00
0.56095512D 01	0.13620740D 00	0.75417721D 01	0.13113651D 00

P* 5.00

0.11183086D 01	-0.50782669D 01	0.13052136D 01	-0.38391199D 00
0.16762030D 01	0.11449550D 01	0.21450493D 01	0.25722965D 00
0.29177071D 01	-0.39328043D 00	0.38600162D 01	-0.18378841D 00
0.53494559D 01	0.14810994D 00	0.71496622D 01	0.13342408D 00
0.99634819D 01	-0.57163228D-01	0.0	0.0

P* 5.10

0.11136147D 01	-0.52958705D 01	0.12929247D 01	-0.38461312D 00
0.16473484D 01	0.12045717D 01	0.20942554D 01	0.25868926D 00
0.28260113D 01	-0.41945041D 00	0.37157778D 01	-0.18577240D 00
0.51113399D 01	0.16057399D 00	0.67924285D 01	0.13563684D 00
0.93982968D 01	-0.63077080D-01	0.0	0.0

P* 5.20

0.11091965D 01	-0.55178155D 01	0.12813707D 01	-0.38527637D 00
0.16203422D 01	0.12655452D 01	0.20468564D 01	0.26007877D 00
0.27409460D 01	-0.44644109D 00	0.35825154D 01	-0.18767595D 00
0.48927818D 01	0.17359952D 00	0.64660616D 01	0.13777720D 00
0.88854669D 01	-0.69356265D-01	0.0	0.0

P* 5.30

0.11050327D 01	-0.57441003D 01	0.12704935D 01	-0.38590438D 00
0.15950273D 01	0.13278684D 01	0.20025512D 01	0.26140239D 00
0.26618762D 01	-0.47424564D 00	0.34591288D 01	-0.18950287D 00
0.46916833D 01	0.18718603D 00	0.61671074D 01	0.13984750D 00
0.84187767D 01	-0.76005714D-01	0.0	0.0

P# 5.40

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.11011039D 01	-0.59747232D 01	0.12602410D 01	-0.38649956D 00
0.15712630D 01	0.13915346D 01	0.19610717D 01	0.26266403D 00
0.25882417D 01	-0.50285737D 00	0.33446527D 01	-0.19125680D 00
0.45062194D 01	0.20133255D 00	0.58925776D 01	0.14185008D 00
0.79928938D 01	-0.83029791D-01	0.0	0.0

P# 5.50

0.10973928D 01	-0.62096828D 01	0.12505658D 01	-0.38706415D 00
0.15489233D 01	0.14565374D 01	0.19221783D 01	0.26386735D 00
0.25195470D 01	-0.53226971D 00	0.32382373D 01	-0.19294117D 00
0.43347971D 01	0.21603773D 00	0.56398775D 01	0.14378724D 00
0.76032243D 01	-0.90432328D-01	0.99322082D 01	-0.10777751D 00

P# 5.60

0.10938834D 01	-0.64489779D 01	0.12414249D 01	-0.38760017D 00
0.15278947D 01	0.15228711D 01	0.18856569D 01	0.26501573D 00
0.24553519D 01	-0.56247622D 00	0.31391323D 01	-0.19455926D 00
0.41760209D 01	0.23129989D 00	0.54067458D 01	0.14566121D 00
0.72457954D 01	-0.98216651D-01	0.94210673D 01	-0.10971685D 00

P# 5.70

0.10905614D 01	-0.66926072D 01	0.12327796D 01	-0.38810950D 00
0.15080751D 01	0.15905302D 01	0.18513151D 01	0.26611233D 00
0.23952649D 01	-0.59347067D 00	0.30466731D 01	-0.19611419D 00
0.40286643D 01	0.24711706D 00	0.51912049D 01	0.14747420D 00
0.69171583D 01	-0.10638561D 00	0.89530151D 01	-0.11160556D 00

P# 5.80

0.10874135D 01	-0.69405696D 01	0.12245942D 01	-0.38859386D 00
0.14893721D 01	0.16595095D 01	0.18189799D 01	0.26716011D 00
0.23389365D 01	-0.62524695D 00	0.29602692D 01	-0.19760891D 00
0.38916461D 01	0.26348698D 00	0.49915197D 01	0.14922833D 00
0.66143087D 01	-0.11494161D 00	0.85233850D 01	-0.11344473D 00

P# 5.90

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10844277D 01	-0.71928641D 01	0.12168366D 01	-0.38905484D 00
0.14717022D 01	0.17298043D 01	0.17884953D 01	0.26816181D 00
0.22860539D 01	-0.65779917D 00	0.28793948D 01	-0.19904623D 00
0.37640102D 01	0.28040723D 00	0.48061632D 01	0.15092570D 00
0.63346205D 01	-0.12388666D 00	0.81281164D 01	-0.11523547D 00

P# 6.00

0.10815931D 01	-0.74494898D 01	0.12094772D 01	-0.38949391D 00
0.14549894D 01	0.18014100D 01	0.17597204D 01	0.26912000D 00
0.22363364D 01	-0.69112162D 00	0.28035804D 01	-0.20042881D 00
0.36449088D 01	0.29787516D 00	0.46337871D 01	0.15256831D 00
0.60757904D 01	-0.13322236D 00	0.77636643D 01	-0.11697893D 00

P# 6.10

0.10788996D 01	-0.77104458D 01	0.12024890D 01	-0.38991242D 00
0.14391650D 01	0.18743225D 01	0.17325275D 01	0.27003708D 00
0.21895319D 01	-0.72520877D 00	0.27324054D 01	-0.20175918D 00
0.35335877D 01	0.31588801D 00	0.44731977D 01	0.15415815D 00
0.58357925D 01	-0.14294996D 00	0.74269234D 01	-0.11867622D 00
0.97204038D 01	0.65546842D-01	0.0	0.0

P# 6.20

0.10763379D 01	-0.79757313D 01	0.11958473D 01	-0.39031162D 00
0.14241662D 01	0.19485378D 01	0.17068010D 01	0.27091529D 00
0.21454129D 01	-0.76005528D 00	0.26654926D 01	-0.20303974D 00
0.34293747D 01	0.33444288D 00	0.43233353D 01	0.15569710D 00
0.56128393D 01	-0.15307040D 00	0.71151655D 01	-0.12032852D 00
0.92685189D 01	0.71030089D-01	0.0	0.0

P# 6.30

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10738994D 01	-0.82453456D 01	0.11895294D 01	-0.39069267D 00
0.14099357D 01	0.20240522D 01	0.16824354D 01	0.27175672D 00
0.21037739D 01	-0.79565602D 00	0.26025024D 01	-0.20427276D 00
0.33316687D 01	0.35353677D 00	0.41832564D 01	0.15718700D 00
0.54053489D 01	-0.16358430D 00	0.68259864D 01	-0.12193694D 00
0.88512474D 01	0.76788564D-01	0.0	0.0

P# 6.40

0.10715765D 01	-0.85192881D 01	0.11835145D 01	-0.39105665D 00
0.13964212D 01	0.21008623D 01	0.16593350D 01	0.27256334D 00
0.20644292D 01	-0.83200603D 00	0.25431288D 01	-0.20546040D 00
0.32399313D 01	0.37316661D 00	0.40521190D 01	0.15862964D 00
0.52119179D 01	-0.17449201D 00	0.65572614D 01	-0.12350264D 00
0.84651733D 01	0.82825268D-01	0.0	0.0

P# 6.50

0.10693619D 01	-0.87975580D 01	0.11777834D 01	-0.39140454D 00
0.13835748D 01	0.21789648D 01	0.16374123D 01	0.27333698D 00
0.20272100D 01	-0.86910054D 00	0.24870953D 01	-0.20660472D 00
0.31536787D 01	0.39332928D 00	0.39291698D 01	0.16002672D 00
0.50312978D 01	-0.18579361D 00	0.63071075D 01	-0.12502674D 00
0.81072863D 01	0.89142915D-01	0.0	0.0

P# 6.60

0.10672488D 01	-0.90801548D 01	0.11723184D 01	-0.39173728D 00
0.13713527D 01	0.22583566D 01	0.16165872D 01	0.27407937D 00
0.19919632D 01	-0.90693498D 00	0.24341515D 01	-0.20770766D 00
0.30724754D 01	0.41402161D 00	0.38137332D 01	0.16137992D 00
0.48623750D 01	-0.19748898D 00	0.60738517D 01	-0.12651036D 00
0.77749254D 01	0.95743943D-01	0.97363343D 01	0.99508369D-01

P# 6.70

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10652313D 01	-0.93670780D 01	0.11671032D 01	-0.39205572D 00
0.13597145D 01	0.23390349D 01	0.15967866D 01	0.27479211D 00
0.19585491D 01	-0.94550492D 00	0.23840704D 01	-0.20877106D 00
0.29959288D 01	0.43524043D 00	0.37052021D 01	0.16269081D 00
0.47041540D 01	-0.20957776D 00	0.58560039D 01	-0.12795460D 00
0.74657324D 01	0.10263053D 00	0.93180818D 01	0.10098816D 00

P# 6.80

0.10633036D 01	-0.96583271D 01	0.11621227D 01	-0.39236067D 00
0.13486231D 01	0.24209969D 01	0.15779434D 01	0.27547673D 00
0.19268407D 01	-0.98480616D 00	0.23366454D 01	-0.20979668D 00
0.29236836D 01	0.45698255D 00	0.36030296D 01	0.16396095D 00
0.45557428D 01	-0.22205942D 00	0.56522333D 01	-0.12936055D 00
0.71776122D 01	0.10980460D 00	0.89294790D 01	0.10243540D 00

P# 6.90

0.10614604D 01	-0.99539016D 01	0.11573629D 01	-0.39265287D 00
0.13380443D 01	0.25042403D 01	0.15599957D 01	0.27613464D 00
0.18967216D 01	-0.10248346D 01	0.22916886D 01	-0.21078618D 00
0.28554181D 01	0.47924477D 00	0.35067218D 01	0.16519182D 00
0.44163400D 01	-0.23493325D 00	0.54613490D 01	-0.13072927D 00
0.69086986D 01	0.11726783D 00	0.85678136D 01	0.10385068D 00

P# 7.00

0.10596968D 01	-0.10253801D 02	0.11528109D 01	-0.39293302D 00
0.13279463D 01	0.25887624D 01	0.15428871D 01	0.27676718D 00
0.18680853D 01	-0.10655864D 01	0.22490285D 01	-0.21174114D 00
0.27908404D 01	0.50202392D 00	0.34158324D 01	0.16638484D 00
0.42852244D 01	-0.24819837D 00	0.52822826D 01	-0.13206182D 00
0.66573253D 01	0.12502170D 00	0.82306721D 01	0.10523460D 00

P# 7.10

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10580083D 01	-0.10558025D 02	0.11484548D 01	-0.39320176D 00
0.13183013D 01	0.26745612D 01	0.15265651D 01	0.27737560D 00
0.18408344D 01	-0.11070578D 01	0.22085086D 01	-0.21266307D 00
0.27296850D 01	0.52531686D 00	0.33299565D 01	0.16754140D 00
0.41617456D 01	-0.26185380D 00	0.51140736D 01	-0.13335922D 00
0.64220011D 01	0.13306746D 00	0.79159016D 01	0.10658779D 00
0.99602471D 01	-0.68150550D-01	0.0	0.0

P# 7.20

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10563907D 01	-0.10866573D 02	0.11442834D 01	-0.39345971D 00
0.13090809D 01	0.27616345D 01	0.15109815D 01	0.27796110D 00
0.18148793D 01	-0.11492452D 01	0.21699852D 01	-0.21355338D 00
0.26717100D 01	0.54912045D 00	0.32487267D 01	0.16866280D 00
0.40453156D 01	-0.27589840D 00	0.49558568D 01	-0.13462248D 00
0.62013887D 01	0.14140617D 00	0.76215773D 01	0.10791085D 00
0.95569694D 01	-0.73066376D-01	0.0	0.0

P# 7.30

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10548401D 01	-0.11179445D 02	0.11402863D 01	-0.39370742D 00
0.13002609D 01	0.28499804D 01	0.14960919D 01	0.27852476D 00
0.17901377D 01	-0.11921452D 01	0.21333271D 01	-0.21441343D 00
0.26166949D 01	0.57343163D 00	0.31718088D 01	0.16975033D 00
0.39354020D 01	-0.29033095D 00	0.48068511D 01	-0.13585257D 00
0.59942857D 01	0.15003868D 00	0.73459745D 01	0.10920440D 00
0.91805847D 01	-0.78196928D-01	0.0	0.0

P# 7.40

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10533527D 01	-0.11496641D 02	0.11364539D 01	-0.39394544D 00
0.12918179D 01	0.29395968D 01	0.14818551D 01	0.27906765D 00
0.17665337D 01	-0.12357545D 01	0.20984134D 01	-0.21524451D 00
0.25644378D 01	0.59824735D 00	0.30988982D 01	0.17080521D 00
0.38315218D 01	-0.30515011D 00	0.46663500D 01	-0.13705047D 00
0.57996097D 01	0.15896569D 00	0.70875444D 01	0.11046908D 00
0.88287775D 01	-0.83544364D-01	0.0	0.0

P# 7.50

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10519253D 01	-0.11818159D 02	0.11327772D 01	-0.39417425D 00
0.12837307D 01	0.30304821D 01	0.14682329D 01	0.27959076D 00
0.17439973D 01	-0.12800699D 01	0.20651333D 01	-0.21604784D 00
0.25147542D 01	0.62356463D 00	0.30297169D 01	0.17182861D 00
0.37332360D 01	-0.32035448D 00	0.45337134D 01	-0.13821711D 00
0.56163833D 01	0.16818773D 00	0.68448935D 01	0.11170550D 00
0.84994724D 01	-0.89110668D-01	0.0	0.0

P# 7.60

0.10505545D 01	-0.12144001D 02	0.11292480D 01	-0.39439432D 00
0.12759793D 01	0.31226346D 01	0.14551900D 01	0.28009499D 00
0.17224641D 01	-0.13250885D 01	0.20333844D 01	-0.21682459D 00
0.24674746D 01	0.64938052D 00	0.29640108D 01	0.17282166D 00
0.36401448D 01	-0.33594256D 00	0.44083601D 01	-0.13935340D 00
0.54437231D 01	0.17770518D 00	0.66167653D 01	0.11291427D 00
0.81908055D 01	-0.94897661D-01	0.99719862D 01	-0.91682804D-01

P# 7.70

0.10492375D 01	-0.12474165D 02	0.11258583D 01	-0.39460609D 00
0.12685451D 01	0.32160527D 01	0.14426935D 01	0.28058125D 00
0.17018743D 01	-0.13708073D 01	0.20030726D 01	-0.21757586D 00
0.24224437D 01	0.67569214D 00	0.29015471D 01	0.17378545D 00
0.35518836D 01	-0.35191282D 00	0.42897614D 01	-0.14046022D 00
0.52808285D 01	0.18751827D 00	0.64020246D 01	0.11409601D 00
0.79010991D 01	-0.10090701D 00	0.95948551D 01	-0.92882262D-01

P# 7.80

0.10479715D 01	-0.12808652D 02	0.11226008D 01	-0.39480997D 00
0.12614109D 01	0.33107349D 01	0.14307129D 01	0.28105035D 00
0.16821726D 01	-0.14172236D 01	0.19741108D 01	-0.21830270D 00
0.23795184D 01	0.70249668D 00	0.28421124D 01	0.17472102D 00
0.34681195D 01	-0.36826364D 00	0.41774361D 01	-0.14153846D 00
0.51269732D 01	0.19762709D 00	0.61996442D 01	0.11525134D 00
0.76288406D 01	-0.10714022D 00	0.92412426D 01	-0.94059002D-01

P* 7.90

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10467537D 01	-0.13147460D 02	0.11194688D 01	-0.39500634D 00
0.12545605D 01	0.34066798D 01	0.14192199D 01	0.28150307D 00
0.16633078D 01	-0.14643347D 01	0.19464185D 01	-0.21900611D 00
0.23385670D 01	0.72979135D 00	0.27855106D 01	0.17562938D 00
0.33885477D 01	-0.38499340D 00	0.40709446D 01	-0.14258894D 00
0.49814968D 01	0.20803163D 00	0.60086927D 01	0.11638084D 00
0.73726631D 01	-0.11359866D 00	0.89092527D 01	-0.95213371D-01

P* 8.00

0.10455820D 01	-0.13490591D 02	0.11164559D 01	-0.39519556D 00
0.12479791D 01	0.35038861D 01	0.14081879D 01	0.28194016D 00
0.16452324D 01	-0.15121381D 01	0.19199214D 01	-0.21968705D 00
0.22994681D 01	0.75757347D 00	0.27315615D 01	0.17651149D 00
0.33128890D 01	-0.40210041D 00	0.39698854D 01	-0.14361248D 00
0.48437977D 01	0.21873176D 00	0.58283241D 01	0.11748512D 00
0.71313297D 01	-0.12028355D 00	0.85971737D 01	-0.96345727D-01

P* 8.10

0.10444538D 01	-0.13838043D 02	0.11135560D 01	-0.39537798D 00
0.12416525D 01	0.36023525D 01	0.13975924D 01	0.28236231D 00
0.16279021D 01	-0.15606313D 01	0.18945503D 01	-0.22034643D 00
0.22621094D 01	0.78584040D 00	0.26800989D 01	0.17736827D 00
0.32408870D 01	-0.41958296D 00	0.38738911D 01	-0.14460989D 00
0.47133275D 01	0.22972723D 00	0.56577686D 01	0.11856477D 00
0.69037187D 01	-0.12719600D 00	0.83034577D 01	-0.97456429D-01

P* 8.20

0.10433671D 01	-0.14189817D 02	0.11107636D 01	-0.39555392D 00
0.12355676D 01	0.37020778D 01	0.13874103D 01	0.28277018D 00
0.16112760D 01	-0.16098121D 01	0.18702412D 01	-0.22098511D 00
0.22263873D 01	0.81458956D 00	0.26309696D 01	0.17820061D 00
0.31723063D 01	-0.43743933D 00	0.37826246D 01	-0.14558194D 00
0.45895849D 01	0.24101770D 00	0.54963249D 01	0.11962036D 00
0.66888113D 01	-0.13433696D 00	0.80267020D 01	-0.98545842D-01
0.97819010D 01	0.75272837D-01	0.0	0.0

P# 8.30

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10423199D 01	-0.14545912D 02	0.11080734D 01	-0.39572367D 00
0.12297122D 01	0.38030609D 01	0.13776201D 01	0.28316440D 00
0.15953157D 01	-0.16596781D 01	0.18469345D 01	-0.22160391D 00
0.21922057D 01	0.84381843D 00	0.25840322D 01	0.17900937D 00
0.31069301D 01	-0.45566775D 00	0.36957766D 01	-0.14652936D 00
0.44721114D 01	0.25260276D 00	0.53433527D 01	0.12065246D 00
0.64856805D 01	-0.14170730D 00	0.77656336D 01	-0.99614333D-01
0.94395744D 01	0.79933901D-01	0.0	0.0

P# 8.40

0.10413102D 01	-0.14906329D 02	0.11054805D 01	-0.39588753D 00
0.12240747D 01	0.39053007D 01	0.13682017D 01	0.28354555D 00
0.15799855D 01	-0.17102274D 01	0.18245750D 01	-0.22220362D 00
0.21594757D 01	0.87352457D 00	0.25391559D 01	0.17979537D 00
0.30445586D 01	-0.47426649D 00	0.36130627D 01	-0.14745290D 00
0.43604866D 01	0.26448189D 00	0.51982667D 01	0.12166163D 00
0.62934818D 01	-0.14930773D 00	0.75190952D 01	-0.10066227D 00
0.91170889D 01	0.84770047D-01	0.0	0.0

P# 8.50

0.10403364D 01	-0.15271066D 02	0.11029801D 01	-0.39604576D 00
0.12186444D 01	0.40087962D 01	0.13591363D 01	0.28391421D 00
0.15652522D 01	-0.17614577D 01	0.18031111D 01	-0.22278499D 00
0.21281147D 01	0.90370561D 00	0.24962195D 01	0.18055940D 00
0.29850074D 01	-0.49323377D 00	0.35342212D 01	-0.14835326D 00
0.42543250D 01	0.27665452D 00	0.50605312D 01	0.12264843D 00
0.61114442D 01	-0.15713888D 00	0.72860331D 01	-0.10169002D 00
0.88129550D 01	0.89782772D-01	0.0	0.0

P# 8.60

0.10393967D 01	-0.15640124D 02	0.11005680D 01	-0.39619861D 00
0.12134111D 01	0.41135464D 01	0.13504063D 01	0.28427090D 00
0.15510847D 01	-0.18133672D 01	0.17824947D 01	-0.22334872D 00
0.20980458D 01	0.93435921D 00	0.24551108D 01	0.18130221D 00
0.29281062D 01	-0.51256782D 00	0.34590106D 01	-0.14923112D 00
0.41532722D 01	0.28911*00D 00	0.49296549D 01	0.12361339D 00
0.59388629D 01	-0.16520127D 00	0.70654861D 01	-0.10269796D 00
0.85258185D 01	0.94973469D-01	0.0	0.0

P# 8.70

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10384895D 01	-0.16013503D 02	0.10982400D 01	-0.39634633D 00
0.12083654D 01	0.42195505D 01	0.13419952D 01	0.28461612D 00
0.15374540D 01	-0.18659539D 01	0.17626810D 01	-0.22389551D 00
0.20691978D 01	0.96548313D 00	0.24157256D 01	0.18202454D 00
0.28736974D 01	-0.53226689D 00	0.33872082D 01	-0.15008715D 00
0.40570020D 01	0.30187762D 00	0.48051868D 01	0.12455705D 00
0.57750930D 01	-0.17349533D 00	0.68565764D 01	-0.10368646D 00
0.82544461D 01	0.10034343D 00	0.98117875D 01	0.86439624D-01

P# 8.80

0.10376133D 01	-0.16391202D 02	0.10959921D 01	-0.39648914D 00
0.12034982D 01	0.43268074D 01	0.13338873D 01	0.28495036D 00
0.15243327D 01	-0.19192161D 01	0.17436281D 01	-0.22442599D 00
0.20415041D 01	0.99707518D 00	0.23779671D 01	0.18272708D 00
0.28216351D 01	-0.55232922D 00	0.33186082D 01	-0.15092199D 00
0.39652139D 01	0.31492662D 00	0.46867121D 01	0.12547992D 00
0.56195429D 01	-0.18202140D 00	0.66585010D 01	-0.10465587D 00
0.79977126D 01	0.10589385D 00	0.94879566D 01	0.87420577D-01

P# 8.90

0.10367668D 01	-0.16773222D 02	0.10938209D 01	-0.39662725D 00
0.11988013D 01	0.44353165D 01	0.13260681D 01	0.28527406D 00
0.15116956D 01	-0.19731521D 01	0.17252969D 01	-0.22494079D 00
0.20149029D 01	0.10291332D 01	0.23417453D 01	0.18341050D 00
0.27717838D 01	-0.57275305D 00	0.32530200D 01	-0.15173626D 00
0.38776308D 01	0.32826618D 00	0.45738491D 01	0.12638252D 00
0.54716699D 01	-0.19077971D 00	0.64705243D 01	-0.10560657D 00
0.77545902D 01	0.11162582D 00	0.91818362D 01	0.88385165D-01

P# 9.00

0.10359487D 01	-0.17159562D 02	0.10917228D 01	-0.39676087D 00
0.11942666D 01	0.45450770D 01	0.13185240D 01	0.28558766D 00
0.14995185D 01	-0.20277602D 01	0.17076509D 01	-0.22544051D 00
0.19893363D 01	0.10616552D 01	0.23069764D 01	0.18407545D 00
0.27240180D 01	-0.59353665D 00	0.31902671D 01	-0.15253057D 00
0.37939964D 01	0.34189546D 00	0.44662459D 01	0.12726534D 00
0.53309747D 01	-0.19977043D 00	0.62919715D 01	-0.10653892D 00
0.75241385D 01	0.11754036D 00	0.88921710D 01	0.89333611D-01

P# 9.10

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10351576D 01	-0.17550222D 02	0.10896945D 01	-0.39689019D 00
0.11898868D 01	0.46560880D 01	0.13112421D 01	0.28589157D 00
0.14877792D 01	-0.20830388D 01	0.16906556D 01	-0.22592570D 00
0.19647504D 01	0.10946391D 01	0.22735821D 01	0.18472255D 00
0.26782208D 01	-0.61467830D 00	0.31301860D 01	-0.15330550D 00
0.37140740D 01	0.35581354D 00	0.43635776D 01	0.12812886D 00
0.51969981D 01	-0.20899365D 00	0.61222227D 01	-0.10745328D 00
0.73054961D 01	0.12363838D 00	0.86178130D 01	0.90266141D-01

P# 9.20

0.10343924D 01	-0.17945202D 02	0.10877330D 01	-0.39701539D 00
0.11856548D 01	0.47683489D 01	0.13042101D 01	0.28618618D 00
0.14764567D 01	-0.21389863D 01	0.16742789D 01	-0.22639690D 00
0.19410946D 01	0.11280829D 01	0.22414897D 01	0.18535241D 00
0.26342836D 01	-0.63617630D 00	0.30726247D 01	-0.15406162D 00
0.36376442D 01	0.37001951D 00	0.42655443D 01	0.12897358D 00
0.50693166D 01	-0.21844939D 00	0.59607078D 01	-0.10835000D 00
0.70978725D 01	0.12992073D 00	0.83577112D 01	0.91182985D-01
0.99621628D 01	-0.77554213D-01	0.0	0.0

P# 9.30

0.10336519D 01	-0.18344502D 02	0.10858354D 01	-0.39713664D 00
0.11815639D 01	0.48818590D 01	0.12974167D 01	0.28647186D 00
0.14655310D 01	-0.21956014D 01	0.16584907D 01	-0.22685464D 00
0.19183219D 01	0.11619848D 01	0.22106311D 01	0.18596558D 00
0.25921052D 01	-0.65802895D 00	0.30174418D 01	-0.15479949D 00
0.35645036D 01	0.38451240D 00	0.41718681D 01	0.12979994D 00
0.49475395D 01	-0.22813760D 00	0.58069018D 01	-0.10922944D 00
0.69005416D 01	0.13638815D 00	0.81109020D 01	0.92084375D-01
0.96484994D 01	-0.81848934D-01	0.0	0.0

P# 9.40

0.10329352D 01	-0.18748122D 02	0.10839989D 01	-0.39725411D 00
0.11776080D 01	0.49966177D 01	0.12908511D 01	0.28674895D 00
0.14549836D 01	-0.22528827D 01	0.16432627D 01	-0.22729940D 00
0.18963878D 01	0.11963430D 01	0.21809427D 01	0.18656263D 00
0.25515912D 01	-0.68023459D 00	0.29645061D 01	-0.15551962D 00
0.34944636D 01	0.39929121D 00	0.40822920D 01	0.13060841D 00
0.48313060D 01	-0.23805816D 00	0.56603206D 01	-0.11009194D 00
0.67128356D 01	0.14304133D 00	0.78765008D 01	0.92970544D-01
0.93511785D 01	-0.86288780D-01	0.0	0.0

P# 9.50

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10322411D 01	-0.19156062D 02	0.10822209D 01	-0.39736795D 00
0.11737810D 01	0.51126243D 01	0.12845031D 01	0.28701781D 00
0.14447970D 01	-0.23108287D 01	0.16285681D 01	-0.22773165D 00
0.18752511D 01	0.12311556D 01	0.21523648D 01	0.18714409D 00
0.25126536D 01	-0.70279158D 00	0.29136950D 01	-0.15622254D 00
0.34273490D 01	0.41435494D 00	0.39965773D 01	0.13139943D 00
0.47202820D 01	-0.24821090D 00	0.55205171D 01	-0.11093786D 00
0.65341394D 01	0.14988086D 00	0.76536942D 01	0.93841725D-01
0.90690962D 01	-0.90874909D-01	0.0	0.0

P# 9.60

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10315688D 01	-0.19568321D 02	0.10804990D 01	-0.39747832D 00
0.11700775D 01	0.52298783D 01	0.12783631D 01	0.28727873D 00
0.14349547D 01	-0.23694383D 01	0.16143821D 01	-0.22815185D 00
0.18548727D 01	0.12664210D 01	0.21248418D 01	0.18771046D 00
0.24752102D 01	-0.72569829D 00	0.28648945D 01	-0.15690874D 00
0.33629965D 01	0.42970256D 00	0.39145029D 01	0.13217344D 00
0.46141585D 01	-0.25859560D 00	0.53870782D 01	-0.11176752D 00
0.63638857D 01	0.15690729D 00	0.74417338D 01	0.94698154D-01
0.88012385D 01	-0.95608411D-01	0.0	0.0

P# 9.70

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10309174D 01	-0.19984900D 02	0.10788307D 01	-0.39758534D 00
0.11664920D 01	0.53483791D 01	0.12724219D 01	0.28753204D 00
0.14254411D 01	-0.24287101D 01	0.16006809D 01	-0.22856043D 00
0.18352163D 01	0.13021374D 01	0.20983213D 01	0.18826224D 00
0.24391839D 01	-0.74895312D 00	0.28179980D 01	-0.15757871D 00
0.33012545D 01	0.44533301D 00	0.38358628D 01	0.13293086D 00
0.45126487D 01	-0.26921198D 00	0.52596216D 01	-0.11258126D 00
0.62015510D 01	0.16412108D 00	0.72399298D 01	0.95540066D-01
0.85466735D 01	-0.10049031D 00	0.99863347D 01	-0.81163444D-01

P# 9.80

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10302859D 01	-0.20405799D 02	0.10772140D 01	-0.39768915D 00
0.11630197D 01	0.54681262D 01	0.12666712D 01	0.28777802D 00
0.14162416D 01	-0.24886430D 01	0.15874424D 01	-0.22895780D 00
0.18162473D 01	0.13383032D 01	0.20727545D 01	0.18879990D 00
0.24045029D 01	-0.77255452D 00	0.27729060D 01	-0.15823292D 00
0.32419814D 01	0.46124523D 00	0.37604659D 01	0.13367209D 00
0.44154867D 01	-0.28005971D 00	0.51377931D 01	-0.11337942D 00
0.60466511D 01	0.17152263D 00	0.70476456D 01	0.96367696D-01
0.83045427D 01	-0.10552155D 00	0.96879771D 01	-0.81997371D-01

P# 9.90

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10296737D 01	-0.20831016D 02	0.10756466D 01	-0.39778988D 00
0.11596558D 01	0.55891191D 01	0.12611026D 01	0.28801694D 00
0.14073422D 01	-0.25492359D 01	0.15746456D 01	-0.22934436D 00
0.17979337D 01	0.13749169D 01	0.20480951D 01	0.18932389D 00
0.23710997D 01	-0.79650092D 00	0.27295255D 01	-0.15887181D 00
0.31850454D 01	0.47743815D 00	0.36881337D 01	0.13439755D 00
0.43224253D 01	-0.29113843D 00	0.50212642D 01	-0.11416231D 00
0.58987383D 01	0.17911227D 00	0.68642933D 01	0.97181277D-01
0.80740554D 01	-0.11070303D 00	0.94043767D 01	-0.82818904D-01

P# 10.00

ZERO	DERIVATIVE	BEND POINT	FUNCTION
0.10290798D 01	-0.21260554D 02	0.10741267D 01	-0.39788765D 00
0.11563957D 01	0.57113572D 01	0.12557085D 01	0.28824907D 00
0.13987298D 01	-0.26104876D 01	0.15622709D 01	-0.22972048D 00
0.17802449D 01	0.14119769D 01	0.20243000D 01	0.18983466D 00
0.23389112D 01	-0.82079081D 00	0.26877694D 01	-0.15949583D 00
0.31303231D 01	0.49391068D 00	0.36187004D 01	0.13510763D 00
0.42332348D 01	-0.30244774D 00	0.49097301D 01	-0.11493026D 00
0.57573975D 01	0.18689029D 00	0.66893293D 01	0.97981044D-01
0.78544815D 01	-0.11603558D 00	0.91345868D 01	-0.83628192D-01

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EFFECT OF A TRAVELLING ATMOSPHERIC PRESSURE DISTURBANCE ON A NARROW LAKE WITH A DEPTH-DISCONTINUITY

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0. Abstract

The effect of a travelling atmospheric pressure disturbance on the water level of a narrow lake with a depth-discontinuity has been studied using the method of characteristics. Both semi-infinite stress-bands and finite stress-bands were considered. The set-up defined as the difference in the water level between the right and left boundaries does not become periodic any time after the disturbance crosses the lake. For semi-infinite stress-bands the water level at the left side is predominantly negative while it is positive on the right side and it becomes both positive and negative for finite stress-band widths.

1. Introduction

The calculations reported here have been originally made in connection with evaluating the role of the Saguenay River Barrier on the water level changes in the St. Lawrence River when atmospheric disturbances travel over the river. The paper was presented at the 51st Annual Meeting of the American Geophysical Union held at Washington, D.C. in April 1970 and a condensed version is published (Murty, 1971). Neither in the presentation of the paper nor in the paper published are the details of the calculations dealt with. These are discussed in this manuscript report.

The effect of a travelling pressure disturbance on the water level in a non-rotating one-dimensional lake of uniform depth has been studied by Harris (1957). His solutions are valid only for very short time intervals after the disturbance crosses

the lake. Rao (1967) treated the same problem and also the case of a rectangular bay of uniform depth (1969) making use of the method of characteristics. For both these cases he showed that the maximum value of the set-up (set-up is defined as the difference of water level at the right and left boundaries of the lake in the case of an atmospheric disturbance travelling from left to right) is periodic some time after the passage of the disturbance. In fact, for the uniform depth cases this periodicity could be proved a priori from the characteristics' equations even before obtaining solutions for the height fields h_L and h_R at the left and right boundaries.

In the present study I consider the case of a lake with a depth-discontinuity. An example of such a discontinuity in nature is the so-called Saguenay River Barrier (the region where the Saguenay River joins the St. Lawrence River) where the depth falls approximately by a factor of four in a distance of a few kilometers. However, our interest here is not a particular case (in which case direct numerical integration could have been used) but to study this problem generally keeping other conditions as simple as possible. For this reason the problem is solved analytically through the use of the method of characteristics.

An atmospheric disturbance travelling over a body of water imparts energy to the water through pressure-gradient and wind-stress forces. For a small-scale system such as that considered here, the pressure-gradient force can be neglected and thus only the wind-stress is considered. The rate at which

the water absorbs energy from the atmospheric disturbance depends upon how close the propagation speed of the disturbance is to the propagation speed of long gravity waves in the lake because resonant coupling is possible when they are equal or approximately so. The relative sizes of the atmospheric disturbance and the lake are also important because the finite size of the lake has to be taken into account if the size of the atmospheric disturbance is comparable to or greater than the size of the lake. This means wave reflections at the left and right boundaries of the lake as well as at the depth-discontinuity must be taken into account.

Certain simplifying assumptions have been made to keep the mathematical problem tractable and adapt the method of characteristics to obtain the solution. For convenience, the depth-discontinuity is assumed to be at mid-length of the lake. The two portions of the lake on either side of this discontinuity are assumed to have uniform depths D_1 and D_2. Hydrostatic approximation has been made for the pressure field and the nonlinear advective terms are suppressed. Platzman (1958) gave a justification for these approximations based on order of magnitude considerations. Though in principle the method of characteristics could be used with a two-dimensional lake (in which case integration must be performed along characteristic surface rather than along characteristic lines), in practice the calculation is complicated. For this reason a narrow lake is considered here in which the earth's rotation effects could be ignored and no transverse motions are permitted. Also, since

our interest is in the transient aspects of the motion and not in the final steady state, bottom frictional effects also are ignored.

2. Formulation of the Problem and the Method of Solution

Consider a lake of length L containing water of uniform density ρ. The origin of a right-handed cartesian co-ordinate system with the x-axis along the length of the lake and the Z-axis pointing upward is placed on the left boundary at the mean water level. Then the depth-discontinuity is at x=L/2 and the right boundary is at x=L. The vertically integrated equations of motion and continuity for regions to the left of the depth-discontinuity (identified through the subscript 1) and to the right of the discontinuity (identified through the subscript 2) are

$$\frac{\partial M_{1,2}}{\partial t} = -gD_{1,2} \frac{\partial h_{1,2}}{\partial x} + R \tag{1}$$

$$\frac{\partial h_{1,2}}{\partial t} = \frac{\partial M_{1,2}}{\partial x} \tag{2}$$

where

$$M_{1,2} \equiv \int_{-D_{1,2}}^{h_{1,2}} U_{1,2} \, dz \tag{3}$$

Here M is the volume transport through a vertical section, U is the horizontal component of velocity along the length of the lake, h is the deviation of the water level from the equilibrium position and R is a forcing function such that

$$R \equiv \frac{\tau}{\rho} \tag{4}$$

where τ is the wind-stress which is assumed to be known and R is the quantity to be prescribed in the actual calculation.

The boundary conditions of the problem are prescribed through assuming perfect reflections at the left and right boundaries of the lake. Thus

$$\begin{aligned} M_1 &= 0 \text{ at } x = 0 \\ M_2 &= 0 \text{ at } x = L \end{aligned} \tag{5}$$

At the depth-discontinuity, continuity of M and h is required.

$$\left.\begin{aligned} M_1 &= M_2 \\ h_1 &= h_2 \end{aligned}\right\} \text{ at } x = \frac{L}{2} \tag{6}$$

Initially the lake is assumed to be at rest.

$$\left.\begin{aligned} M_1 &= M_2 = 0 \\ h_1 &= h_2 = 0 \end{aligned}\right\} \text{ at } t = 0 \tag{7}$$

Let C_1 and C_2 be the speed of long gravity waves in regions 1 and 2 such that

$$C^2_{1,2} \equiv g\, D_{1,2} \tag{8}$$

Since this is a non-dispersive system (with the earth's rotation ignored) all the gravity waves propagate with the same speed C_1 in region 1 and with the same speed C_2 in region 2.

Addition and subtraction of equations (1) and (2) and then use of (8) gives four equations for the four unknowns M_1, M_2, h_1 and h_2 which can be expressed in the following compact

form:

$$\frac{d}{dt}\left(M_{1,2} \pm C_{1,2}\, h_{1,2}\right) = R$$

for (9)

$$\frac{dx}{dt} = \pm\, C_{1,2}$$

This equation states that the quantity $M_{1,2} \pm C_{1,2} \cdot h_{1,2}$ is constant along the characteristics $dx/dt = \pm\, C_{1,2}$. Since both the regions of the lake have uniform depths individually, the characteristics in both regions are straight lines with slopes C_1 and C_2. Had the nonlinear term $U \frac{\partial U}{\partial x}$ been included in the equation of motion, the method of characteristics would still be applicable but the characteristics would no longer be straight lines. In fact, in such a case they have to be determined as a part of the solution with each integration. Through a scale analysis it can be shown that the omission of the nonlinear term is a minor drawback of the model compared to the omission of bottom friction.

The calculations were performed both for semi-infinite and finite stress-bands travelling over deep to shallow (designated as case I) and shallow to deep water (designated as case II). The left side of Figure 1 shows an atmospheric disturbance of the semi-infinite stress-band type moving from left to right for case II. In these calculations we will assume that the atmospheric disturbance travels only from left to right because the results for case I (case II) in which the disturbance travels from right to left can be easily inferred from the

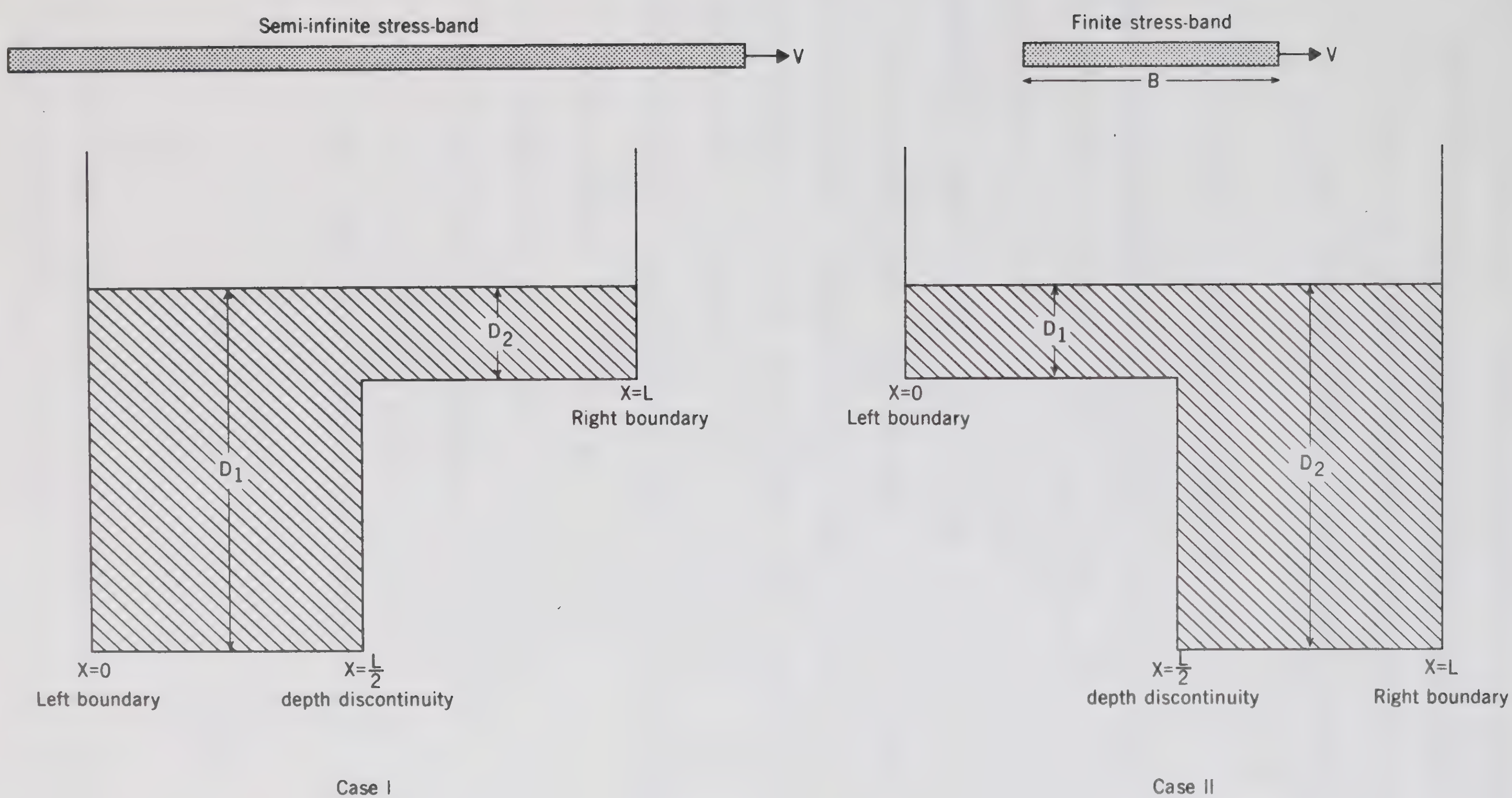


Fig. 1 A schematic representation of the geometry of the problem. Left side shows a semi-infinite stress band moving over deeper to shallower water and right side shows a finite stress band moving over shallower to deeper water.

results corresponding to the travel of the disturbance from left to right for case II (case I). The conditions imposed on the surface stress for the semi-infinite stress-band case which are valid for both cases I and II are

$$R = \begin{cases} 0 \text{ for } t \leq \frac{x}{V} \\ 1 \text{ for } t \geq \frac{x}{V} \end{cases} \tag{10}$$

Next we will express the surface-stress conditions for a finite stress-band of width B. Since this is a linear problem, the solutions for the water levels at the left and right boundaries and for the set-up can be obtained by superposing two semi-infinite stress-bands, one positive and the other negative, both of the same intensity and both moving with the same speed V in the same direction but with their jumps separated by a distance B. The solutions for the negative stress-band can be obtained by replacing the time-variable t by $t-t_o$ in the solutions for the positive stress-band. Here $t_o \equiv B/V$ is the time interval that elapses between the passage of the leading and the trailing edges of the stress-band over a given point of the lake. Hence the conditions imposed on the wind-stress for the finite stress-band case are

$$R = \begin{cases} 0 \text{ for } t \leq \frac{x}{V} \text{ and } t \geq \frac{x}{V} + t_o \\ 1 \text{ for } \frac{x}{V} \leq t \leq \frac{x}{V} + t_o \end{cases} \tag{11}$$

In all these calculations the origin of time t=o is taken at the moment the stress-band is over the left boundary of the lake. The leading edge of a semi-infinite stress-band crosses

the right boundary of the lake at $t_o=L/V$ after which the whole lake is under the influence of the wind-stress. However, for the finite stress-band, as can be seen from equation (11), a given point in the lake distant x from the left boundary is influenced by the wind-stress, only during the interval x/V to $\frac{x}{V} + t_o$.

Although we could have obtained solutions for any depth ratios of the two regions, we did the computations only for two cases, namely $C_1/C_2=2$ and 1/2. The first case is one in which the region to the left of the discontinuity is deeper than the region to the right by a factor of four. In the second case the situation is reversed. Detailed algebraic solutions were obtained for the water level h_L at the left boundary and the water level h_R at the right boundary in terms of the wind-stress R, length L of the lake, the speed V of the atmospheric disturbance and either C_1 or C_2, which ever is greater depending upon the case.

3. The Characteristic Diagrams

The characteristic diagram is a representation of the characteristics (which identify the long gravity waves) in the x-t plane. Several different characteristic diagrams (subcases) are possible for a given case depending upon the ratio of V to C_1 or C_2. Figure 2 shows the characteristic diagram for $V/C_1=3$ and $C_1/C_2=2$. The dashed-dotted line shows the position of the leading edge of the stress-band which is usually referred to as the stress-jump line. The sloping lines in the diagram are the characteristics. Although this diagram is drawn for

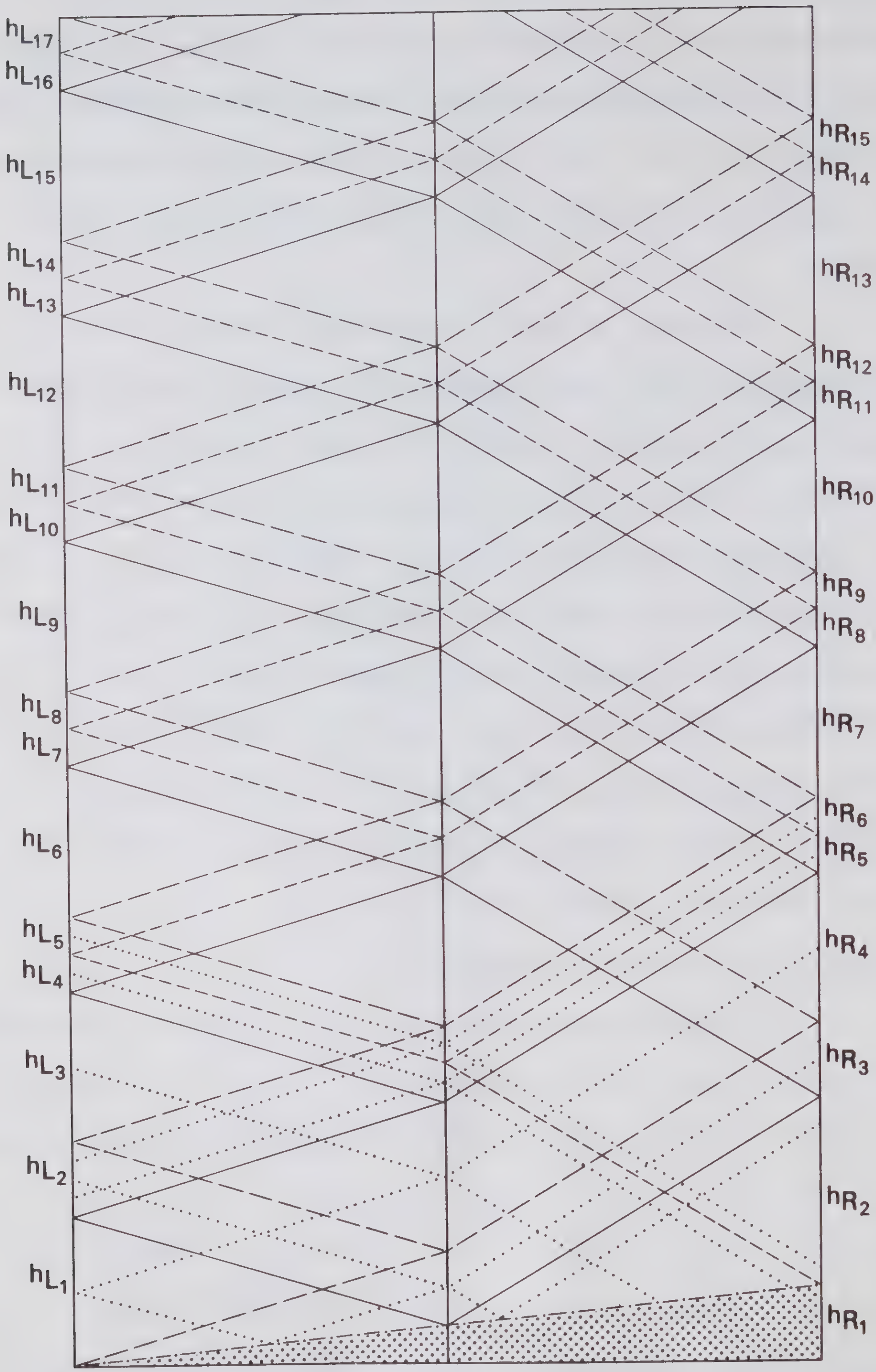


Fig. 2 Characteristic diagram for sub-case 1 ($\infty > \frac{V}{\bar{C}_1} > 2$) of case I. The diagram is actually drawn for $\frac{V}{\bar{C}_1} = 3$.

$V/C_1=3$, it can be seen that the nature of the diagram remains the same for $\infty > \frac{V}{C_1} > 2$ except that as V/C_1 increases, the slope of the stress-jump line decreases until at $V/C_1=\infty$, the stress-jump line merges with the abscissa. Since $V>C_{1,2}$ for this sub-case, the stress-band propagates faster than the free gravity waves in both regions.

When the stress-band arrives over the left boundary of the lake it generates a free gravity wave that propagates in region 1 with uniform speed C_1. Since in this sub-case $V>C_1$, the stress-band arrives at the depth-discontinuity earlier than the free gravity wave that has originated at the left boundary. The stress-band generates two gravity waves at the discontinuity, one travelling with speed C_1 in region 1 and the other travelling with speed C_2 in region 2. These gravity wave systems thus created arrive at the respective boundaries in course of time and get reflected. When the gravity wave that has originated at the left boundary arrives at the discontinuity, it creates another two gravity waves. When the stress-band arrives at the right boundary it creates another system of gravity waves.

Thus, essentially we can distinguish among three families of gravity waves based on the location of the source of their creation: the L-family (originated at the left boundary), D-family (originated at the discontinuity) and the R-family (originated at the right boundary). Using these three families of characteristics, the characteristic diagram could be built up in advance before we attempt to obtain solutions for h_L and h_R.

The different possibilities of intermingling among these three families of characteristics and also the stress-band represent the different sub-cases. In principle, for each case an infinite number of sub-cases are possible. However, from a practical point of view we have limited our discussion to $3 \geq \frac{V}{C_{1,2}} \geq \frac{1}{3}$. It so happens that $\infty > \frac{V}{C_{1,2}} > 2$ is only one sub-case. The possible sub-cases for case I in this range are listed in the Table.

Out of these twelve sub-cases, two sub-cases, namely $\frac{V}{C_1} = 1$ and $\frac{V}{C_2} = 1$ are not considered here because the method of characteristics fails when $V = C_1$ or C_2 and the solutions for h_L and h_R are discontinuous. Figures 3 and 4 show the characteristic diagrams for sub-cases 2 and 5 of case I and Figure 5 shows the diagram for sub-case 12 of case II. In sub-case 2 of case I there are only two families of characteristics because the R-family merges with the L-family at $t = \frac{3}{2} \frac{L}{C_1}$ and $x = \frac{L}{2}$ as can be seen from Figure 3. Since the diagrams are different for each sub-case, the solutions for h_L and h_R are different. Note that if $D_1 = D_2 = D$ then $C_1 = C_2 = C$ and only three sub-cases are possible, namely $\frac{V}{C} \gtrless 1$. If $V/C = 1$, the method of characteristics fails; however, the maximum value of the set-up is continuous (Rao, 1967).

4. Detailed Solutions

The detailed solutions for sub-case 1 of case I will be described here along with the technique of integration along the characteristics to obtain expressions for h_L and h_R. First the solutions on the left boundary of the lake will be obtained.

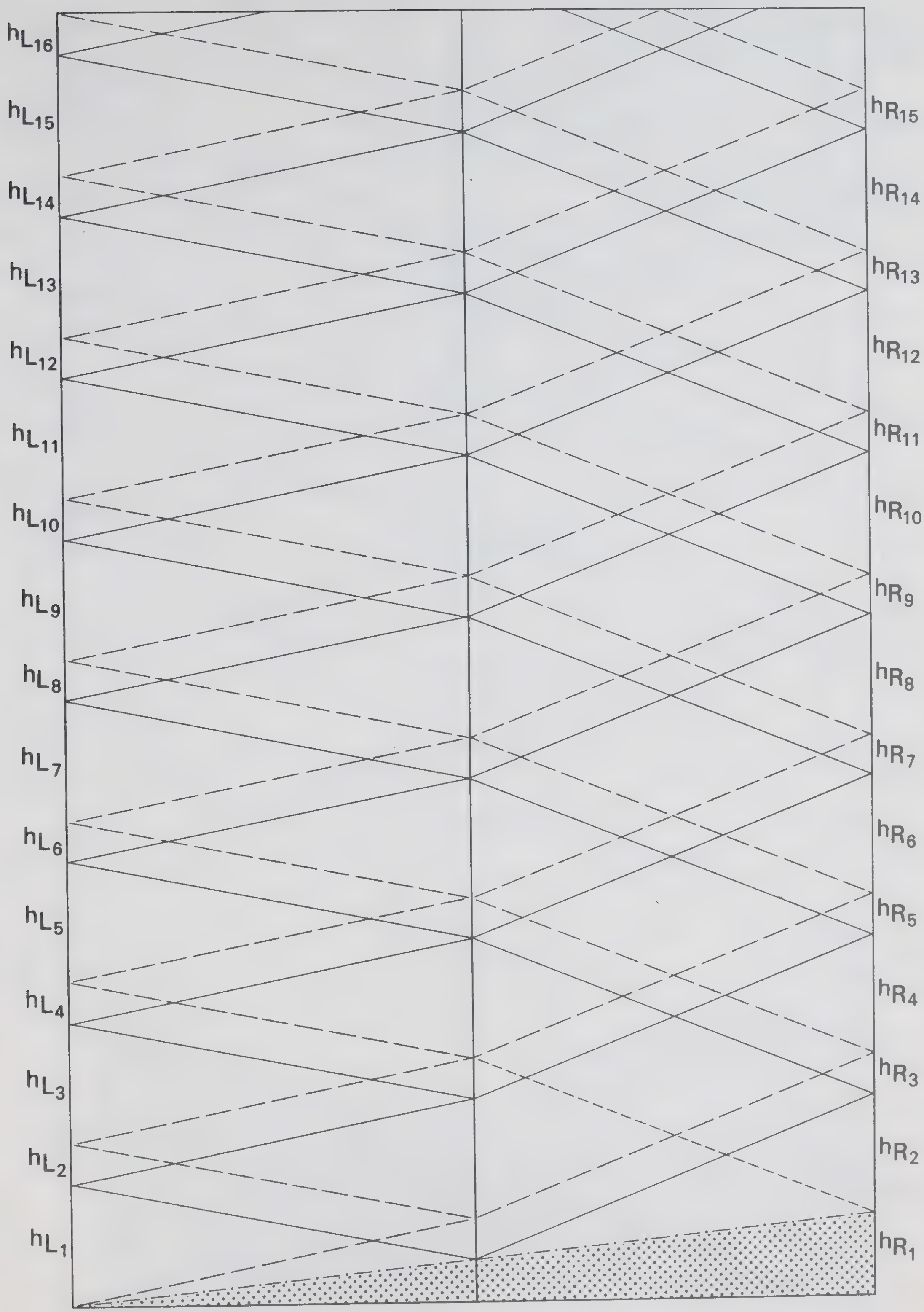


Fig. 3 Characteristic diagram for sub-case 2 ($\frac{V}{C_1}=2$) for case I.

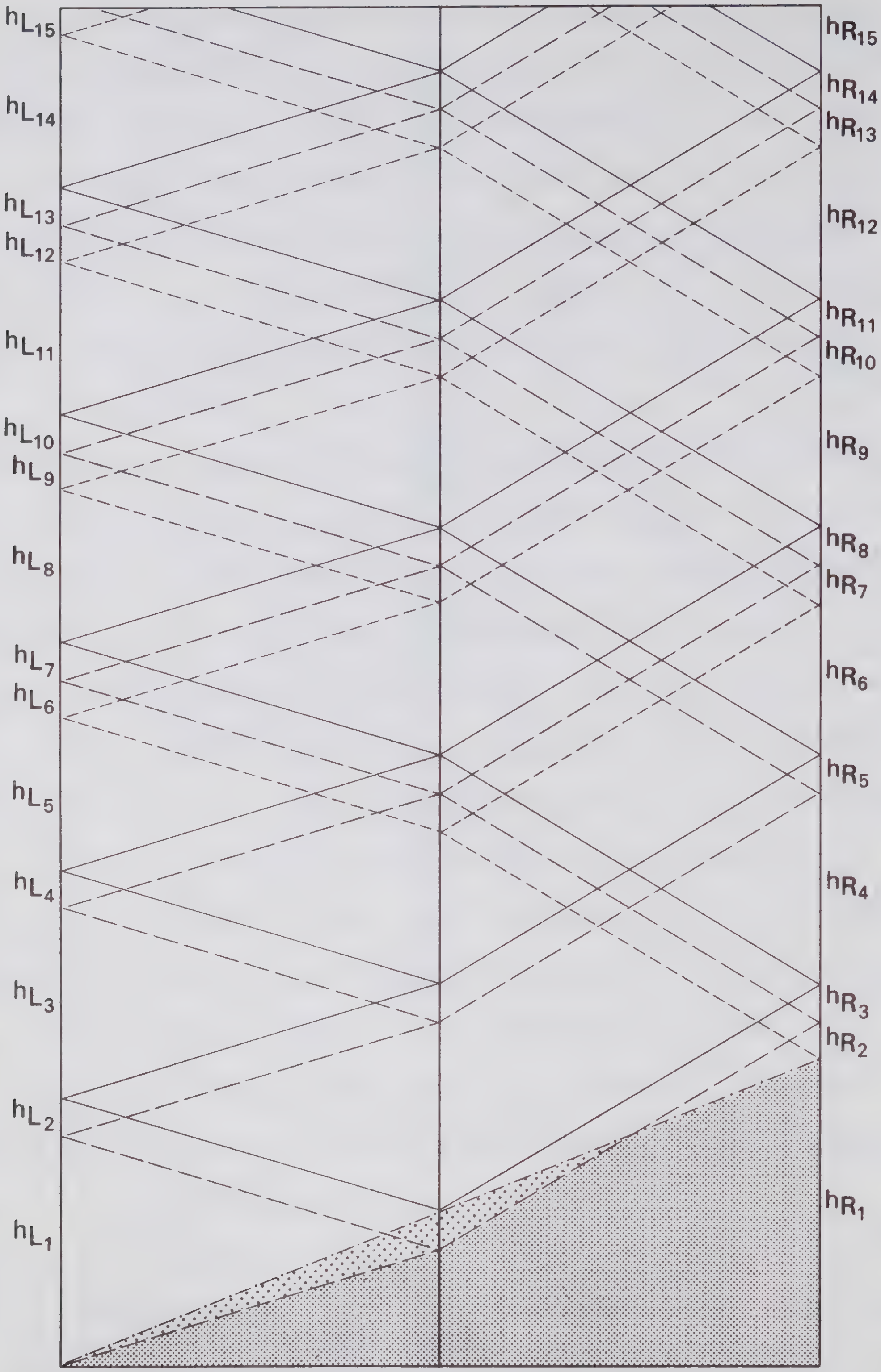


Fig. 4 Characteristic diagram for sub-case 5 $(1 > \frac{V}{C_1} > \frac{2}{3})$ of case I.

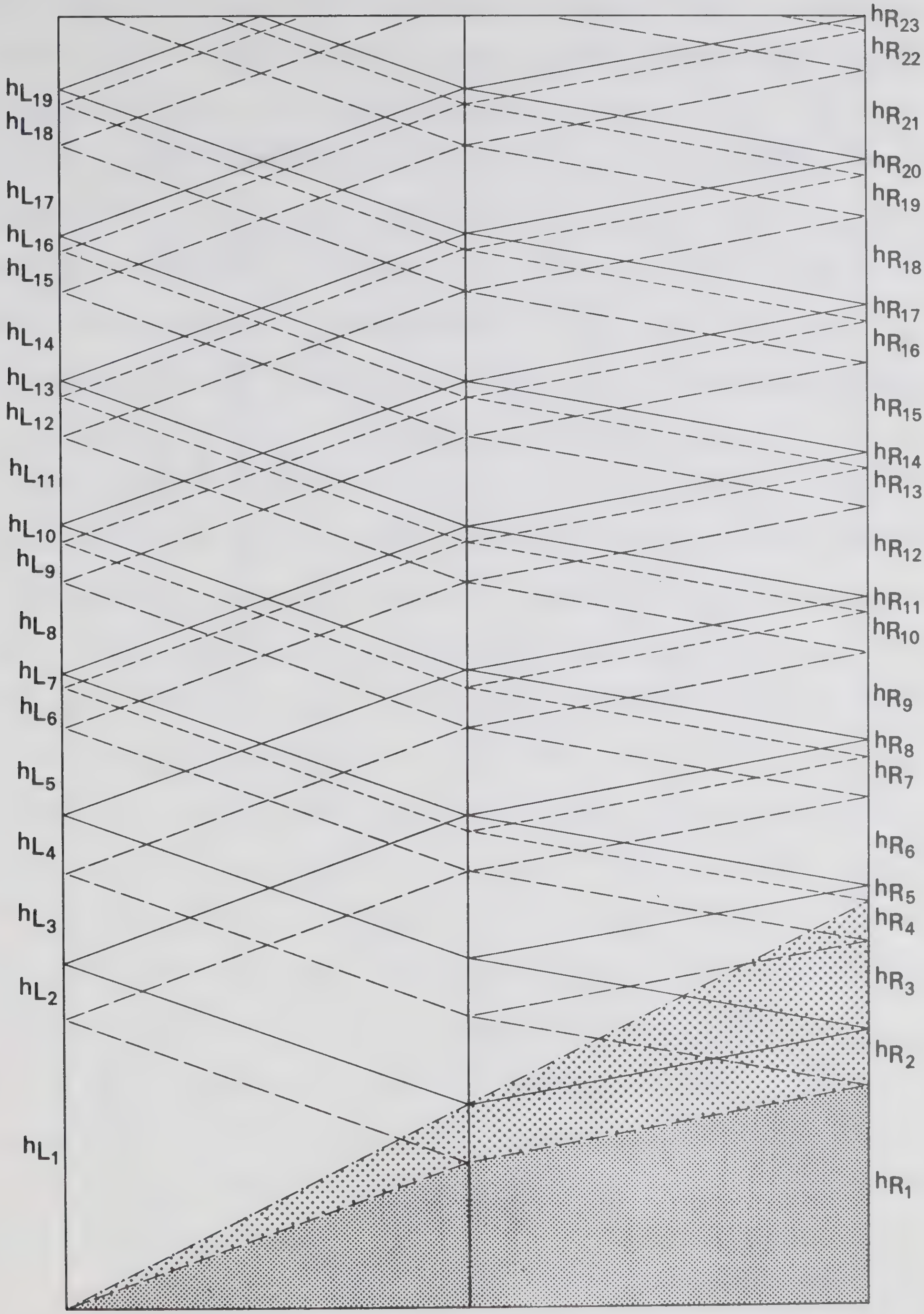


Fig. 5 Characteristic diagram for sub-case 12 ($\frac{V}{C_1} = \frac{1}{3}$) of case II.

Table

Sub-case for case I ($C_1 = 2C_2$)

Sub-case Number	Range of $\frac{V}{C_1}$
1	$\infty > \frac{V}{C_1} > 2$
2	2
3	$2 > \frac{V}{C_1} > 1$
4	1
5	$1 > \frac{V}{C_1} > \frac{2}{3}$
6	$\frac{2}{3}$
7	$\frac{2}{3} > \frac{V}{C_1} > \frac{1}{2}$
8	$\frac{1}{2}$
9	$\frac{1}{2} > \frac{V}{C_1} > 0.4$
10	0.4
11	$0.4 > \frac{V}{C_1} > \frac{1}{3}$
12	$\frac{1}{3}$

Since the lake is assumed to be at rest till the stress-band passes over it, $h_L = 0$ at $t = 0$. As can be seen from Figure 2, an expression for h_L that is valid for $0 \leq t \leq \frac{L}{2V} + \frac{L}{2C_1}$ can be derived through integration of equation (9) along the characteristic AB which intersects the stress-jump line at the point A in region I. The solution for h_L which is valid in the above time interval is designated as h_{L_1}. The solution for h_L valid for the time interval $\frac{L}{2V} + \frac{L}{2C_1} \leq t \leq \frac{L}{C_1}$ and designated by h_{L_2} can be obtained through integration along the characteristics FD, DA, and DL. Of these, FD and DL are negative characteristics (since dx/dt is negative) and DA is a positive characteristic. The solutions h_{L_1} and h_{L_2} should match at the time $t = \frac{L}{2V} + \frac{L}{2C_1}$ which is the upper time limit for h_{L_1} and the lower time limit for h_{L_2}. Otherwise the solutions are discontinuous and have no relevance. Solutions for h_{L_3}, h_{L_4} etc. could be built up through integration along the proper characteristics.

For this sub-case 1 of case I (as for some other but not all sub-cases) the water near the right boundary of the lake is at rest till the stress-band passes over it at $t = \frac{L}{V}$. Thus $h_{R_1} = 0$ for $0 \leq t \leq \frac{L}{V}$. The solution h_{R_2} for $\frac{L}{V} \leq t \leq \frac{L}{2V} + \frac{L}{2C_2}$ can be obtained through integration along the characteristics PD, DL and DA. The solutions for h_{R_3} and h_{R_4} can be constructed in a similar manner by integrating along the proper characteristics.

To illustrate the actual integration process along the characteristics, consider the integration along BA to obtain the solution for h_{L_1}. Equation (9) can be written as

$M_{1_B} - C_1 h_{1_B} - M_{1_A} + C_1 h_{1_A} = R(t_B - t_A)$. From the boundary and initial conditions

$$M_{1_B} = M_{1_A} = h_{1_A} = 0$$

Since $h_{1_B} = h_{L_1}$ the above equation becomes

$$- C_1 \, h_{L_1} = R(t_B - t_A) \tag{12}$$

To determine $t_B - t_A$ one can use the information that A is the point of intersection of the stress-jump line x=V.t with the negative characteristic $dx/dt = -C_1$. This then gives

$$V.t_A = C_1(t_B - t_A)$$

Noting that t_B can be replaced by t where t denotes any time in the interval of $0 \leq t \leq \frac{L}{2V} + \frac{L}{2C_1}$, equation (12) can be written as

$$h_{L_1} = \frac{-R.V.t}{C_1(C_1+V)} \tag{13}$$

This solution satisfies the condition $h_{L_1} = 0$ at $t = 0$.

The solution for h_{L_2} requires integrations along FD, DA and DL. Integration along FA gives

$$M_{1_F} - C_1.h_{1_F} - M_{1_D} + C_1.h_{1_D} = R(t_F - t_D)$$

but

$$M_{1_F} = 0; \; h_{1_F} = h_{L_2} \text{ and } t_F - t_D = \frac{L}{2C_1}$$

Hence

$$h_{L_2} = \frac{-M_{1_D}}{C_1} + h_{1_D} - \frac{RL}{2C_1^2} \tag{14}$$

Integration along DA gives

$$M_{1_D} + C_1.h_{1_D} - M_{1_A} - C_1.h_{1_A} = R(t_D-t_A)$$

Noting that $M_{1_A} = h_{1_A} = 0$ and determining t_D-t_A making use of the property that A is the point of intersection of the stress-jump line with the positive characteristic AD we get

$$M_{1_D} + C_1.h_{1_D} = \frac{R(\frac{L}{2} - V.t_D)}{(C_1 - V)} \tag{15}$$

Integration along DL gives

$$M_{2_D} - C_2.h_{2_D} - M_{2_L} + C_2.h_{2_L} = R(t_D-t_L)$$

But

$$h_{2_D} = h_{1_D};\ h_{2_L} = 0;\ M_{2_L} = 0 \text{ and } M_{1_D} = M_{2_D}$$

Also determining t_D-t_L in a manner as before, the above expression becomes

$$M_{1_D} - \frac{C_1}{2}\,h_{1_D} = \frac{-R(L-2V.t_D)}{(C_1+2V)} \tag{16}$$

Solving (15) and (16) for M_{1_D} and h_{1_D} we get the solutions for the volume transport and the water level at the discontinuity

$$M_{1_D} = \frac{R(L-2V.t_D)\ (2V-C_1)}{2(2V+C_1)\ (C_1-V)} \tag{17}$$

and

$$h_{1_D} = \frac{R(L-2V.t_D)}{(C_1-V)\ (C_1+2V)} \tag{18}$$

Substitution of (17) and (18) into (14) gives the solution for h_{L2}.

$$h_{L2} = \frac{R\left\{L\left(1+\frac{V}{C_1}\right) - 2Vt\right\}(3C_1-2V)}{2C_1(C_1-V)\ (C_1+2V)} - \frac{RL}{2C_1^2} \tag{19}$$

which is valid for

$$\frac{L}{2V} + \frac{L}{2C_1} \le t \le \frac{L}{C_1}$$

The solutions for h_{L_2} and h_{L_1} match correctly at $t = \frac{L}{2V} + \frac{L}{C_1}$. In a similar manner the solutions for h_{L_3}, h_{L_4} etc. and h_{R_1}, h_{R_2} etc. can be constructed. These solutions are summarized below for this sub-case. For i=3 and 4 only

$$h_{L_i} = \frac{1}{3} \underset{\left(t-\frac{L}{C_1}\right)}{h_{L_{i-2}}} - \frac{8}{3}\frac{R}{C_1}\left(\frac{Vt-\frac{L}{2}\left(1+\frac{V}{C_1}\right)}{(2V+C_1)}\right) - \frac{RL}{3C_1^2}$$

$$i = 3:\quad \frac{L}{C_1} \le t \le \frac{L}{2V} + \frac{3}{2}\frac{L}{C_1}$$

$$i = 4:\quad \frac{L}{2V} + \frac{3}{2}\frac{L}{C_1} \le t \le \frac{L}{V} + \frac{3}{2}\frac{L}{C_1}$$

These give the upper and lower time limits during which the solutions for h_{L_3} and h_{L_4} are valid. Note that the upper time limit for h_{L_3} coincides with the lower time limit for h_{L_4}, as it should be. The solution for h_{L_5} is given by

$$h_{L_5} = \frac{1}{3} \underset{\left(t-\frac{L}{C_1}\right)}{h_{L_2}} + \frac{2}{3} \underset{\left(t-\frac{3}{2}\frac{L}{C_1}\right)}{h_{R_2}} - \frac{5}{3}\frac{RL}{C_1^2}$$

for $\frac{L}{V} + \frac{3}{2}\frac{L}{C_1} \le t \le \frac{2L}{C_1}$

For i = 6 and 7 only, the solutions are

$$h_{L_i} = \frac{1}{3} h_{L_{i-3}}\left(t-\frac{L}{C_1}\right) + \frac{2}{3} h_{R_{i-4}}\left(t-\frac{3}{2}\frac{L}{C_1}\right) - \frac{5}{3}\frac{RL}{C_1^2}$$

$$i = 6: \quad \frac{2L}{C_1} \leq t \leq \frac{L}{2V} + \frac{5}{2}\frac{L}{C_1}$$

$$i = 7: \quad \frac{L}{2V} + \frac{5}{2}\frac{L}{C_1} \leq t \leq \frac{L}{V} + \frac{5}{2}\frac{L}{C_1}$$

Now we are in a position to write the general solutions on the left side. For $i \geq 8$ (i takes only integer values)

$$h_{L_i} = \frac{1}{3} h_{L_{i-3}}\left(t-\frac{L}{C_1}\right) + \frac{2}{3} h_{R_{i-5}}\left(t-\frac{3}{2}\frac{L}{C_1}\right) - \frac{5}{3}\frac{RL}{C_1^2}$$

Depending upon the time limits these general solutions fall into three types. For i=8, 11, 14, 17, etc. (general solution of first kind) the lower time limit is

$T'_{8+3J} = \frac{L}{V} + \frac{L}{2C_1}(5 + 2J)$; the upper limit = $T^u_{8+3J} = \frac{L}{C_1}(3 + J)$

where

$J = 0,1,2,3,$ etc.

For i=9, 12, 15, 18 etc. (general solution of second kind)

the lower time limit = $T'_{9+3J} = \frac{L}{C_1}(3 + J)$

the upper time limit = $T^u_{9+3J} = \frac{L}{2V} + \frac{L}{2C_1}(7 + 2J)$

For i=10, 13, 16, 19, etc. (general solution of third kind)

the lower time limit = $T'_{10+3J} = \frac{L}{2V} + \frac{L}{2C_1}(7 + 2J)$

the upper time limit = $T^u_{10+3J} = \frac{L}{V} + \frac{L}{2C_1}(7 + 2J)$

This completes the solutions on the left side of the lake. Next we will summarize the solutions for the water level on the right side of the lake.

$$h_{R1} = 0 \text{ for } 0 \leq t \leq \frac{L}{V}$$

$$h_{R2} = \frac{4R(L-Vt)}{C_1(C_1-2V)} \text{ for } \frac{L}{V} \leq t \leq \frac{L}{2V} + \frac{L}{C_1}$$

$$h_{R3} = \frac{2RV}{C_1}\left\{\frac{2Vt-L(1+\frac{2V}{C_1})}{(C_1+2V)(V-C_1)}\right\} + \frac{2RL}{C_1^2}$$

for $\frac{L}{2V} + \frac{L}{C_1} \leq t \leq \frac{3}{2}\frac{L}{C_1}$

The solutions for h_{R4} and h_{R5} are similar. Thus for i=4 and 5 only

$$h_{R_i} = \frac{4}{3} h_{L_{i-3}\,(t-\frac{3}{2}\frac{L}{C_1})} + \frac{2}{3}\frac{R}{C_1}\left\{\frac{2Vt-L(1+\frac{2V}{C_1})}{(2V+C_1)}\right\} + \frac{8}{3}\frac{RL}{C_1^2}$$

The time limits are given by

$$i = 4: \quad \frac{3}{2}\frac{L}{C_1} \leq t \leq \frac{L}{2V} + \frac{2L}{C_1}$$

$$i = 5: \quad \frac{L}{2V} + \frac{2L}{C_1} \leq t \leq \frac{L}{V} + \frac{2L}{C_1}$$

The solution for h_{R_6} is given by

$$h_{R_6} = \frac{4}{3} h_{L_2}\left(t-\frac{3}{2}\frac{L}{C_1}\right) - \frac{1}{3} h_{R_2}\left(t-\frac{2L}{C_1}\right) + \frac{10}{3}\frac{RL}{C_1^2}$$

for $\frac{L}{V} + \frac{2L}{C_1} \leq t \leq \frac{5}{2}\frac{L}{C_1}$

The solutions for h_{R_7} and h_{R_8} are similar and are given by the following expression. For i=7 and 8 only

$$h_{R_i} = \frac{4}{3} h_{L_{i-4}}\left(t-\frac{3}{2}\frac{L}{C_1}\right) - \frac{1}{3} h_{R_{i-5}}\left(t-\frac{2L}{C_1}\right) + \frac{10}{3}\frac{RL}{C_1^2}$$

The time limits are given by

i = 7: $\frac{5}{2}\frac{L}{C_1} \leq t \leq \frac{L}{2V} + \frac{3L}{C_1}$

i = 8: $\frac{L}{2V} + \frac{3L}{C_1} \leq t \leq \frac{L}{V} + \frac{3L}{C_1}$

Now the general solution on the right side can be written. For $i \geq 9$

$$h_{R_i} = \frac{4}{3} h_{L_{i-4}}\left(t-\frac{3}{2}\frac{L}{C_1}\right) - \frac{1}{3} h_{R_{i-6}}\left(t-\frac{2L}{C_1}\right) + \frac{10}{3}\frac{RL}{C_1^2}$$

Depending upon the time limits, these general solutions fall into three categories. For i=9, 12, 15, 18, etc. (general solution of the first kind) we have the following limits;

the lower time limit $= T'_{9+3J} = \frac{L}{V} + \frac{L}{C_1}(3 + J)$

the upper time limit $= T^u_{9+3J} = \frac{L}{2C_1}(7 + 2J)$

For i=10, 13, 16, 19, etc. (general solution of the second kind)

the lower time limit = $T'_{10+3J} = \frac{L}{2C_1}(7 + 2J)$

the upper time limit = $T^u_{10+3J} = \frac{L}{2V} + \frac{L}{C_1}(4 + J)$

For i=11, 14, 17, 20, etc. (general solution of the third kind)

the lower time limit = $T'_{11+3J} = \frac{L}{2V} + \frac{L}{C_1}(4 + J)$

the upper time limit = $T^u_{11+3J} = \frac{L}{V} + \frac{L}{C_1}(4 + J)$

This completes the solutions on the right side.

The setup is calculated from the relation

$$\Delta h_t = h_{R_t} - h_{L_t}$$

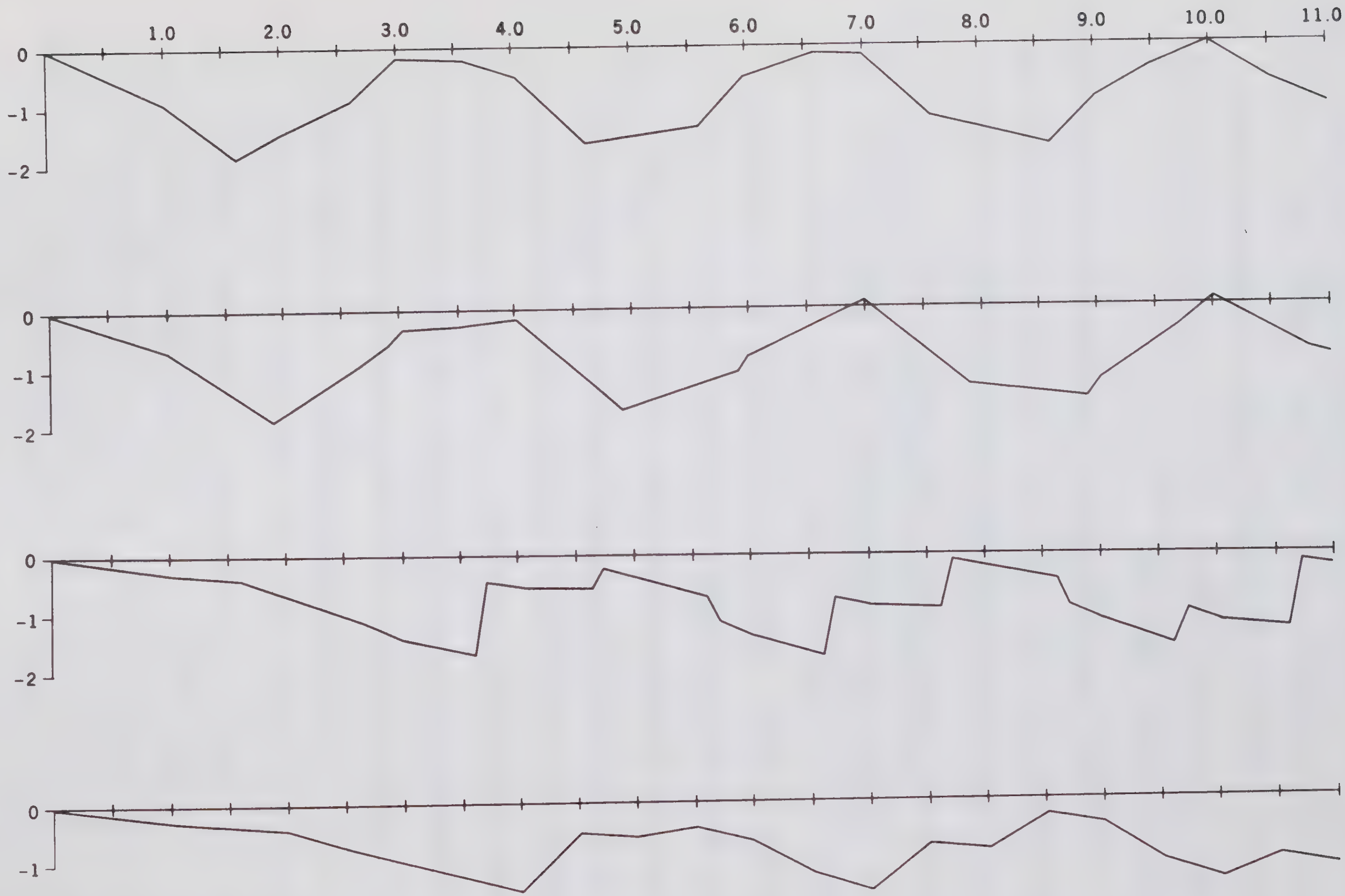
This completes the solutions for sub-case 1 of case I. In a similar manner the solutions for all sub-cases of cases I and II for the semi-infinite stress-band could be determined. The method of obtaining the solutions for the finite stress-band starting from those for the semi-infinite stress-band has been described in Section 2. The solutions for the finite stress-band were determined for band widths of 2L, 1.5L, L, 0.75L, 0.5L and 0.25L where L is the length of the lake.

5. Results

Most of the results of the computations are presented in Figures 6 to 16 all with the same format, namely the amplitude of the water level as ordinate and the time as abscissa. The numbers marked along both the abscissa and the ordinate are nondimensional quantities. To get the corresponding dimensional

numbers, for cases I and II, the ordinate values have to be multiplied by RL/C_1^2 and RL/C_2^2 respectively. The abscissa values have to be multiplied by $L/2C_1$ for case I and $L/2C_2$ for case II. Note that the values of V quoted in the diagrams are non-dimensional, being expressed as a ratio to C_1 for case I and as a ratio to C_2 for case II.

Figure 6 shows the water level variation with time at the left boundary of the lake for a semi-infinite stress-band moving over deep to shallow water for four different speeds. The interesting result is that the water level is predominantly negative. On this diagram the water level becomes positive only for $V = 2.5$ around $t = 7$ and $t = 10$. Since the figures are self-explanatory, lengthy descriptions will be omitted. Since bottom frictional effects are not included in this computation our results become less reliable with increasing time. In every case the computations have been made for V ranging from 10 to 1/3 but only those cases which showed interesting results are included in the diagram. Figure 7 shows the variation of water level at the right boundary of the lake for a semi-infinite stress-band moving over deep to shallow water for four different speeds. It is immediately clear from this diagram that the water level is predominantly positive. In fact, in this diagram, it becomes negative only for $V = 1.5$ around $t = 3.6$. It is clear from Figures 6 and 7 that the water level, even in the absence of friction, does not become periodic, although from some values of V it assumes a quasi-periodic nature.



Semi-infinite stress band HL Case I V=10, 2.5, 0.45 and 1/3

Fig. 6 Variation of water level at the left boundary of the lake in case I for a semi-infinite stress-band.

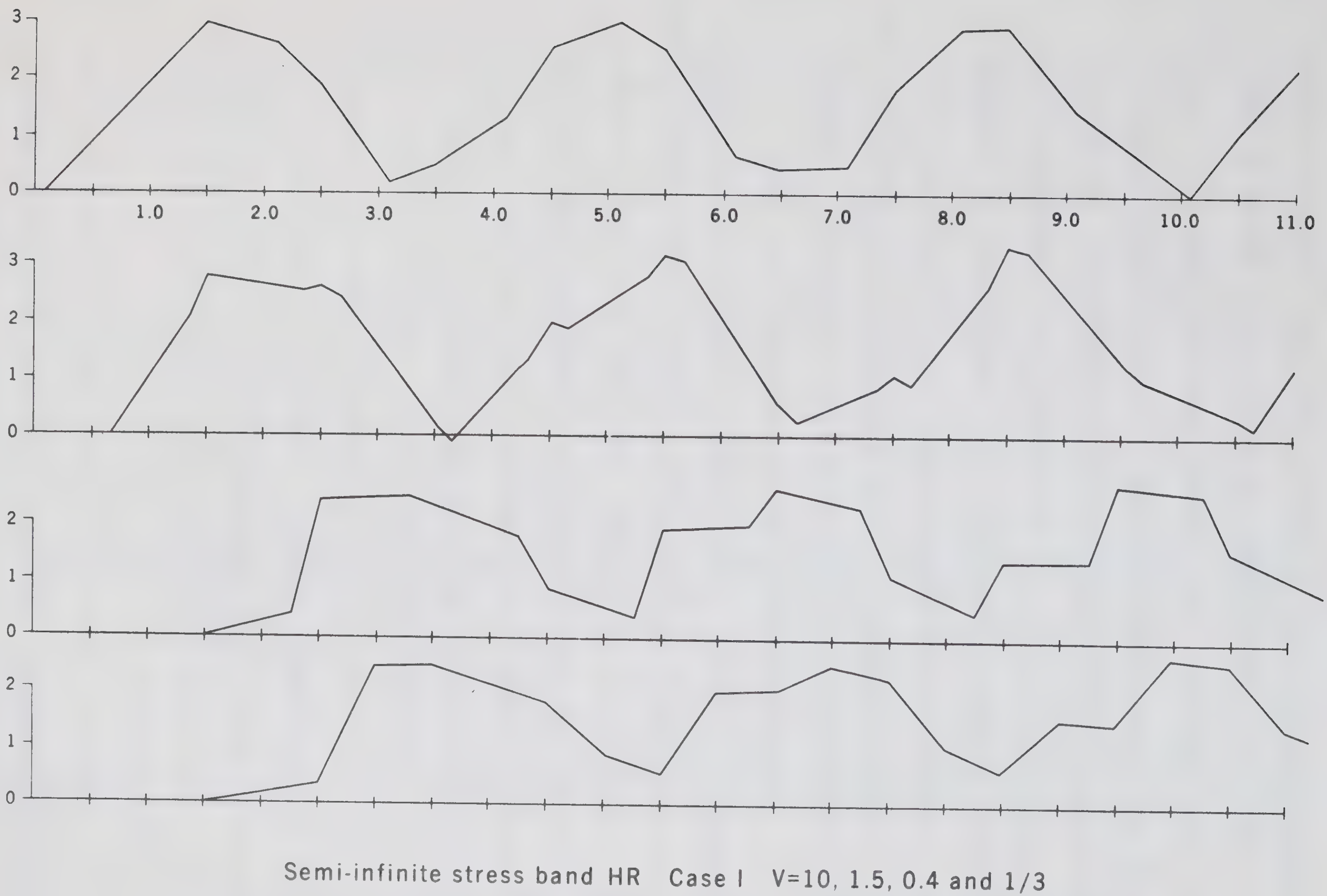


Fig. 7 Variation of water level at the right boundary in case I for a semi-infinite stress-band.

Figure 8 shows the time variation of h_L for a finite stress-band of width 0.4L in case I for four different speeds. First of all it is clear that the water level could be both positive and negative. The second important point is that the nature of the water level variation is drastically different for different speeds of movement of the stress-band. In this diagram the maximum range of the water level variation occurs for V = 0.75 probably because of the possibility of resonant coupling between the stress-band and the gravity wave in region I. The cases of V = 1 and 1/2 cannot be treated by the method of characteristics for both cases I and II. Figure 9 shows the situation at the right boundary of the lake corresponding to Figure 8 for speeds V = 1.5,2/3,0.6 and 0.4. Again, as on the left side the water level is both positive and negative but in contrast to the left side the range of water level variation on the right side is uniformly high for the four different speeds. By comparison with the previous figures it can be seen that the qualitative nature of the variation in case II is quite different from case I. Figure 10 shows the water level h_L in case II for a finite stress-band of width 0.1L for four different speeds. Figure 11 shows the corresponding water level variation at the right boundary of the lake. The range of variation is considerably lower than at the left boundary.

Figure 12 shows h_L in case I for V = 1/3 for four different band widths. It is interesting to note that the amplitude of variation decreases with the decrease of width of

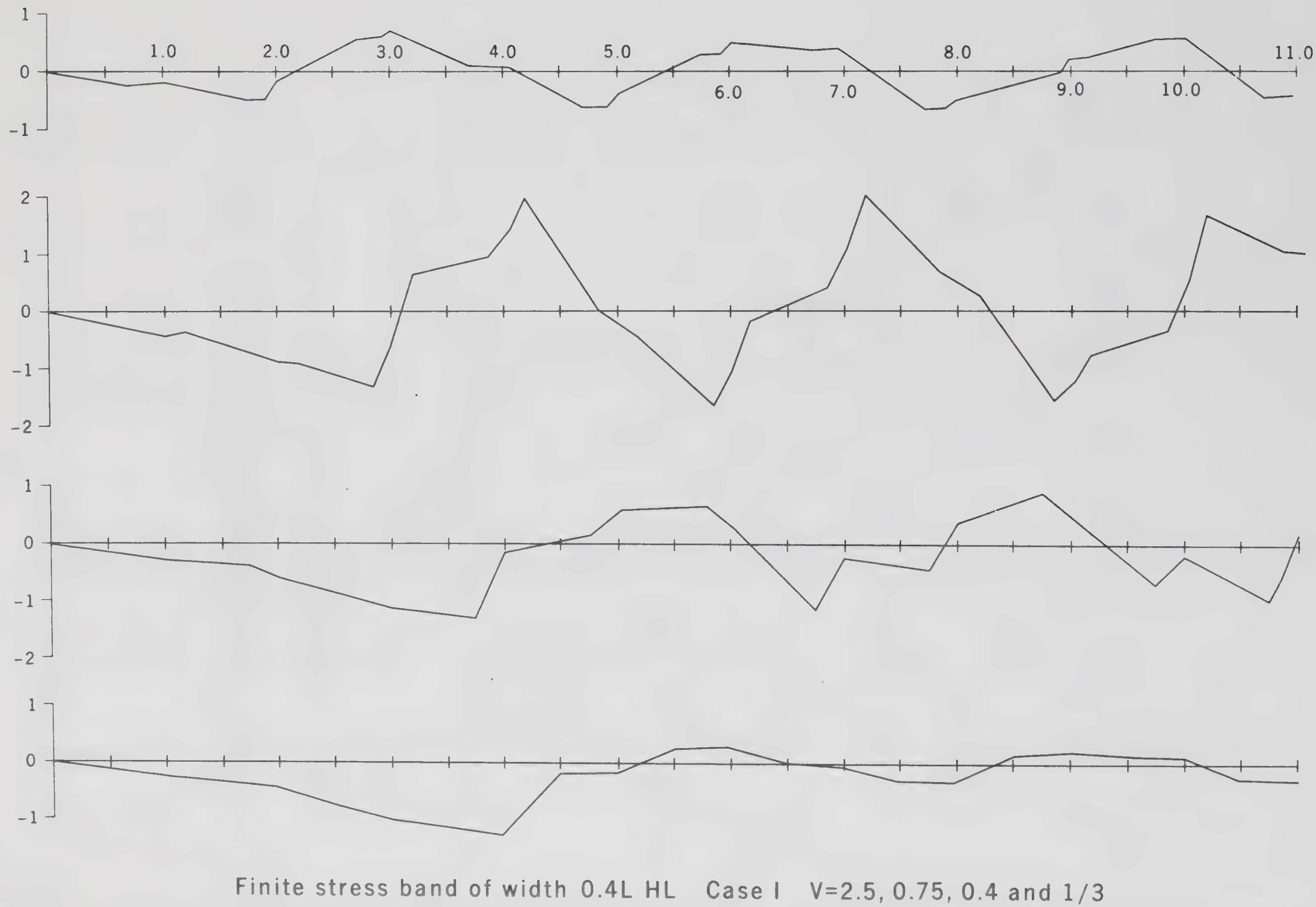
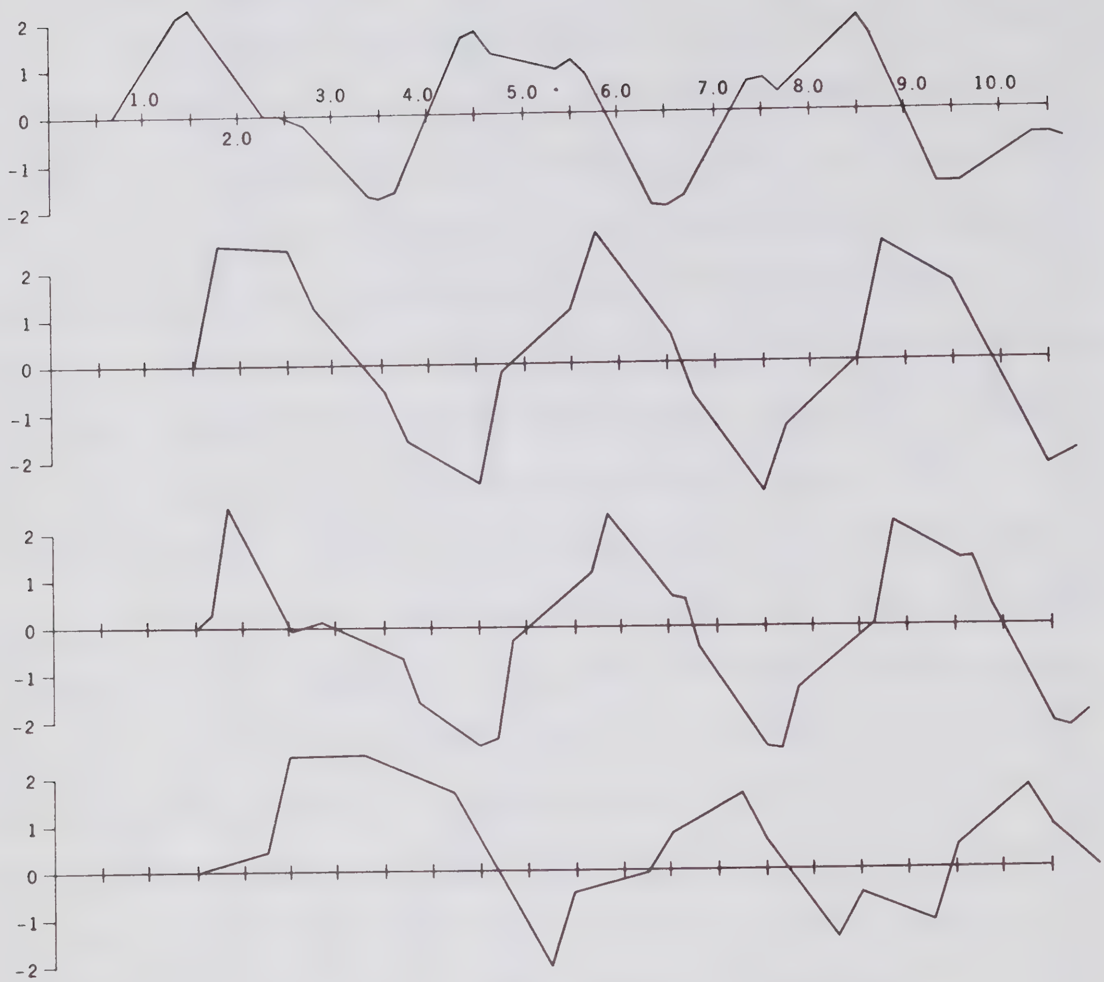
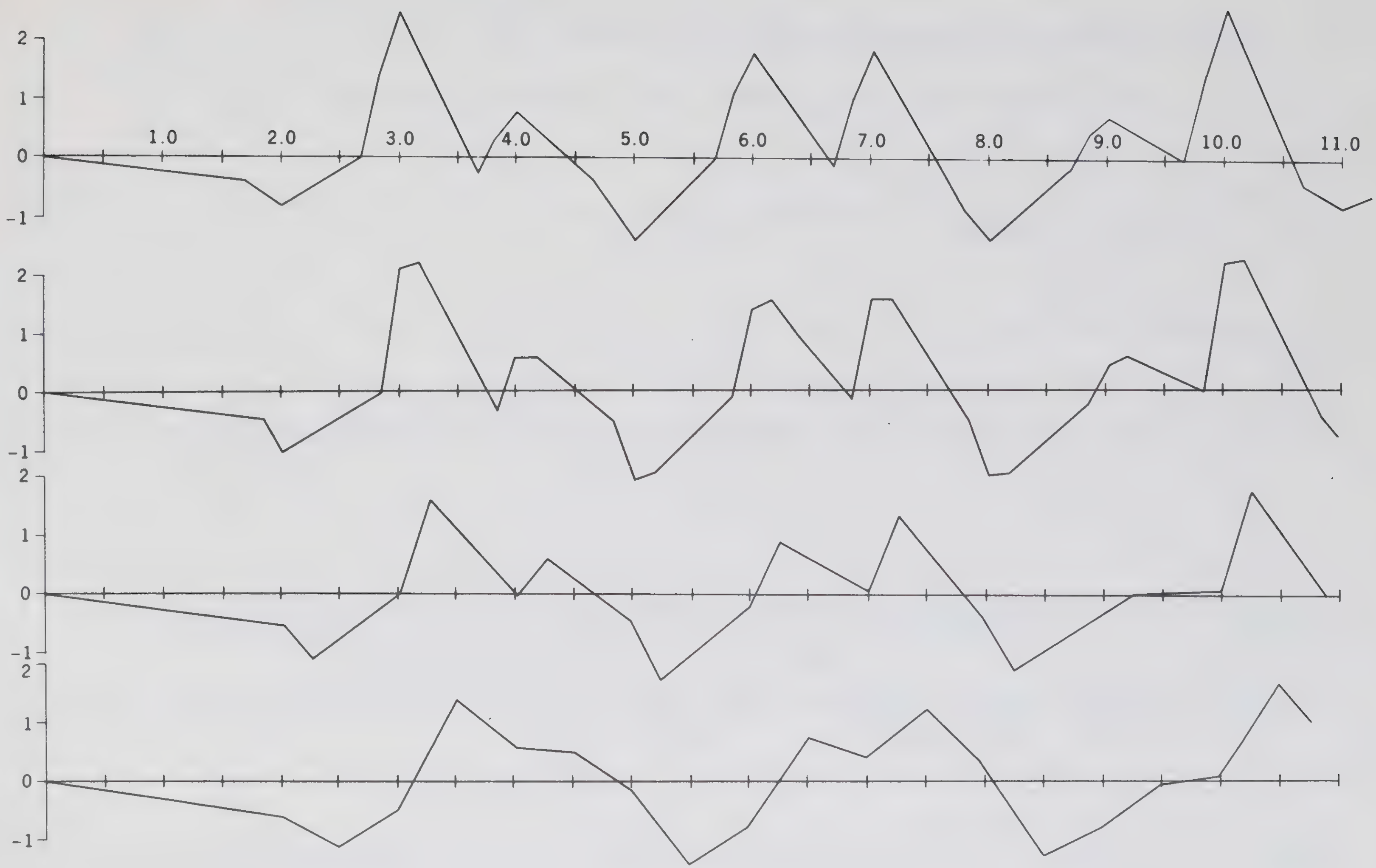


Fig. 8 Variation of water level at the left boundary in case I for a finite stress-band of width 0.4L where L is the length of the lake.



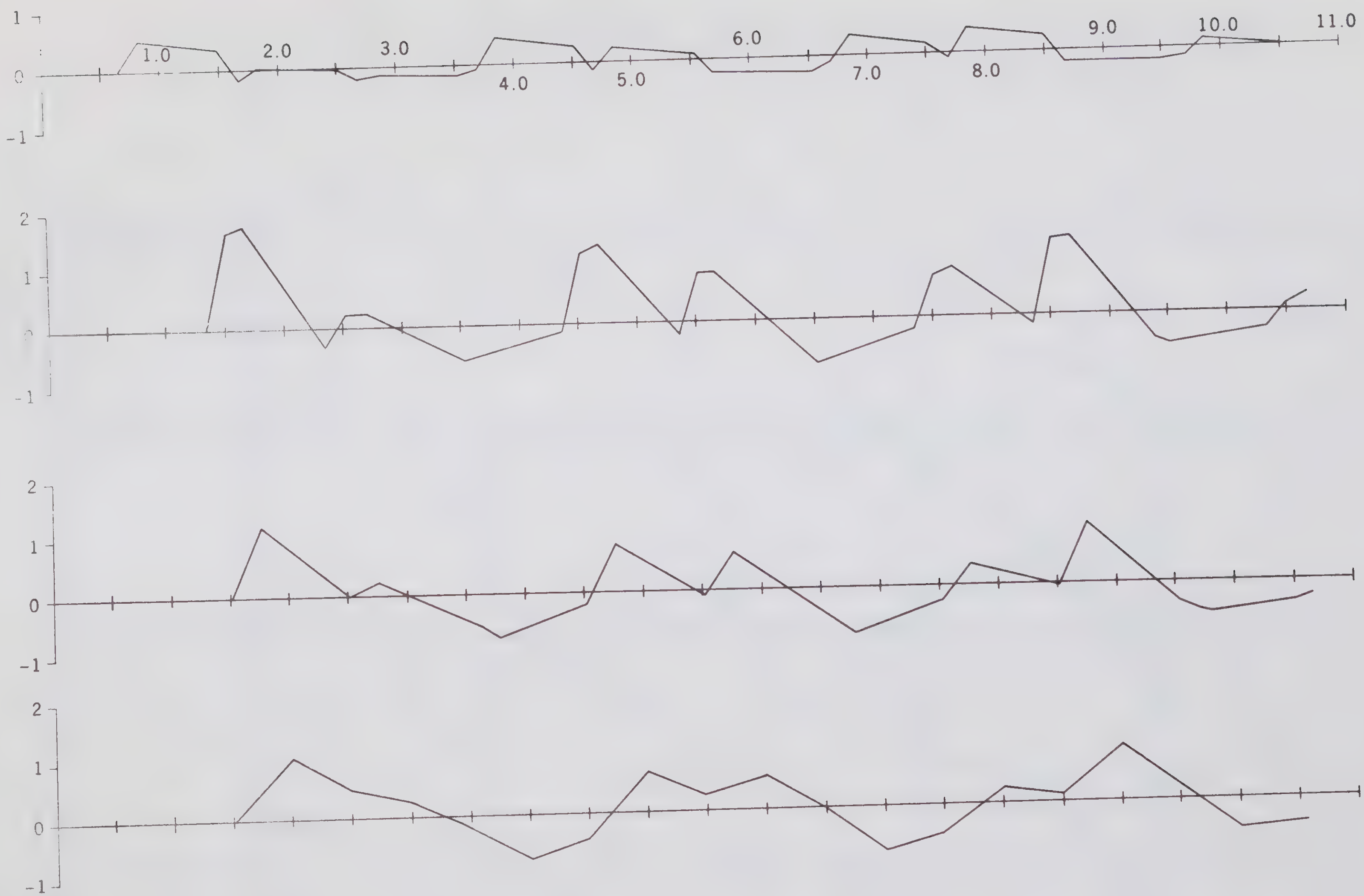
Finite stress-band of width 0.4L HR Case I V=1.5, 2/3, 0.6 and 0.4

Fig. 9 Water level variation at the right boundary in case I for a finite stress-band of width 0.4L.



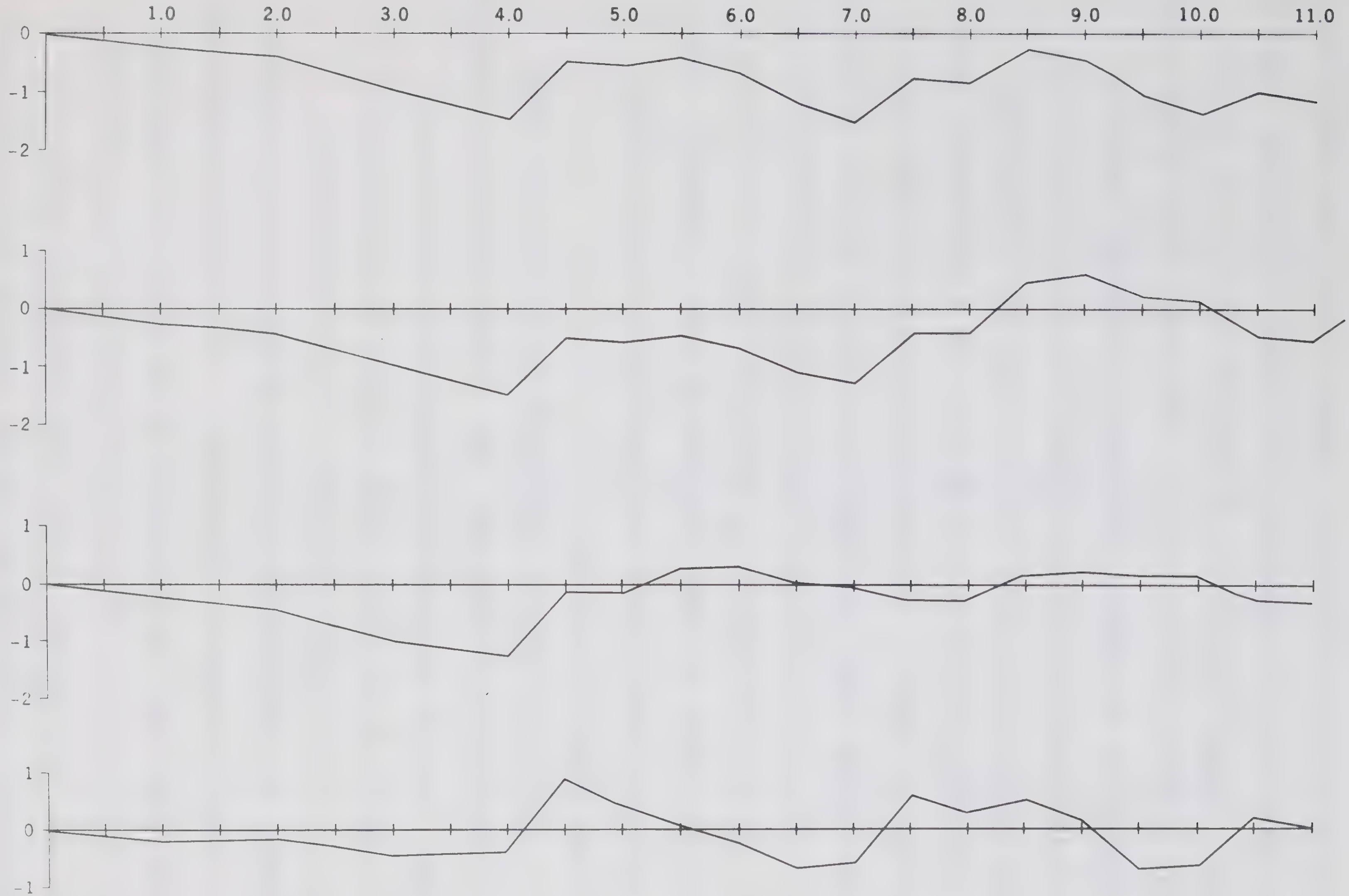
Finite stress band of width 0.1L HL Case II V=0.75, 0.6, 0.4 and 1/3

Fig. 10 Water level variation at the left boundary in case II for a finite stress-band of width 0.1L.



Finite stress band of width 0.IL HR Case II V=1.5, 0.6, 0.4 and 1/3

Fig. 11 Water level variation at the right boundary in case II for a finite stress-band of width 0.1L.

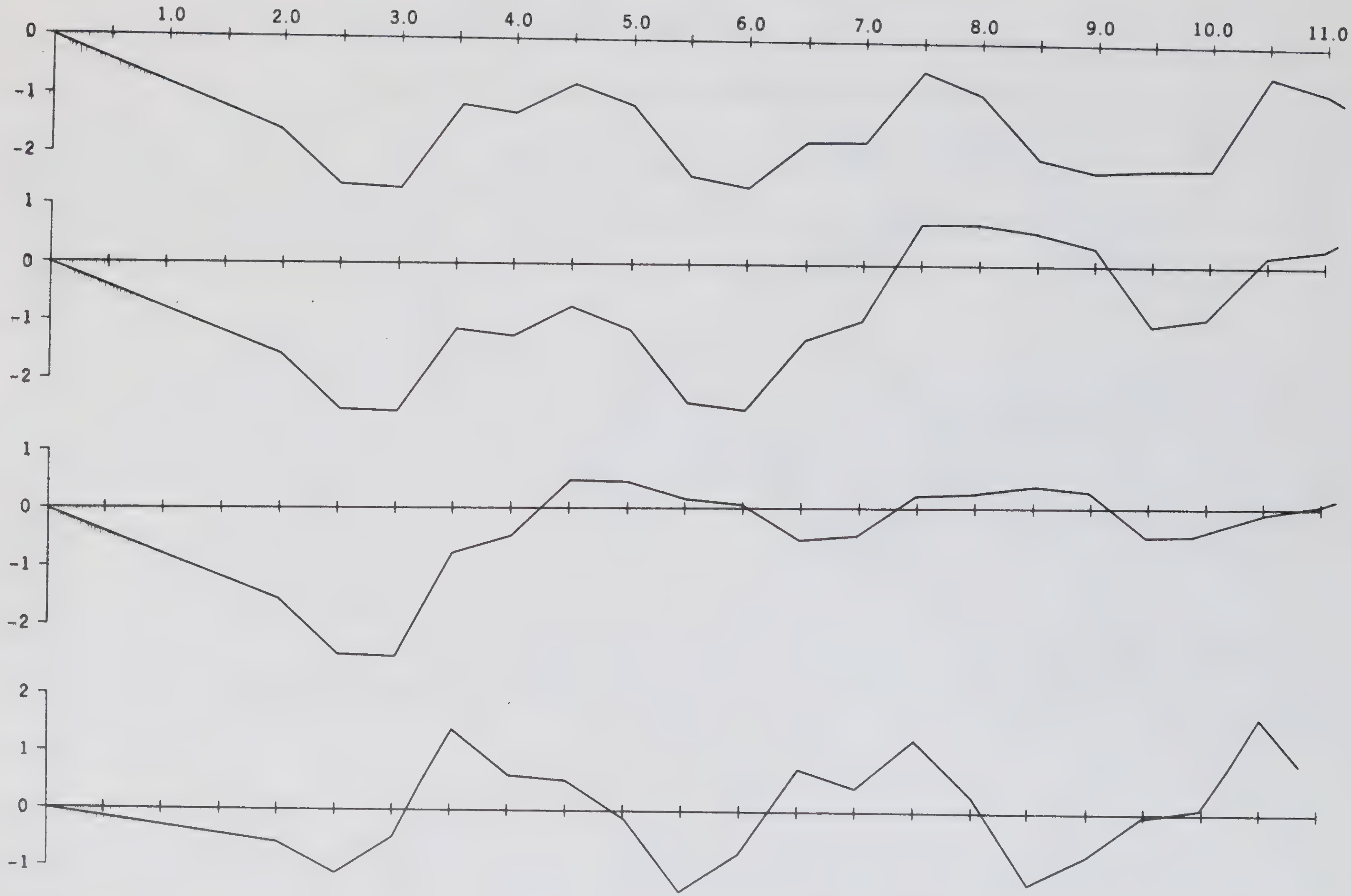


HL Case I V=1/3 Band width 00, 0.8L, 0.4L and 0.1L

Fig. 12 Variation of water level at the left boundary in case I for different band widths.

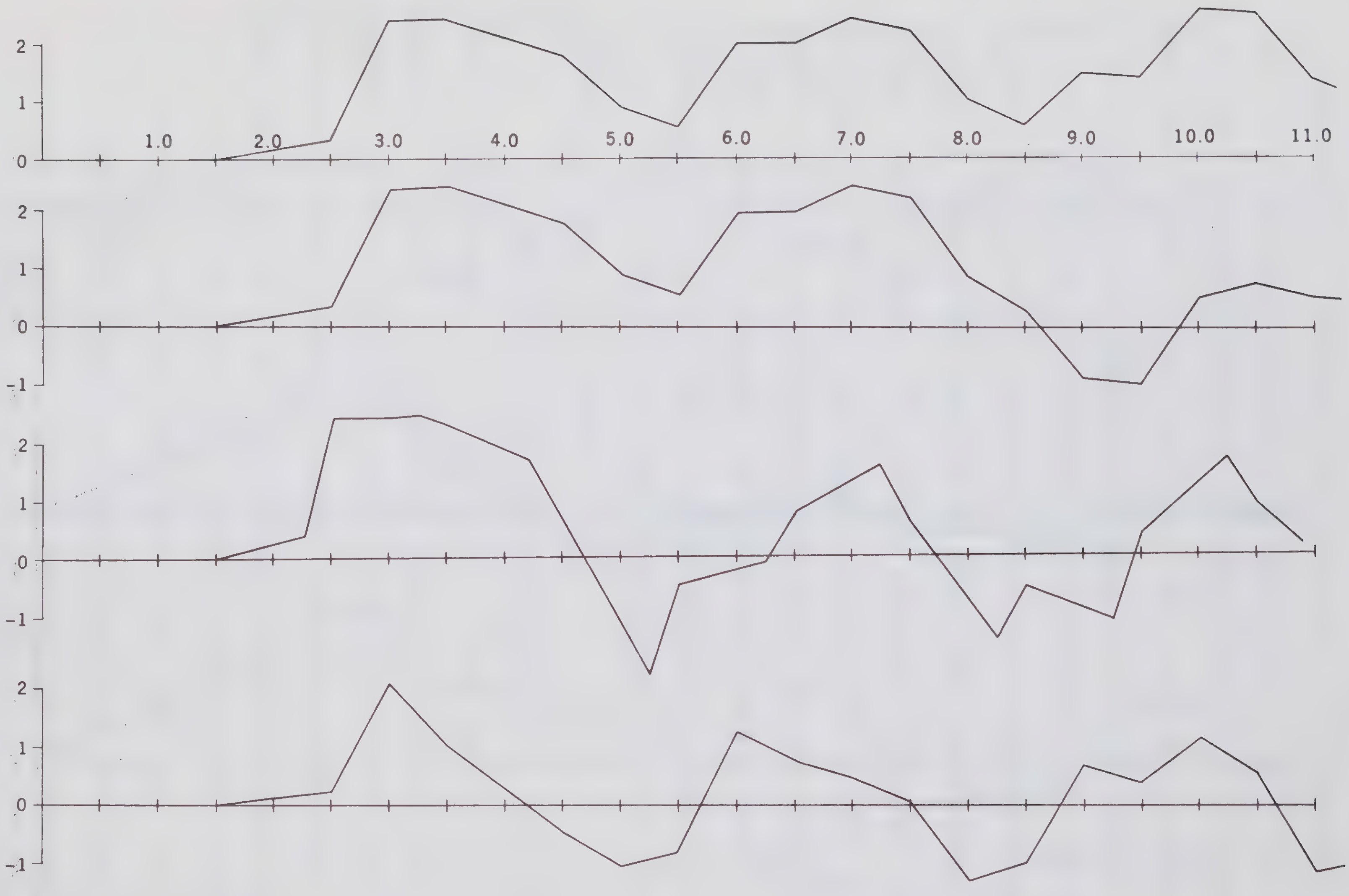
the band until 0.4L and it increases again slightly for 0.1L. Thus the width of the band with respect to the dimension of the lake is important in determining the amount of resonant coupling possible. Figure 13 shows h_L in case II for V = 1/3 for the same band widths as in Figure 12. The range of variation on the whole appears to be greater in Figure 13 than in Figure 12. Thus the amplitude of the water level variation at the left boundary appears to be more for a stress-band moving over shallow to deep water compared to that for a band moving over deep to shallow water. By analogy we expect the amplitude of variation at the right boundary to be greater in case I than in case II. This is exactly the case as can be seen from Figures 14 and 15. Figure 16 compares h_L and h_R in cases I and II for a finite stress-band of width 0.4L moving with a speed V = 1/3. It can be seen that h_R for case I and h_L for case II have higher amplitude of variation.

In conclusion the important results of this study could be summarized as follows. For a semi-infinite stress-band the water level at the left side could be predominantly negative while at the right side it could be predominantly positive. As the speed of movement of the stress-band approaches the speed of long gravity waves either in regions I or II, resonant coupling is possible. For finite stress-bands of width about half the dimension of the lake, the amplitude of the water level variation could be smaller than that for either wider or narrower bands. The additional complexity in reflections introduced due to the depth discontinuity does



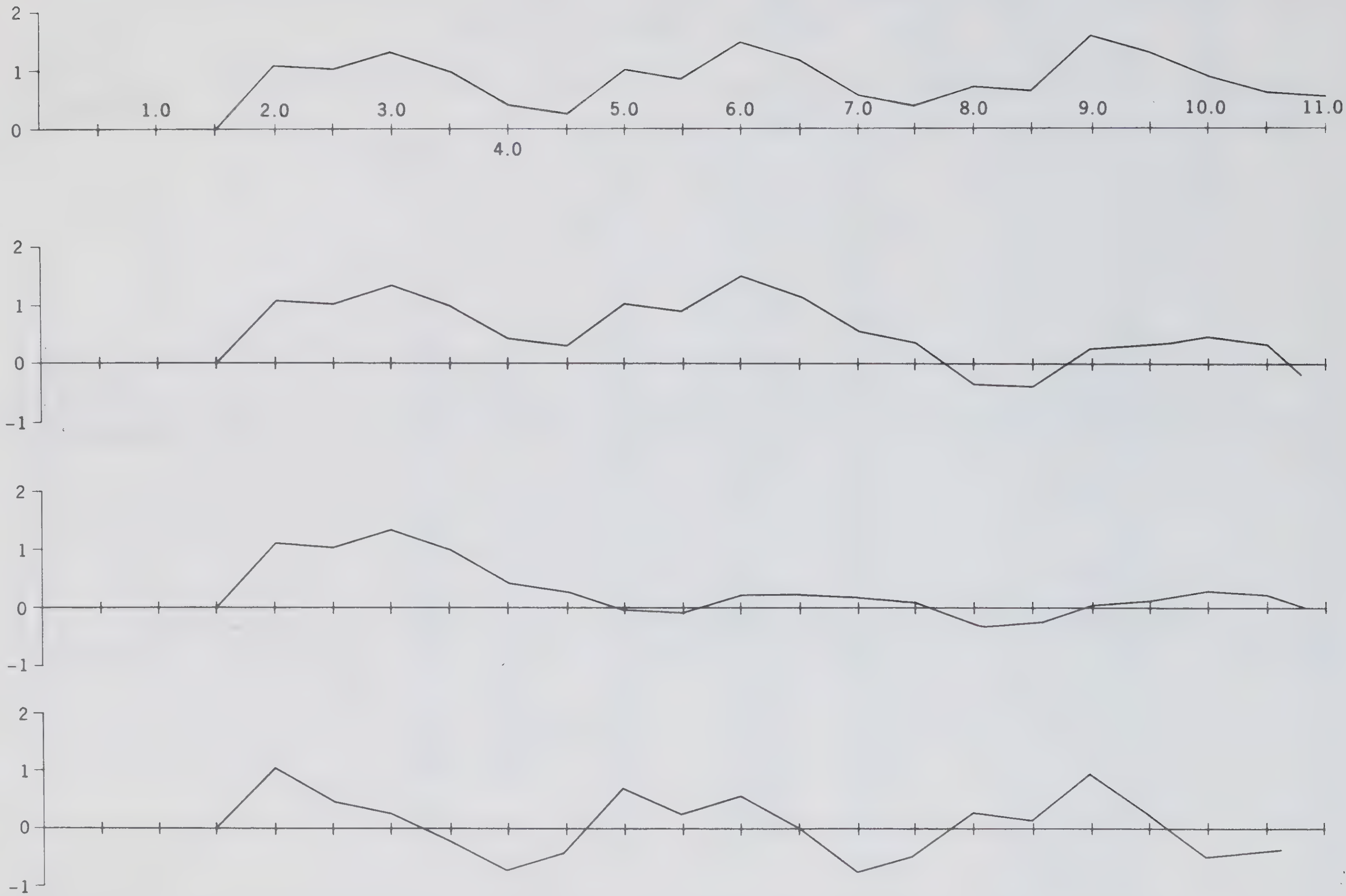
HL Case II V=1/3 Band width 00, 0.8L, 0.4L and 0.1L

Fig. 13 Variation of water level at the left boundary in case II for different band widths.



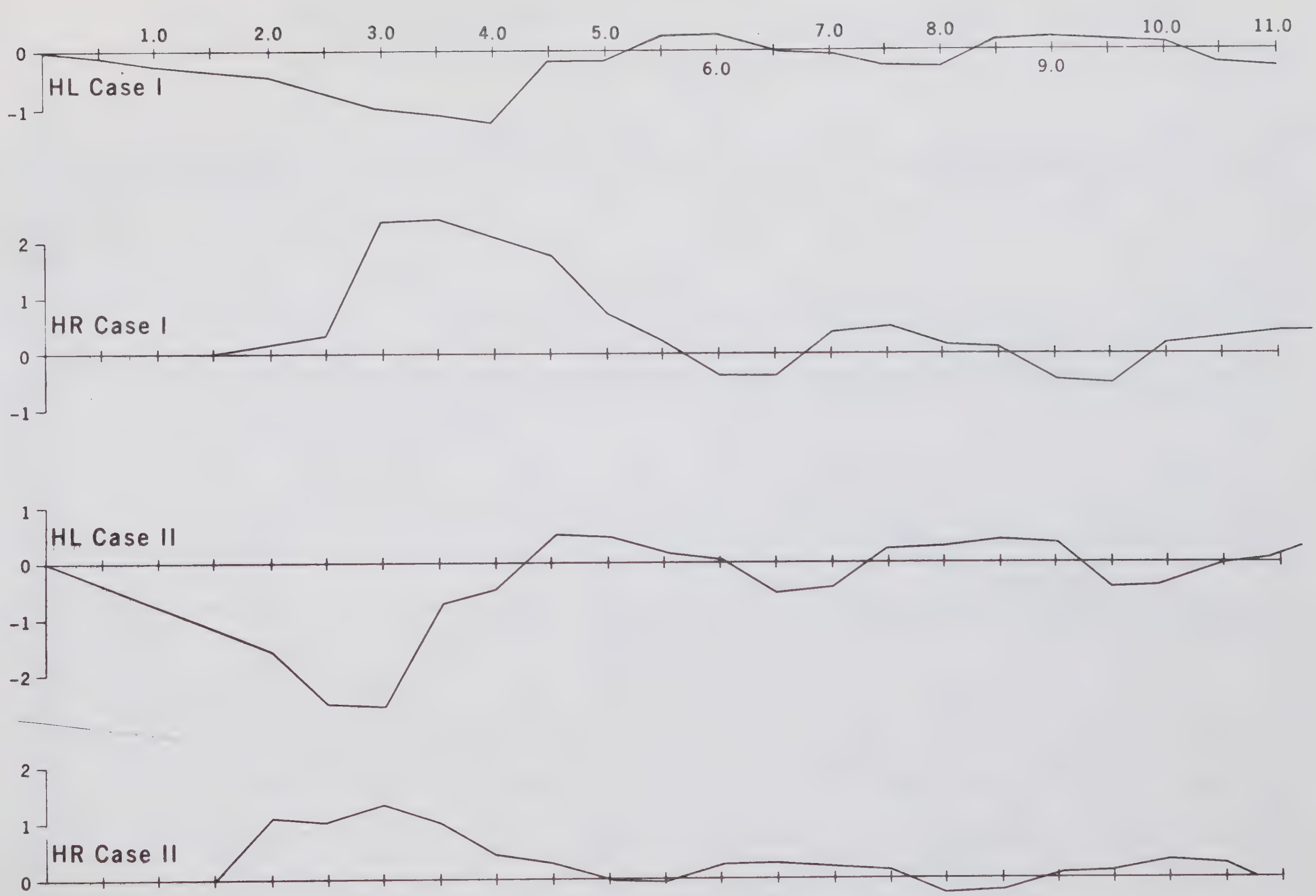
HR Case I V=1/3 Band width 00, 0.8L, 0.4L and 0.1L

Fig. 14 Variation of water level at the right boundary in case I for different band widths.



HR Case II V=1/3 Band width of 00, 0.8L, 0.4 and 0.IL

Fig. 15 Variation of water level at the right boundary in case II for different band widths.



Finite stress band of width 0.4L V=1/3

Fig. 16 Variation of water level at left and right boundaries in cases I and II for a finite stress-band of width 0.4L.

not allow exact periodicity of the setup variation even in the absence of friction although quasi-periodicity is achieved in some cases.

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JAMES BAY

MANUSCRIPT
REPORT SERIES

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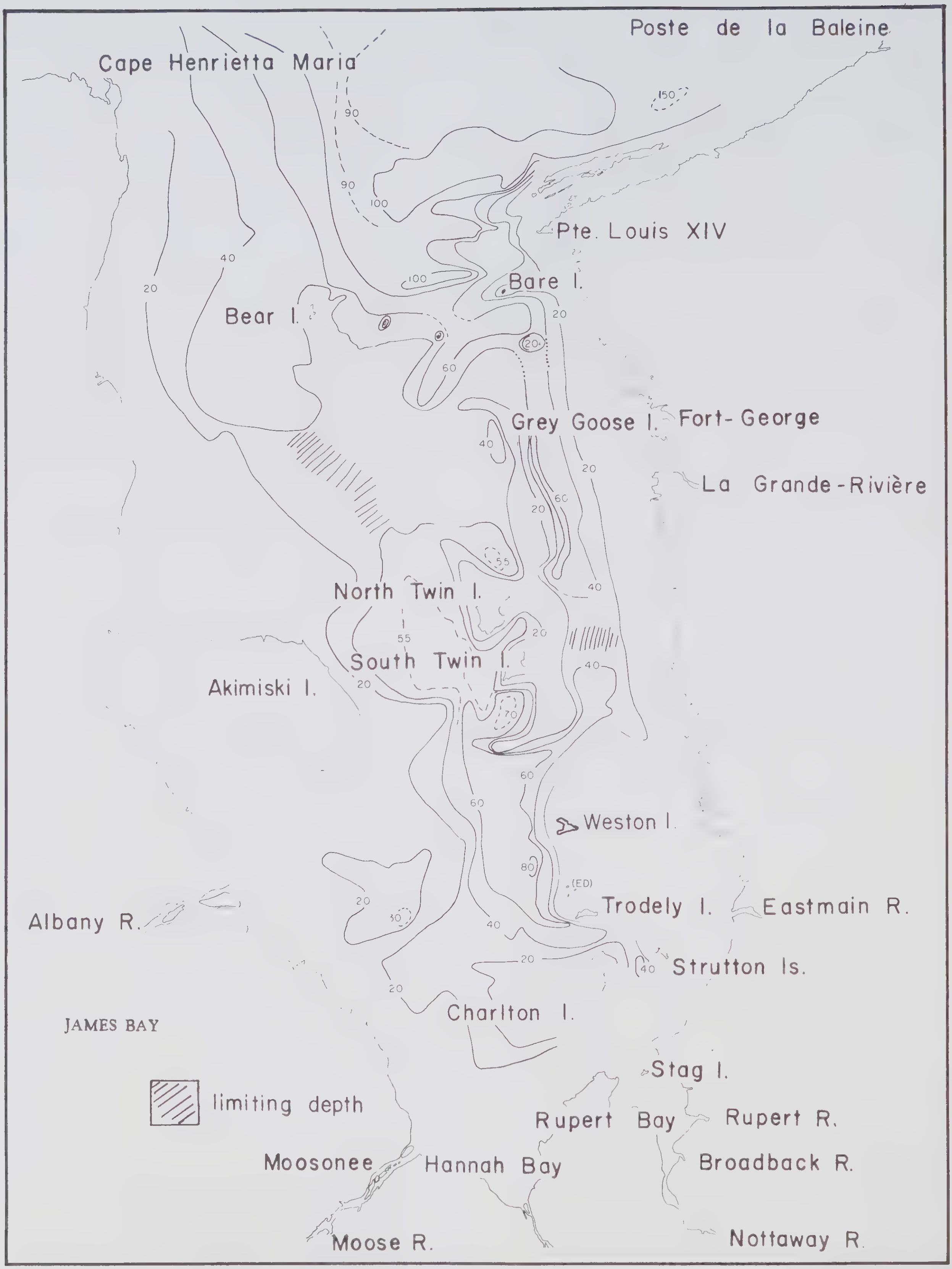
1972



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T.S. Murty

1972

Foreword

The reports contained in this number of our manuscript report series were prepared in response to a request to Federal authorities from the government of the Province of Quebec for an initial consideration of the possible influence of the James Bay Project on the environment; the three reports are directed to the marine environment and comprise but a portion of the total activity. Specifically, the study was to be directed to a question concerning the area to be developed first, i.e. whether construction should begin on the southern rivers or on the northern rivers. It was realized from the outset that we could not, for a number of reasons, contribute significantly to the resolution of the question; nevertheless, it was deemed useful that we attempt to understand the oceanography of James Bay as well as available data would permit.

A.E. Collin,
Director,
Marine Sciences Branch.

December 2, 1971
Ottawa

Contents

On the Oceanography

of

James Bay

F.G. Barber

Contents

1. Introduction

The examination of certain aspects of the oceanography of James Bay (frontispiece) contained in the following was carried out in a consideration of the possible influence on the water of James Bay of the development of the hydroelectric potential there. The work constitutes more of a review or reappraisal of knowledge of the region than it does an impact study, but was believed necessary in our initial approach to the problem. The review aspect emphasizes the importance to James Bay of processes operating in adjacent waters (Figure 1) and the fact that all of these waters generally reflect to similar degree the influence of similar internal and external factors; in consideration of this, the word "system" has been used to include all the region (Barber, 1967, p20).

It was not surprising that an assessment of the influence of the development would prove almost intractable for, while data on the region are few, our understanding of both the qualitative and quantitative changes which may occur through oceanographic processes is limited. The consideration applies to the system as a whole as well as to James Bay, for not only are certain major features not understood but also much of the "understanding" is not well-founded; we have a great deal to learn about our northern waters generally. Part of the understanding could result through a study of the changes actually wrought on the system by the project. It is recommended therefore

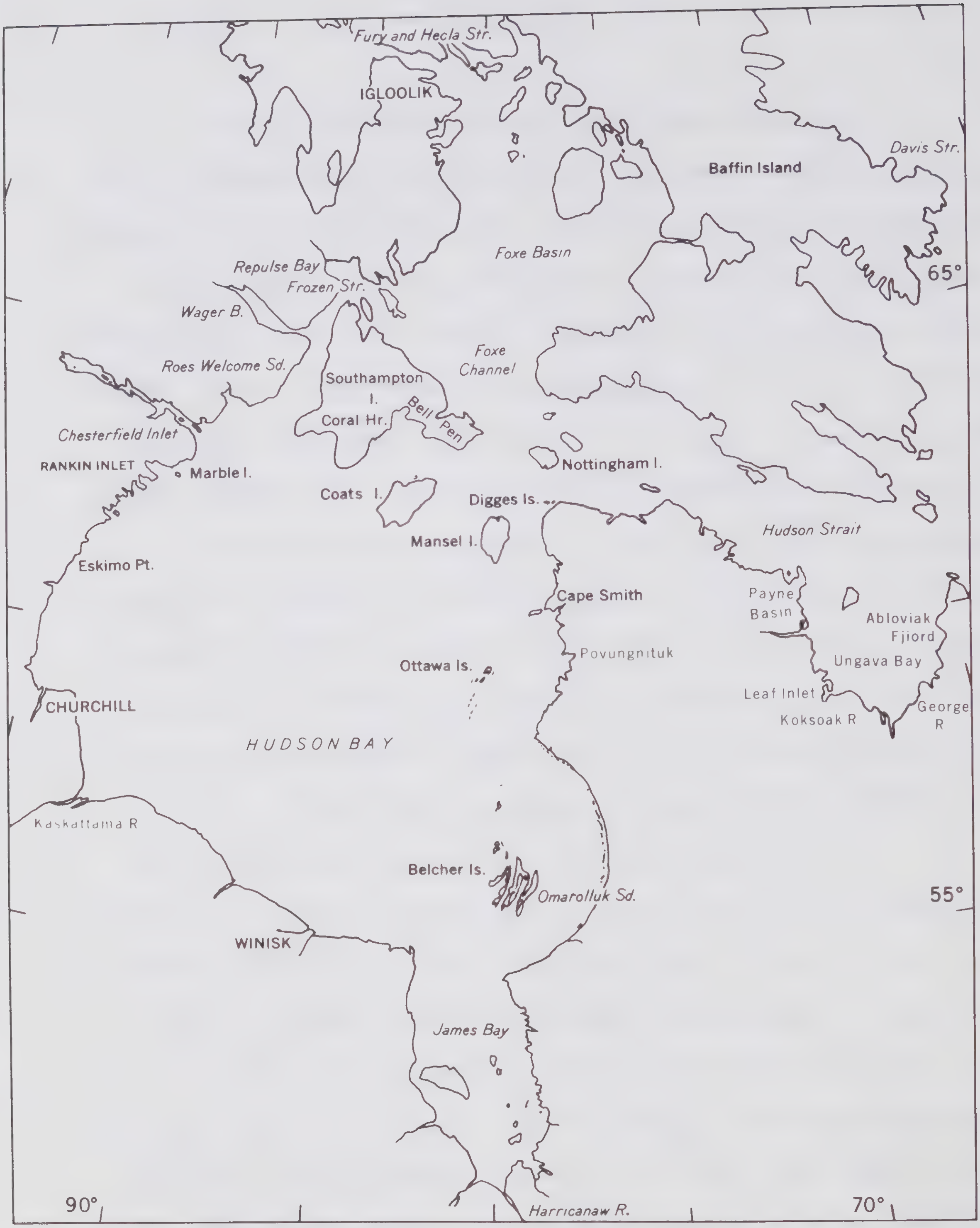

Figure 1. Some place names within the system.

that consideration be given this so as to ensure that an adequate description of the system as it existed prior to development is available. Of course this will not ensure that post-development changes will be related in a causal way to the development, for if a measurable change does occur over a decade or two, or longer, other influences may appear by that time to be of equal or greater significance (Anon., 1971a, p45).

It will be evident in the following that the data requirement is rather formidable, not only in oceanography (Hamill, 1969, p37) but also in hydrography and perhaps in geography. Robinson (1968) reviewed knowledge of the geography of the surrounding land areas where two sub-regions could be delineated (p202). One, "the South Coast Lowland", extends southeast of Churchill to west and south of James Bay where along the coast (p217),

> ...there is a flat strip five to ten miles wide, with the widest parts generally being to the north. The coastal zone is treeless, but grass or marshes are common. Storm beaches, a few feet high, are the only topographic features. Tidal flats may be exposed for one to six miles; even at high tide shallow water extends far offshore. Much of the monotonous coast has been unapproachable from the sea even by small ships; this being one of the main reasons for the lack of settlement. Deeper water may be found at the drowned river mouths, but shifting sand bars and minor deltaic deposits are navigation hazards.

The other, the "East Coast Upland" (p222) extends from east of James Bay to the north coast of Hudson Bay a north-south distance of 750 miles. James Bay constitutes almost a third of this, i.e. 230 miles (389 km), and is up to about 100 miles

(185 km) wide with an average depth of about 32 m. It seems likely that given adequate hydrographic data, distinct sub-regions would be recognizable within the bay. The project will likely influence the priority for such data so that the required knowledge of the bathymetry so important to an understanding of the oceanography may become available reasonably soon.

As yet only limited information is available concerning the project beyond that reported in newspapers, eg. Anonymous, 1971b, which in the main have been about the development of hydroelectric potential on the three southern rivers, Broadback, Nottaway and Rupert. A parallel development is proposed for certain of the northern rivers with diversion of water into La Grande-Rivière, including part of Grande-Rivière de la Baleine (Anon., 1971c). Considerable interest in possible long-term climatic effects of the project arose subsequent to newspaper, radio and television commentary based on expressions of two well-known Canadian scientists (R.W. Stewart and L.M. Dickie), which in turn led some to the conclusion that marked change was to be expected, eg. Time Magazine for September 27 (Anon., 1971d). In a subsequent letter to the magazine, Stewart and Dickie (1971) clarified their views somewhat and seemed to suggest that although it was not possible to predict the influence of the project, it was possible to predict that it will have one. Apparently the influence as they view it will be only on the local climate, or microclimate, and could occur in a number of ways, some rather subtle, requiring

detailed and extensive data in order to recognize the change and process (Landsberg, 1970, p1269). While it is by no means certain whether man's activities have yet influenced global climate (Frisken, 1971), it is the experience that this has not yet occurred (Landsberg, 1970) and may not be likely (Anon. 1970a, p97).

In the following it is assumed that a major feature of the oceanography of the region, the annual ice cover, is due to the global atmospheric circulation and hence not likely influenced by man. The ice cover limits the exchange of heat with the atmosphere so that both the gain of heat during the open season and the loss of heat during the period of ice cover are much smaller than were the area without an ice cover. It is believed that for the region, i.e. Hudson Bay and James Bay, the gain is about equal to the loss although smaller, so that a small deficit occurs which is balanced by an advection from the ocean. In this regard the main parts of the system may be even more uncoupled from the world ocean than is the Arctic Ocean, where a perennial ice cover is general and the balance is maintained by advection from the world ocean and an excess of export over import of ice. It is believed that the export and import of ice from and to our system are small and about equal, so that the amount of ice which melts is equivalent to that which forms there, although considerable redistribution of the ice occurs within the system.

It is possible that the project could influence this redistribution of ice (Murty, 1971) and some attention is given this aspect later. The treatment generally is quite inadequate and does not provide significant insight to those factors of the circulation which might influence ice distribution. This is of particular significance when we realize that not only is the ice cover unstable but also in some years a portion very nearly survives through a summer, eg. in 1969 (see section 2.3). If the project were to lower the average air temperature in summer through an impact on local climate, the ice cover would persist for a longer time causing a further decline in average temperature. It may be that through observation and study it could be determined whether the project will constitute a "major environmental disturbance" (Kershaw, 1971) or whether our considerations which suggest little impact are, in fact, "miscalculations" (Heyneman, 1971).

1.1 Climate

> In contrast to the broad expanses of Arctic tundra that surround Hudson Bay the sub-Arctic lands bordering James Bay are partially forested and thus protected from strong winds. As a result two of the most distinguishing features of Hudson Bay winter climate - wind chill and blowing snow - are not nearly as evident near James Bay. With this important exception and the fact that James Bay is several degrees of latitude farther south the factors that influence the climate of James Bay are essentially the same as those outlined for Hudson Bay.
>
> Along southern coastlines more than 100 inches of winter snowfall are evenly distributed during the months November through March. Fort George, on the east coast, receives almost as much snow but approximately one half of the annual total falls during November and December. Lesser amounts are measured in the mid-winter months

when almost solid ice cover over James Bay cuts off the supply of moisture to the air moving over it. Throughout the winter the snow is soft and deep in the woods but drifted and hard packed over the exposed Bay ice.

In general the lands bordering James Bay are free of snow from late in May until the middle of October. During this period precipitation, mostly in the form of rain, accounts for more than one-half of the 25 to 30-inch annual total. At Moosonee, in fact, rain or drizzle are reported on approximately 15 days of each month in summer. As would be expected from these figures, cloudy days are frequent. While Moosonee and Fort George receive less than 240 hours of bright sunshine in July, stations at approximately the same latitudes in the prairie provinces record close to 320 hours. Moosonee ranks above Churchill in the number of thunderstorms that occur in summer with an average of three or four per month. The frequency of thunderstorms decreases markedly to the north and east however, where the cold waters of Hudson and James Bays tend to inhibit shower development.

Although reports from the weather stations at Moosonee and Fort George do not fully substantiate it, fog is probably quite prevalent over James Bay in June and July when sea ice is still present.

As is the case with Hudson Bay, the cold waters are more influential than latitude in determining summer air temperatures along the shores of James Bay. July temperatures at Moosonee and Moose Factory average about 60°F while daily maxima are near 70°F. Corresponding temperatures for Fort George are about 5 degrees lower. Along the east coast of James Bay daytime high temperatures over 85°F are unusual. At Moosonee, on the other hand, where southwesterly winds bring warm air from the heart of the continent, 90°F temperatures have been recorded in all months from May to August. Sharp and rather frequent temperature fluctuations may be expected during these months depending on whether the winds are off the land or off the water.

Freezing temperatures have been reported in every month of the year at weather stations on the shores of James Bay. While there are wide local variations in the incidence of frost, depending on the proximity of the Bay or on the presence of lakes or muskeg, the first frosts of autumn usually occur early in September. Average daily temperatures generally remain above 32°F, however, until late October.

> January is the coldest month in the James Bay region with average temperatures in the neighbourhood of -5° to -10°F. An extreme minimum temperature of -52°F has been recorded at Moosonee and temperatures of -40°F are not uncommon. Readings as low as -20°F may be expected on one day in four at the height of the winter season. In contrast to the Hudson Bay area where above freezing temperatures are rare in winter, James Bay has the occasional mild spell in January and February. An example of such mild conditions occurred in February 1954, when maximum temperatures at Moosonee exceeded 40°F on five successive days, culminating in a reading of 51°F on the 19th.
>
> In summary, the climate of James Bay is not as severe as that of Hudson Bay. Winters are long and cold however, and summers cool, and on an annual basis the lands surrounding James Bay are colder than most in Canada at similar latitudes.

That the "lands surrounding James Bay are colder than most in Canada at similar latitude" is probably due to the "deep southward penetration of Arctic climate in eastern Canada in winter" caused by alteration of "global wind patterns" by geographic features. The exerpt and the quotes are from Thompson (1968) who also provided climatological data for a number of locations around Hudson Bay and James Bay. Stressed in the article (p267) is the effect of ice and relatively cold water on the local climate whereby winter conditions become quite continental and those of summer quite maritime. Earlier Burbidge (1951) discussed the influence of the open water of Hudson Bay on continental polar air moving over the region. Apparently the influence is greatest over the eastern portion in mid-summer and in early winter prior to freeze-up; although the tempering influence does persist past freeze-up into January (Hagglund and Thompson, 1964). It is clear that a persistent change in the average extent of ice cover would be followed by

a change in climate there. As noted, it is generally believed that ice extent in the northern hemisphere is determined largely by global atmospheric patterns (see also Fletcher, 1969); however, in Hudson Bay the persistence of ice is particularly sensitive to warm air from the south (Mackay, 1952) or, as we shall see, to cold air from the north.

1.2 Ice cover and a possible consequence

The "deep southward penetration of Arctic climate into eastern Canada in winter" is such that ice thickness over much of Hudson Bay approaches that for Arctic regions generally, although for James Bay it is considerably less (for example see Bilello and Bates, 1971).

> The greater part of James Bay is frozen over in the winter. By the end of January, local inhabitants have been known to go across the ice from the eastern shore to almost every island lying in the middle of the bay. At the head of the bay, in mid-winter, frequent journeys are made across the ice in a direct line between Charlton Island (Lat. 52°00'N., Long. 79°25'W.) and Moose River and between Charlton Island and Rupert River. The movement of ice in spring is greatly affected by the direction of the wind. If southerly winds predominate, the wind and outflow of the rivers acting together clear the southern part of the bay by the middle of June. But if northerly winds predominate, the ice will remain in the bay and seriously obstruct navigation till the sun becomes strong enough to melt it.
>
> Coasting vessels are put into commission during the last days in June, and make their passage from one post to another in lanes of open water between the coast and the ice in the centre of the bay. (Anon., 1965, p441).

Larnder (1968) stressed the variability of the ice cover in Hudson Bay, both in its formation and breakup, which she noted differed "widely from year to year and from one locality to

another" (p319) and, "Although, as already noted, the ice in James Bay generally melts or moves out of the southern two-thirds of the bay by early July, the ice that moves into its northern part from Hudson Bay may be so heavy, close-packed and persistent" (p335) that local transportation may be limited. Danielson (1971, p102) also considered that the northern "parts of James Bay receive ice from Hudson Bay" and implied that like southwest Hudson Bay much more ice melts there than forms there (p98). From the charts of ice conditions provided annually by the Meteorological Service it is possible to obtain an appreciation of the extent to which breakup can vary. For example, observations on July 9 of each of 1968 and 1969 (Figure 2) indicated quite a wide divergence in extent of ice cover. This may be due to variations in direction of persistent winds early in the summer which can accumulate or disperse ice. This in turn can influence the surface albedo and hence the amount of insolation eventually absorbed. A pattern of circulation associated with mixed freshwater moving seaward at the surface probably also influences the distribution of ice, but not likely to the same extent as can this early wind.

As James Bay is a region of annual, rather than perennial ice cover and as open water generally occurs at a relatively early time in summer, a seasonal variation in temperature and salinity may be anticipated. In the approach to James Bay the annual sequence of events is, in the main features, likely similar to that suggested

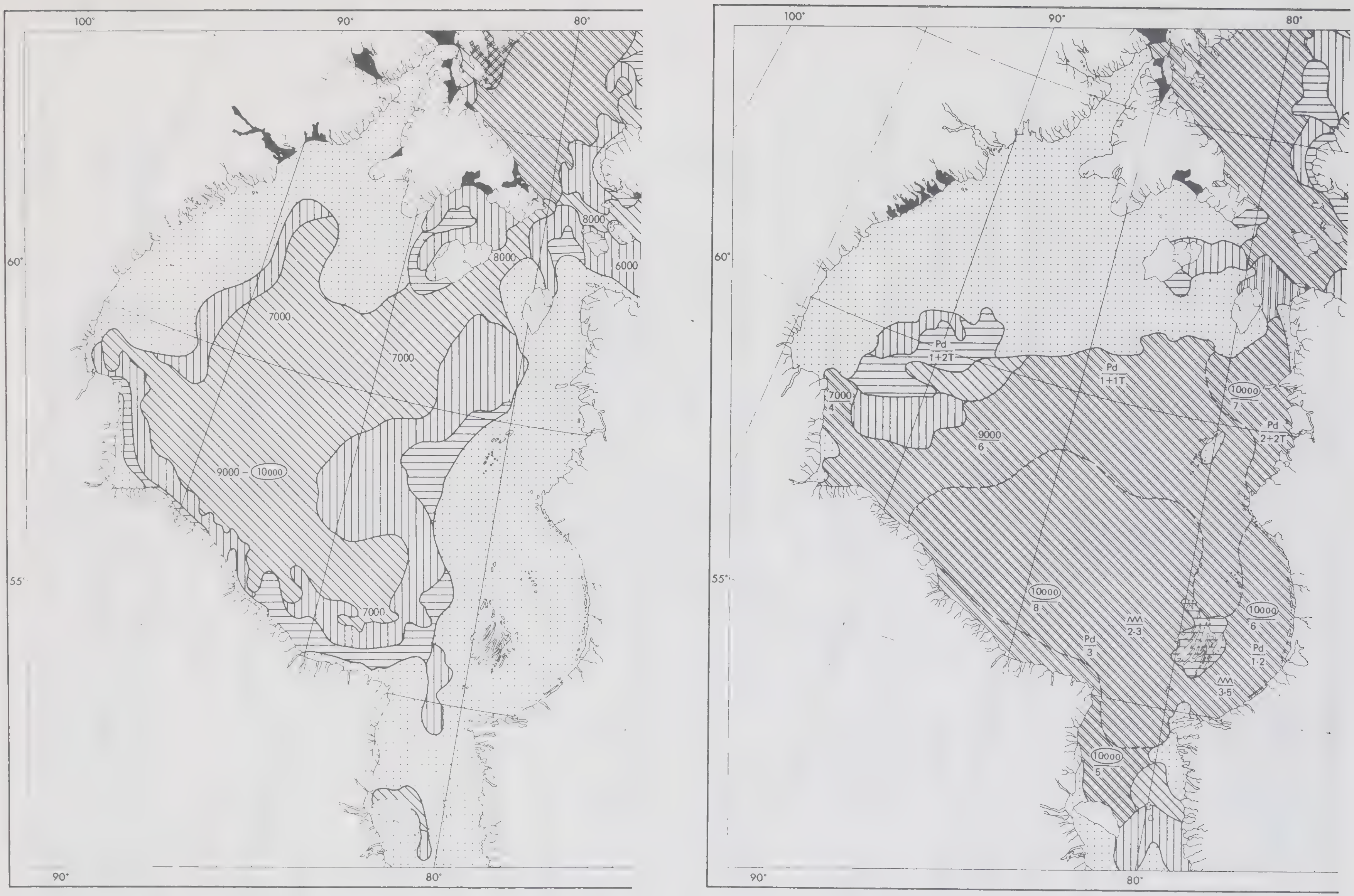

Figure 2. The extent of ice cover on July 9 of 1968 and 1969 (from Anon., 1970e p13; 1971e p13). (a) 1968. (b) 1969.

for a more northerly location in Hudson Bay (Figure 3), where the variation of salinity within a perennial surface layer is largely related to change in the ice cover. Within James Bay, the salinity structure is likely strongly influenced by the amount of freshwater from runoff and in the southern portion it may be that in winter a layer of freshwater exists under the ice (see also section 2.2.2). The existence of such a layer would be a consequence of the marked decrease of mixing at the surface due to a reduction in the transport of energy, from wind and exchange processes, across the sea surface resulting from the existence of an ice cover. Tidal currents would provide some energy for mixing but as these are not likely much above about 1 knot away from shore (Godin, 1971), it is possible the layer persists. Thus, in the absence of observations, the possible late winter distribution of salinity and temperature (Table 1) for the deep water west of Trodely Island is highly speculative, perhaps even fanciful. The small maximum temperature in a halocline is characteristic of arctic seas, as is the halocline itself. It seems almost certain that the halocline exists in winter, so that the water below a surface layer may be effectively uncoupled from local surface processes, i.e. convective processes are limited to depths above the halocline. This situation is believed to exist throughout the year over most of James Bay, so that the deeper water probably is from Hudson Bay with relatively little change.

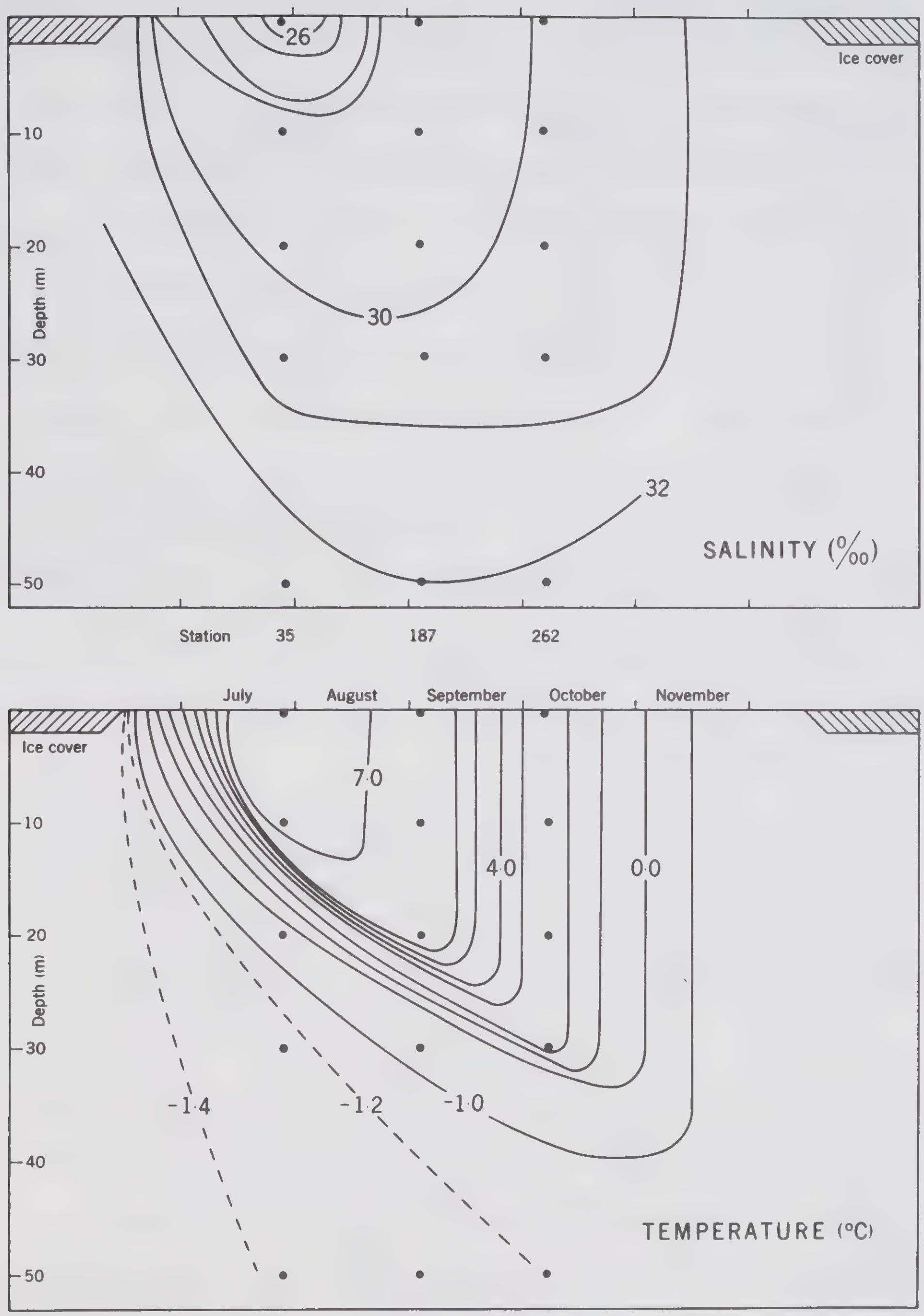

Figure 3. The annual sequence in the near-surface at a location in Hudson Bay (from Barber, 1967, p29). (a) Salinity. (b) Temperature.

Table 1 A possible distribution of salinity and temperature under the ice cover at a point just west of Trodely Island.

Depth (m)	Temp. (°C)	Sal. (o/oo)
2	0.0	0
4	0.0	0
10	0.4	15
20	-0.8	25
30	-1.3	27
50	-1.3	27
75	-1.3	27

1.3 Bathymetry

Most knowledge of the bathymetry (frontispiece) is contained in Canadian Hydrographic Service chart number 5800 and pilotage information is available in another publication of the Service (Anon., 1965) from which the following was extracted (p441):

> The low, flat shores of the bay are fringed with wide mud flats. The bay is filled with numerous islands, rocks and shoals, with no harbours for large vessels. In the fairway which, in the outer part, lies close westward of the middle of the bay, there are general depths of from 10 to 40 fathoms ($18^{m}3$ to $73^{m}2$) to within about 20 miles of the head of the bay.
>
> As in the case of Hudson Bay itself, the eastern shore of James Bay is very irregular in outline, with many islets and rocks lying close off it, while the western shore is even and almost free of islands. There is, however, Akimiski Island lying close off the middle of the western shore. In the eastern half of the bay are many islands and shoals. The western shore of the bay affords no harbours, but there are several fairly good harbours on the eastern shore for vessels drawing up to 12 feet ($3^{m}7$) of water.

Limiting depths exist in certain parts of the bay so that the water in some areas may be isolated from that at similar depth in Hudson Bay. For example, it is thought that for a station occupied in the deep water off

Trodely Island a limiting depth of 40 m exists so that a salinity of about 27 $^{o}/oo$ could be expected there. The actual situation is quite uncertain and it is possible to interpret available bathymetric data as though there were shallows, less than 20m, extending continuously off the east shore from Cape Jones (Pte. Louis XIV) to Bare Island southward through Grey Goose Island, the Twin Islands, Weston and Charlton Islands to the east coast in the vicinity of Rupert Bay, so that a depth of 60 m would be isolated to the eastward. However, the interpretation here (frontispiece) indicates just one such area eastward of the shallows between Grey Goose Island and the Twin Islands, but with deeper areas south of Bare Island and north of Weston Island. Depths of 80 m occur almost as far south as Trodely Island; a depth which it was noted is not likely continuous to Hudson Bay. The areas of limiting depth of about 40 m are believed to exist between Bear Island and Akimiski Island and to the east of South Twin Island.

With regard to the water which may occur in the northern approach to James Bay, oceanographic data indicate that a limiting depth of between 50 to 75 m occurs between the Belcher Islands and Cape Henrietta Maria (Barber, 1967, p15), thus the deeper water north of Pte. Louis XIV would be slightly isolated from Hudson Bay (an interpretation of more recent, but still incomplete, data on Canadian Hydrographic Service field sheet 025A suggests a limiting depth of 80 m) so that the range of salinity observed at

depth in the approach, i.e. 31.7 to 32.0 $^{o}/oo$, is somewhat less than is observed in the open areas of Hudson Bay generally, i.e. up to 33.0 $^{o}/oo$ at 100 m (Figure 4).

1.4 Surface wind

Knowledge of surface wind within the system is based largely on observations at coastal stations, which indicate that winds are strong in all but the summer months when they are:

> ...generally lighter and are variable in direction with a higher proportion of onshore winds at coastal stations, the effect of local sea-breeze circulations.
>
> (Anon., 1965, p395).

Danielson (1971, p97) remarked on the influence of persistent north winds which move ice southward and evidence provided by Archibald (1969) suggests that wind strength may decrease from north to south, being less in southern James Bay than elsewhere in the system. It is the author's (limited) experience that storms can occur for short periods at any time during the summer with winds from the southeast to southwest. At Churchill in late September strong northwest winds may persist for 4 to 5 days and in early October in 1961 in western Hudson Strait strong wind (290°, 50 knots) was experienced. Occasionally it is possible to see in the oceanographic distributions, correlations with surface wind. Of particular interest here is the influence of wind during the summer, i.e. when it is weak and variable with a "high proportion of onshore winds". As cloud and fog are frequent over the water such

Figure 4. The distribution of salinity at the surface (a) and 50 m (b) in Hudson Bay and James Bay (from Barber, 1967) and the distribution in Hudson Bay of salinity (c) and temperature (d) at 100 m (from Barber and Glennie, 1964).

a wind would tend to increase the frequency of cloud and fog at coastal stations; an increase in each has been observed in both Hudson Bay and James Bay (Thompson, 1968). However, the main movement of air at the surface is from west to east so that a relatively warm air mass is moved over the southern-most parts of the system. It is useful to consider the likely surface temperature toward the end of July based on the 1961 experience (Figure 5) by which time considerable warming of the water had occurred over northern Hudson Bay, but with little or none in the southern part due to the ice cover there. Some warming has also occurred on land by this time, for example to the southwest (Figure 6). A warm air moving west to east from the land over the ice covered area would likely become relatively stable with which would be associated a decrease in surface wind. The implication is that this small-scale region of atmospheric stability through a decrease of surface wind, increase of cloud and fog and with the high albedo of ice cover, contributes to the stagnation of the ice in the southwest usually observed (Anon., 1970e). Also implied is a lack of water movement due to other factors; however, this may result as a consequence of the James Bay circulation (Murty, 1971).

1.5 Freshwater from runoff

It seems likely that the pattern of runoff to James Bay will continue to be influenced by man, either through the increasing development of hydroelectric potential as has already occurred to some extent on the

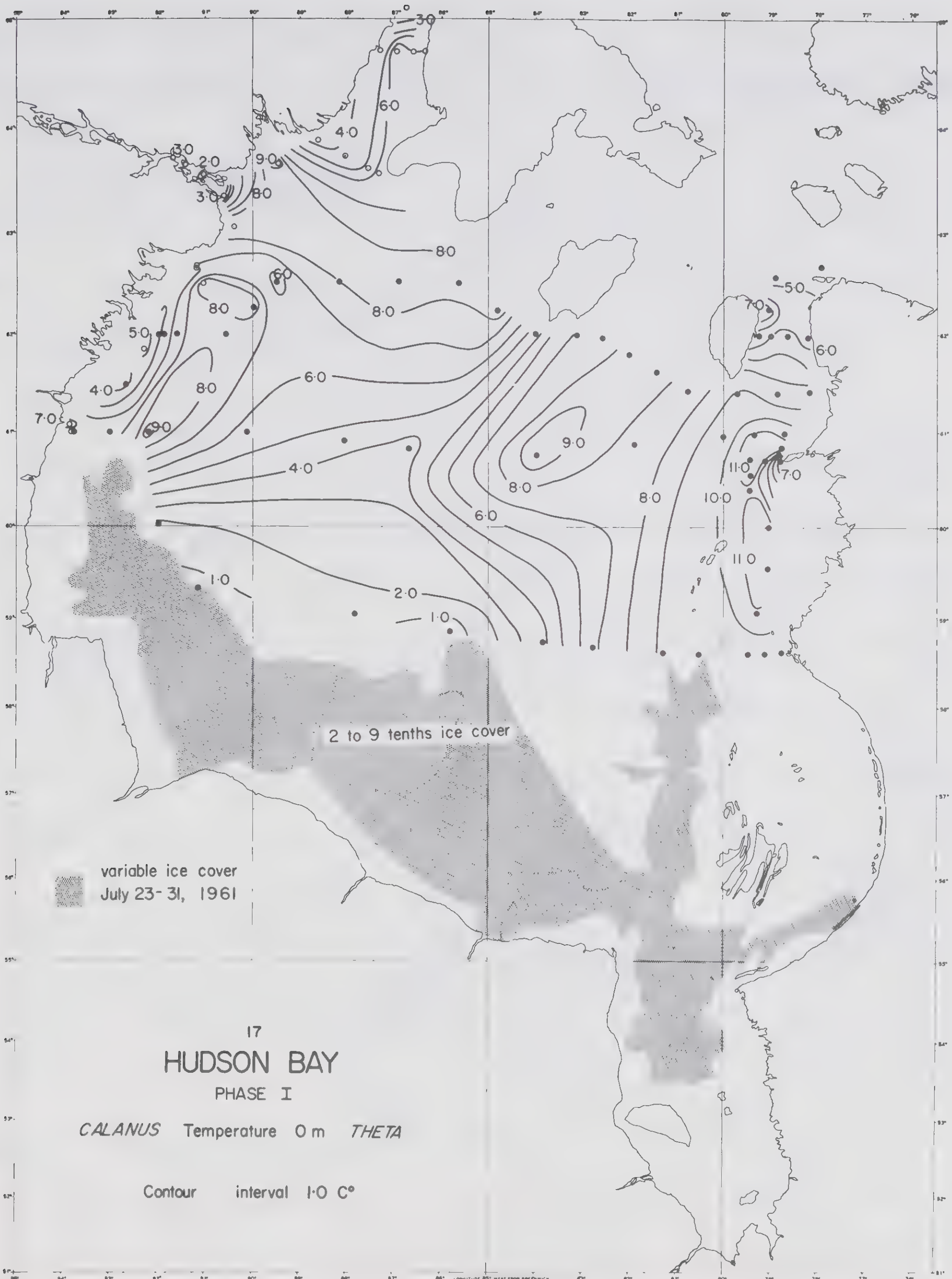

Figure 5. The likely distribution of surface temperature in Hudson Bay toward the end of July based on temperature data (Barber and Glennie, 1964 Figure 17) and ice data (Anon., 1962) in 1961.

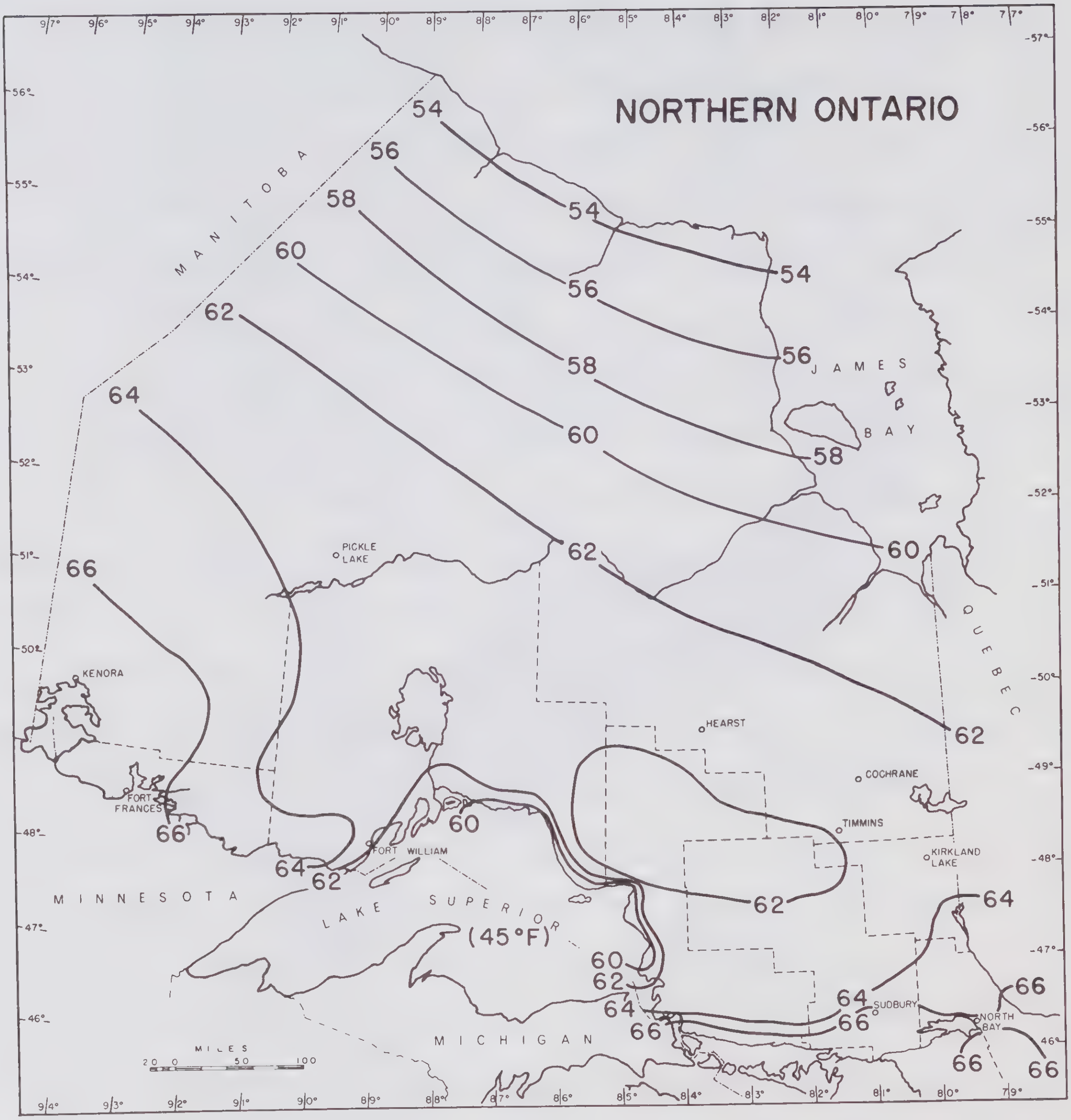

Figure 6. Mean daily temperatures (°F) in northern Ontario for July (Chapman and Thomas, 1968 their figure 8).

Moose River (Robinson, 1968, p219), or perhaps through diversion of water southward (p219). At the present time the annual runoff volume appears to be about $3x10^{11}m^3$, that is if there were no diversions. The value was obtained by extrapolation of the data on runoff from the Québec rivers (Figure 7) over the total drainage basin including James Bay. A check was obtained through the estimation of a value for the annual excess of precipitation over evaporation (P-E) over the total region. Hydroelectric development, in the absence of diversion, would not be expected to influence the annual value appreciably (except that it modified P-E), but would markedly influence the present pattern of runoff (Figure 8), which varies from very high levels in spring to extreme low levels in March and occasionally in summer. Black remarked (1968, p843):

> The rivers become quite shallow by July, even during wet weather. At this time the low water exposes numerous bars, bouldery shoals and rocky outcrops which render stretches of river difficult to navigate, ... A short period of navigation is possible before freeze-up when a second high-water period occurs during heavy autumn rains, ...

Presumably the rivers are not used for navigation during the period of low runoff, i.e. in March, at which time the flow can vary between a fifth to more than an eighth of the much more variable spring volumes. No doubt this large spring runoff has a significant influence on conditions in James Bay, including the distribution of ice, salinity and currents, particularly as it occurs when the ice cover is still extensive. However, even though

Table 2 A tabulation of 1968 surface water data from six stations indicating the name, size of basin, mean flow and the page reference to the source (Anon., 1971f). Other data are available (Anon., 1967a; 1970b; c; d).

Name (St. No.)		Area (Sq. Miles)	Mean Flow (cfs)	Page
La Grande-Rivière (092704)		37,600	60,000	363
Eastmain (090601)		17,100	31,400	352
Grande-Rivière de la Baleine (093803)		16,500	23,700	379
	sub-total	71,200	115,100	
Rupert (081002)		15,800	30,300	345
Broadback (080801)		6,610	11,500	343
Nottaway (080701)		22,200	36,600	333
	sub-total	44,610	78,400	
	TOTAL	115,810	193,500	

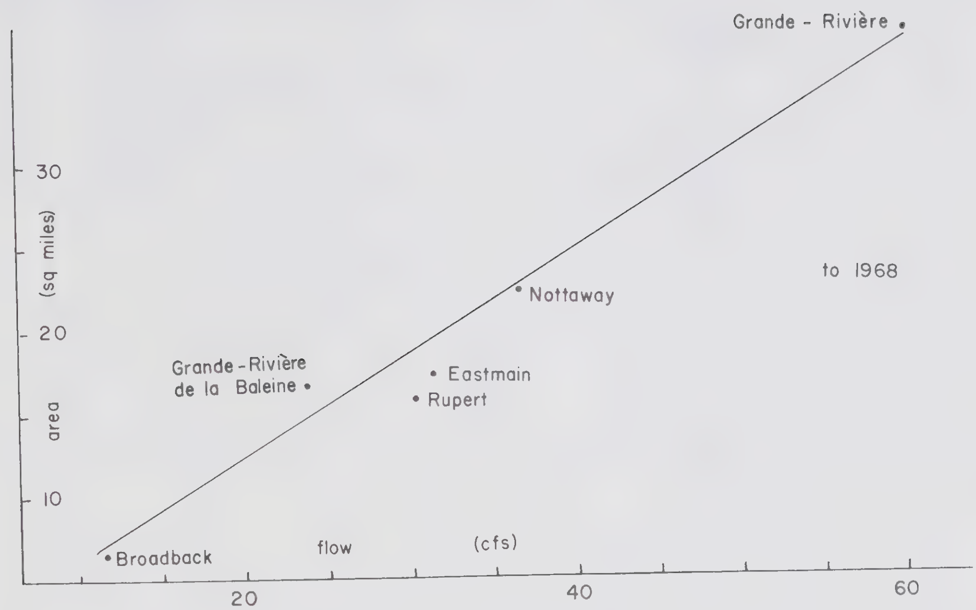

Figure 7. A presentation of the data of Table 2. The extrapolation referred to in the text is based on the "best fit" of the straight line.

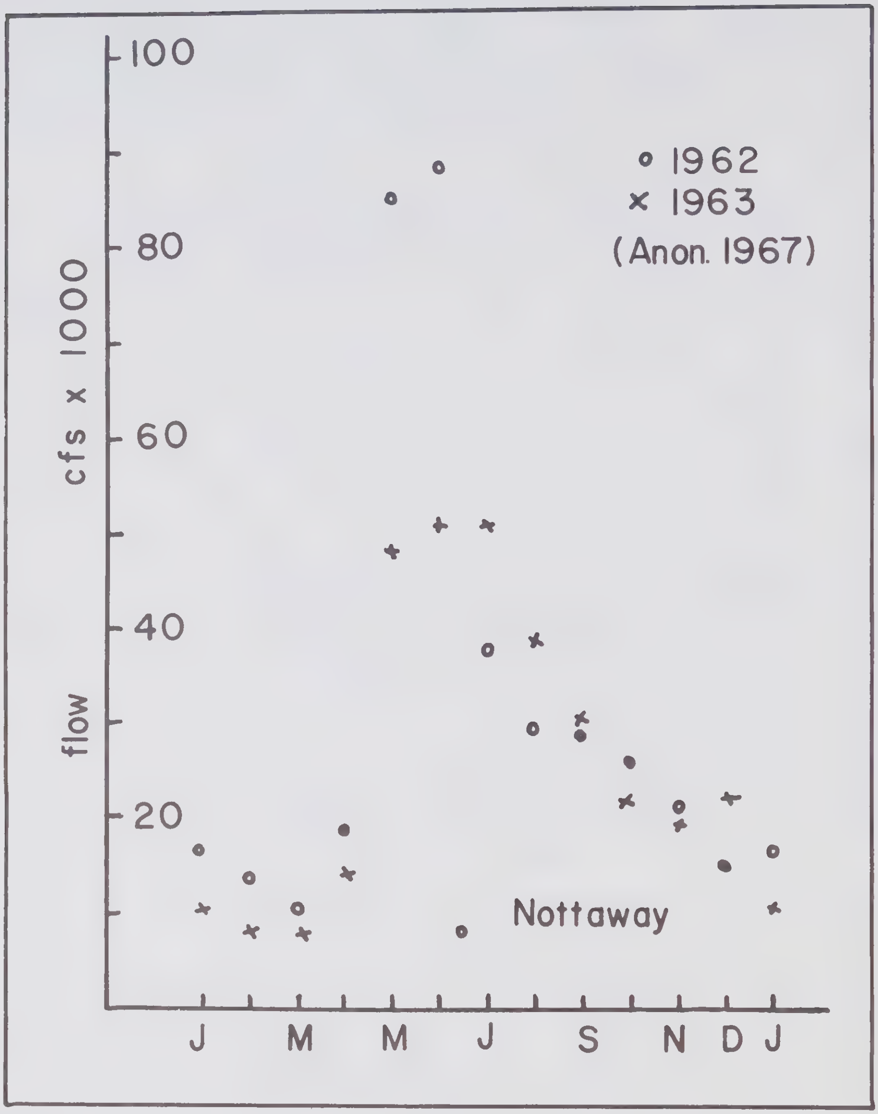

Figure 8. A presentation of the monthly values of runoff for the Nottaway River for 1962 and 1963 (Anon., 1967a).

data on the flow from the Québec rivers are quite adequate, lack of oceanographic data limit their application. For the purpose here the significant annual values appear to be:

1)	amount from rivers and excess of P-E over the bay	$3x10^{11}m^3$
	but not including Grande-Riviére de la Baleine	$0.02x10^{11}m^3$
2)	amount from rivers to be developed	$1.88x10^{11}m^3$
	i.e. the sum of the southern rivers	$0.78x10^{11}m^3$
	and of the northern rivers	$1.1x10^{11}m^3$

Thus the development will increase the total annual value slightly and markedly influence the pattern for about half the runoff to the bay. With the potential for development of the southern and west coast rivers the runoff to the bay could be almost completely smoothed or at least reflect a demand for energy, which may be maximum in winter, i.e. at a time when the runoff prior to development, was at a minimum.

2. The system

2.1 Data in James Bay

It is curious that James Bay has not been examined by oceanographers to nearly the same extent as have areas much further north. Apparently it is considered a difficult area, particularly for vessels suited to ocean survey, so that during the development of the 1961 oceanographic programme

for Hudson Bay the work in James Bay was limited to two sections across the entrance. Earlier, the negative results of an investigation into the fisheries potential of Hudson Bay by Hachey (1931a) considerably deflated interest in the region generally (Hunter, 1968, p373). The 1961 material and the 1959 work of "Calanus" (Table 3) constitute the only significant data yet observed there, although temperature and salinity data were observed in the Moose River with tidal observations (Langford, 1963, p90), but these have not been located. All of the available material were observed during the period of open water, though some of the "Calanus" stations were occupied at a time (Table 4) when some ice cover occurred (see data report for existence of ice nearby at the time the station was occupied). Distributions based on the "Calanus" data were presented by Grainger (1960) and these were also utilized in a description of Hudson Bay (Barber, 1967; 1968a). The recent compilation of information about the region (Beals, 1968) proved useful in a number of ways, for it is "an impressive assemblage of facts not readily found elsewhere" (Jackson, 1970, p841).

2.2 Review

James Bay is part of a general system (Figure 1), comprising it, Hudson Bay and Foxe Basin, connected to the world ocean through Fury and Hecla Strait and Hudson Strait. Fury and Hecla Strait is likely much less important than is Hudson Strait to the character of the water within the system, for not only is Hudson Strait relatively wide

Table 3 A tabulation of the data available indicating the year obtained, name of ship, cruise reference number and a reference.

Year	Ship	CRN	Reference
1958	"Calanus"	320	Grainger, 1960
1959	do	321	do
1961	"Theta"	337	(Anon., 1964a; b)

Table 4 A tabulation of the "Calanus" 1959 data in James Bay indicating the station number and day. The latitude of station 59-4 has been taken as 52°30' and that of 59-15 to be 53°56'.

Number	Day	Number	Day
1	June 20	57	Aug. 26
2	22	58	26
4	22	57*	29
6	23	59	29
7	25	60	29
8	26	61	30
9	27	62	30
11	30	63	30
13	July 1		
15	10		

*Apparently a re-occupation.

and deep, but also strong mixing due to tides occurs there. Thus the water reflects to large extent an influence of the Atlantic Ocean and an influence of a process occurring within the system, and even though James Bay is 600 to 1,000 miles away from these influences they are still strongly evident there. In the present context however, distance need not be particularly significant, for factors such as the general location and the associated climate and pattern of runoff in such a uniquely shallow region as James Bay can be paramount.

The foregoing assumes certain physical properties of water which are of prime significance. For example, at salinities generally encountered in the ocean a temperature of maximum density does not exist at any temperature warmer than the freezing point (Figure 9), as exists for freshwater at 4°C. It will be shown that much of the water is less than oceanic salinity, specifically less than 24.7 $^{o}/oo$, so that a point of maximum density can occur at temperatures warmer than that of ice formation at the surface. At such low salinities, surface cooling at temperatures close to freezing would lead to stability. Furthermore, at those locations where the surface layer is fresh or nearly so, the amount of salt made available to the surface layer through freezing is negligible. Thus, the evolution of a surface mixed layer through the winter would not relate to the formation of denser water at the ice-water interface (Barber, 1968b). Also of utmost significance, here as elsewhere, is the

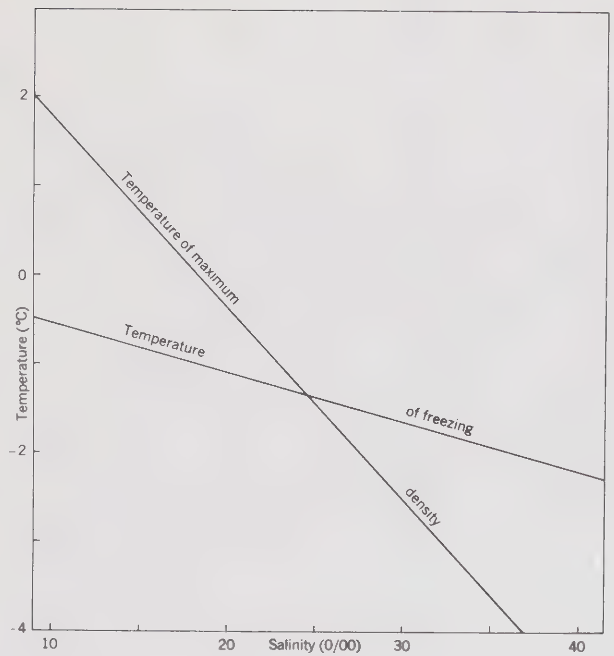

Figure 9. Temperature of the freezing point and temperature of the maximum density as a function of salinity (from Barber, 1967b).

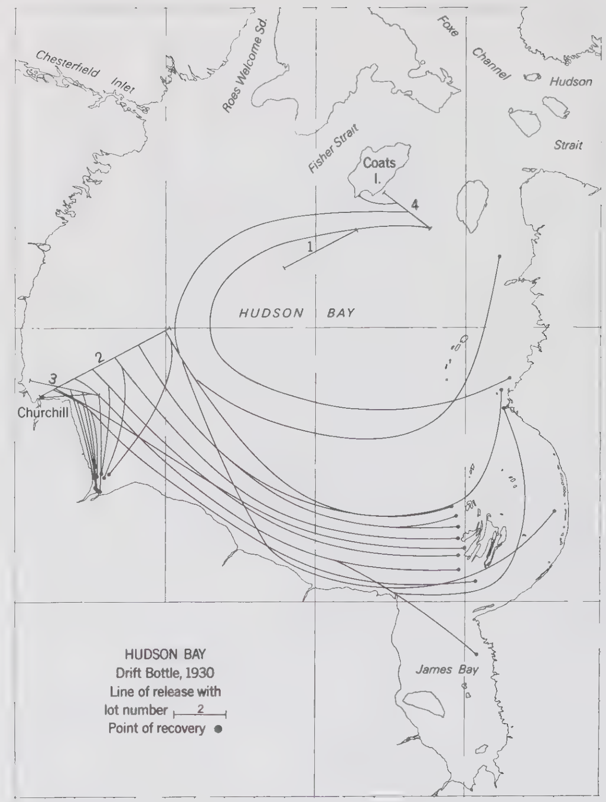

Figure 10. A re-presentation of the drift bottle data of Hachey (1935), (from Barber, 1967).

fact that ice formed at the surface remains there, as it is less dense than water, providing a very different boundary than is the air-sea interface.

2.2.1 Currents

It may be significant that only one of the drift bottles released in 1930 (Hachey, 1935) was returned from James Bay (Figure 10). If it is significant, then it is envisaged that the inflow from Hudson Bay occurs at depth rather than the surface and comprises a cold, relatively saline water. On the other hand, Grainger (1960) inferred an anti-clockwise surface movement with water entering from Hudson Bay along the west coast. Data observed in the autumn (October, 1961) suggest the development of a well-defined movement from James Bay north along the east coast of Hudson Bay and to considerable depth (for example see salinity at 100 m, Figure 72 of Barber and Glennie, 1964), which was partly due to an increase in the amount of freshwater there in October over August (Figure 11). Thus the pattern of circulation in these areas may exhibit a time-dependence in association with the distribution of freshwater from runoff.

Presumably a portion of this runoff would have absorbed a significant amount of shortwave radiation and would enter the bay at a temperature above that of the surface water there, so that it would constitute a gain of sensible heat. Presumably too, much of the outflow at the surface would have, in the absence of ice, absorbed heat and would constitute a loss. The heat transported

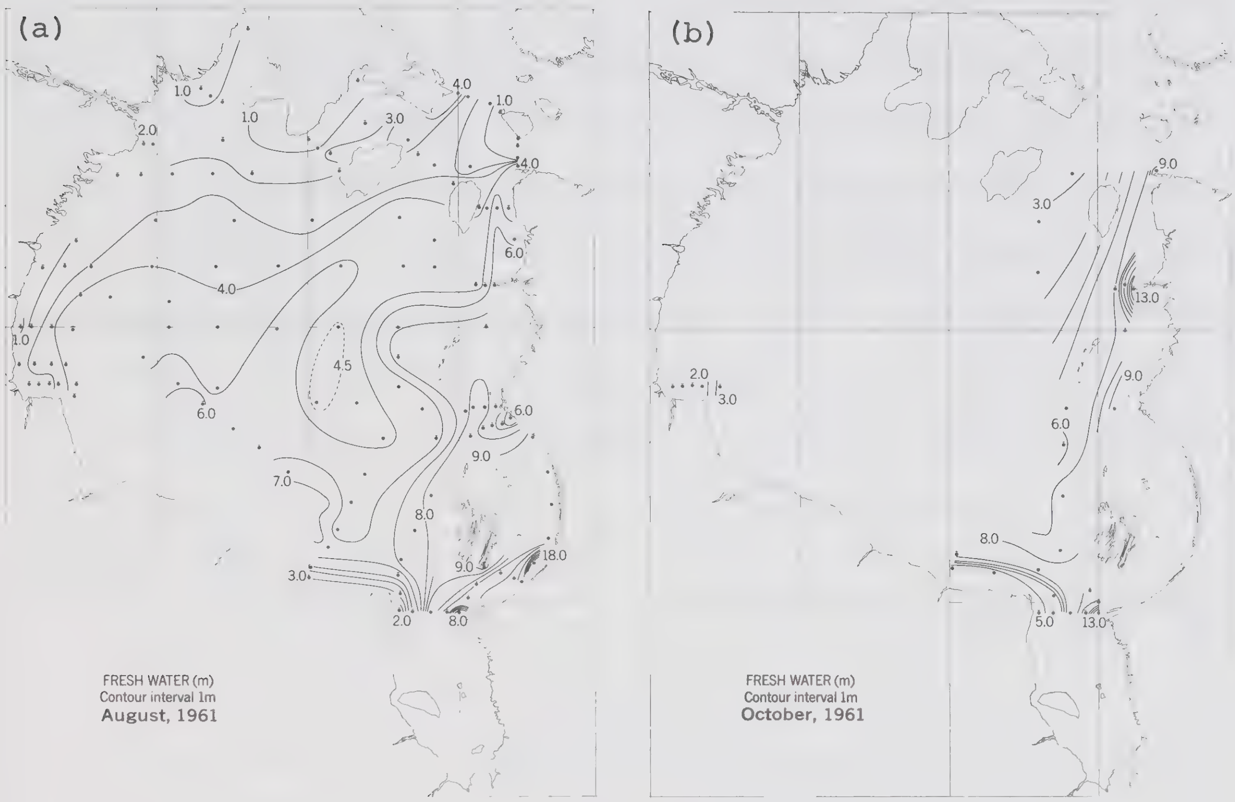

Figure 11. The distribution of the depth of freshwater (m) in August and October, 1961 (from Barber, 1967). (a) August. (b) October.

with the inflowing deep water would be very much less so that in the section across the entrance a loss of heat occurs due to transport. The loss may be balanced by an excess of export over import of ice, but there is no direct evidence that either occurs, and it may be that it is balanced by the input of heat with the runoff. In this circumstance the annual sum of the flux terms would on the average be zero. Danielson (1969, p162) considered that a variation in the flux occurred over the bay ranging from a deficit of about 100 g cal cm^{-2} $year^{-1}$ in the north to about 6000 in the south and leading to an average gain of about 2500 g cal cm^{-2} $year^{-1}$. A rough heat budget for a position in the northern part of the bay was attempted and although a value which might be considered better than Danielson's was not achieved, a comparison of the actual with the potential heat storage for two periods was made from existing data. Between August 11 and 29 (1959) an increase in heat storage of 6500 g cal cm^{-2} occurred, which is about twice the amount to be expected in the absence of advection. This suggests a movement out of the bay of the warmed water. During the period August 29 to October 2 the mixed layer depth increased to 25 m and the heat storage was unchanged. This would have been expected in the absence of advection. The data therefore suggest a variation in a movement of surface water from the bay, again suggesting a time-dependence in the outflow.

Such a time-dependence could be expected to have a distinct influence on the pattern of surface movement

seaward of James Bay, but direct evidence is not available. Some secondary evidence has been mentioned, for example drift bottle data and the distribution of freshwater, but there is one other which should be noted even though quite speculative. This relates to the distribution of ice within Hudson Bay during the summer which exhibits lingering ice, or a pattern of last ice, in the southwest (Anon., 1970e; Danielson, 1971) due in part to movement of ice from the north during the early part of the breakup period. Later the ice appears to stagnate in the southwest with little or no tendency to move and, in particular, no tendency to move with anti-clockwise movement around the coasts presumed to exist.

2.2.2 Salinity

Various distributions based mainly on the 1959 data are included in the final group of figures (Figures 21 to 24). Generally, the material was difficult to interpret and in some distributions certain of the data are not included.

The distribution of salinity at the surface and at 50 m (Figure 4) within Hudson Bay and James Bay is based on both the 1959 and 1961 data. Surface salinity was generally low and ranged from 32.5 $^{o}/oo$ in northernmost Hudson Bay, to 27 $^{o}/oo$ in the approach to James Bay and to less than 10 $^{o}/oo$ at the head of the bay, most of the decrease occurring in the southern part. Salinity values in excess of 31 $^{o}/oo$ were observed at 50 m in the approach

to the bay and a marked north-south gradient extended well into the bay.

The 1959 data are utilized in the illustration of the longitudinal distribution of surface and bottom salinity (Figure 12) which emphasizes both the strong gradient at the surface toward the head and the marked vertical stratification or structure. It seems likely that stratification is a permanent feature of the oceanography of the bay; a feature which may become more pronounced seaward of the major rivers in winter to the extent that freshwater occurs under the ice. Such a circumstance has been observed in Kugmallit Bay (Barber, 1968b) just east of the delta of the Mackenzie River and in Tuktoyaktuk Harbour. The spatial distribution of the freshwater layer seaward of Kugmallit Bay was not observed, but it was visualized that the freshwater moved eastward close to the coast within a surface layer and with little mixing with deeper salt water. As entrainment of salt was minimal the normal estuarine type of circulation did not exist and it was possible to discern an exchange process due to tides. In James Bay, however, the rms value of the tidal current appears somewhat larger than in Kugmallit Bay so that tidal mixing may be considerable. This would tend to remove stratification so that the layer of freshwater suggested here may exist only in the immediate vicinity of the various estuaries. Nevertheless, in order to emphasize the lack of data and the considerations in

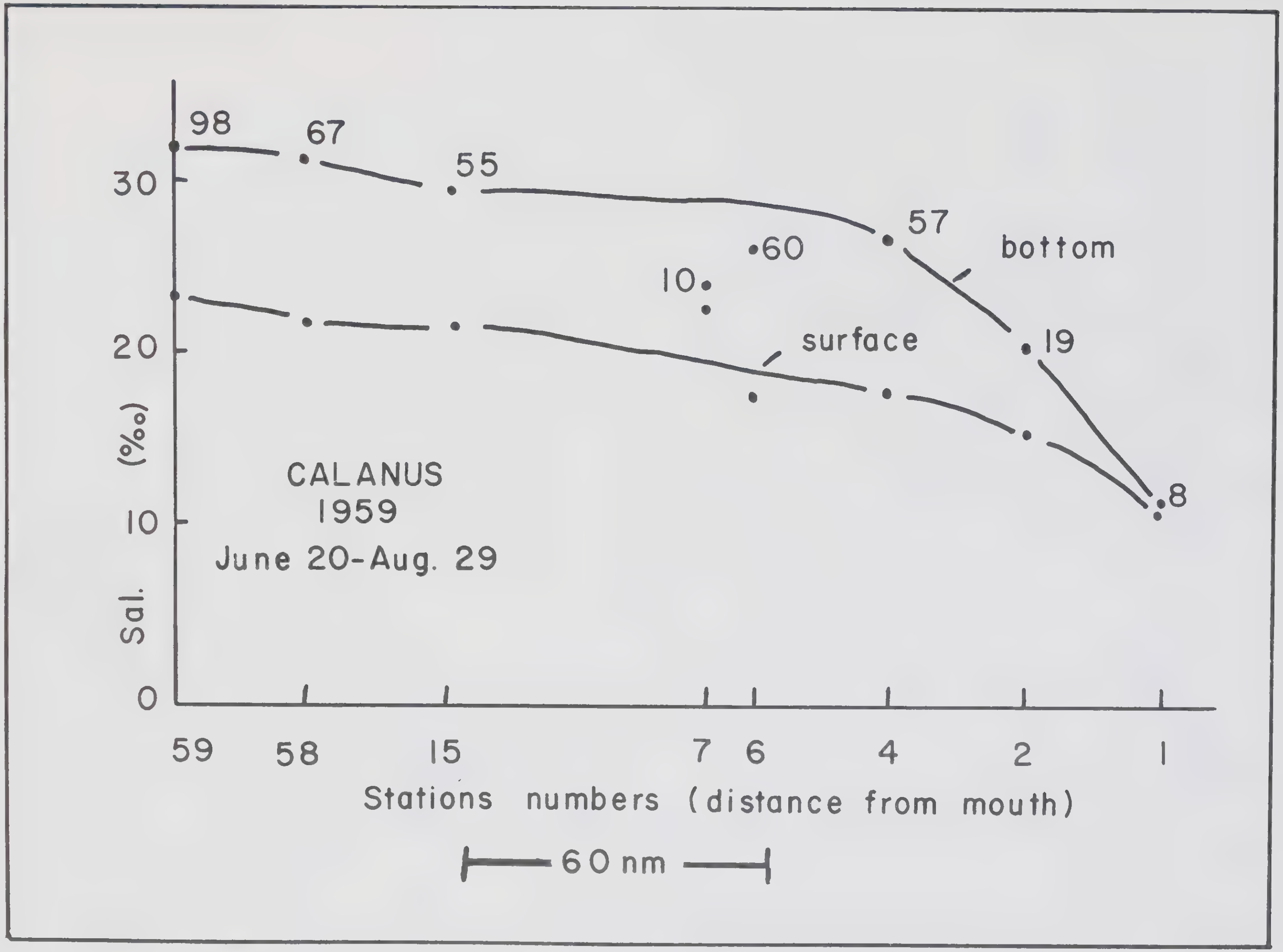

Figure 12. A longitudinal distribution of surface and bottom salinity based on "Calanus" data during the period June 2 to August 29, 1959. The depth in metres of the near-bottom sample at each station is indicated.

other sections about the coupling to Hudson Bay, the layer is shown (Table 1) as extending to Trodely Island.

It was suggested earlier (section 1.2) that in the approach to James Bay a surface layer probably existed within which seasonal changes of temperature and salinity could be recognized; the salinity change was due to variations in the annual ice cover. Observations at about one location in a section across the mouth of the bay (Figure 13) support the view that such a change occurs, but the extent to which it reflects the influence of the annual ice cover as opposed to annual runoff is not known, expect that at positions in the section close to the west and east coasts, a variation in salinity occurred which is believed due to runoff (Figure 24).

2.2.3 Secchi disk depth

The distribution of Secchi disk depth in James Bay in 1959 and Hudson Bay in 1961 during the period of navigation of those years is shown in Figure 14. Least values occurred in James Bay where depths less than 2 m were observed in the south and where in general the depth appears to have been less than 5 m. In the approach to James Bay values to 10 and 15 m occurred. Over much of Hudson Bay readings were close to 15 m with a significant number to 20 m and an occasional (2) reading to 25 m.

The relatively low values in James Bay may be due to a sediment load associated with the large inflow of freshwater from runoff, although other factors could be of equal or greater significance. Langford (1963)

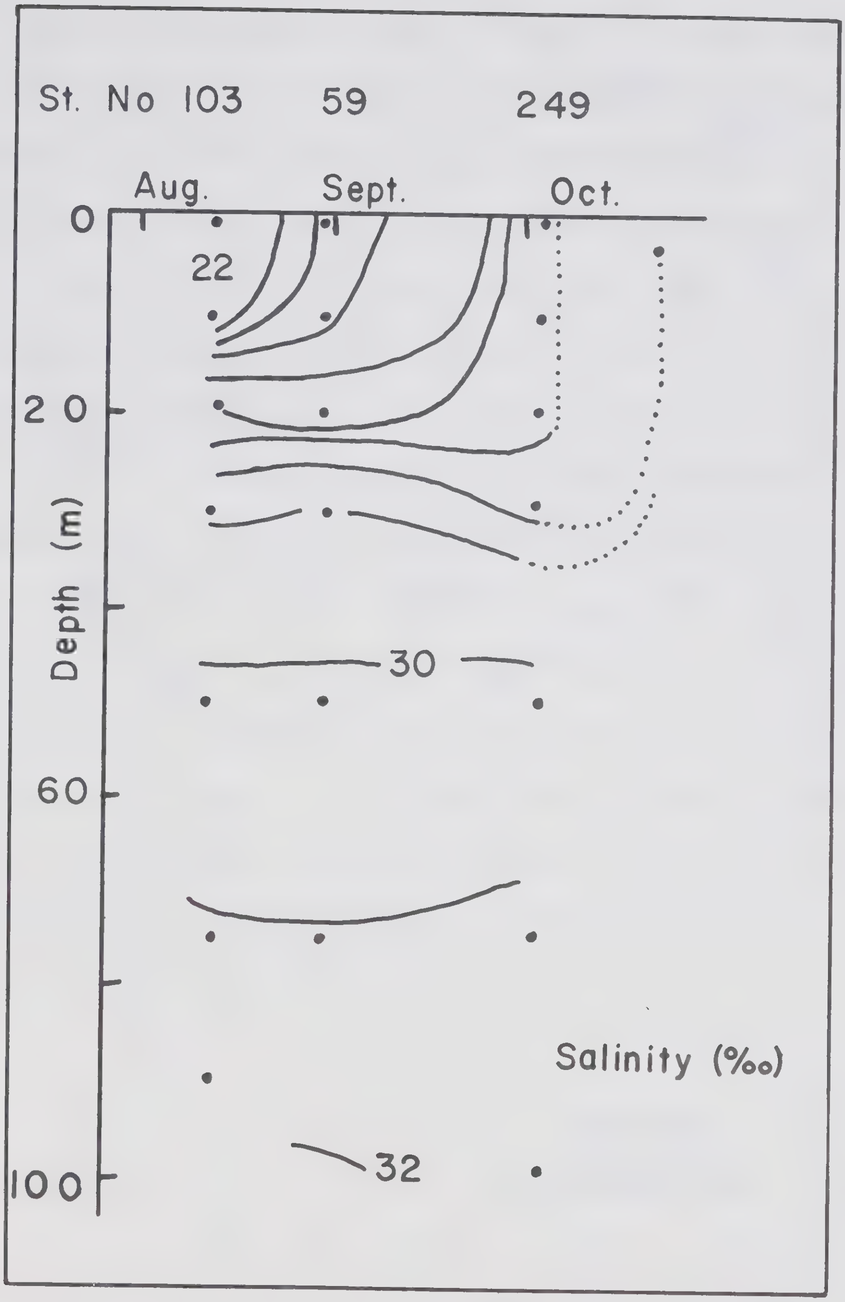

Figure 13. The distribution of salinity at about one location in the mouth of James Bay derived from "Calanus" station 59 in 1959 and "Theta" stations 103 and 249 in 1961.

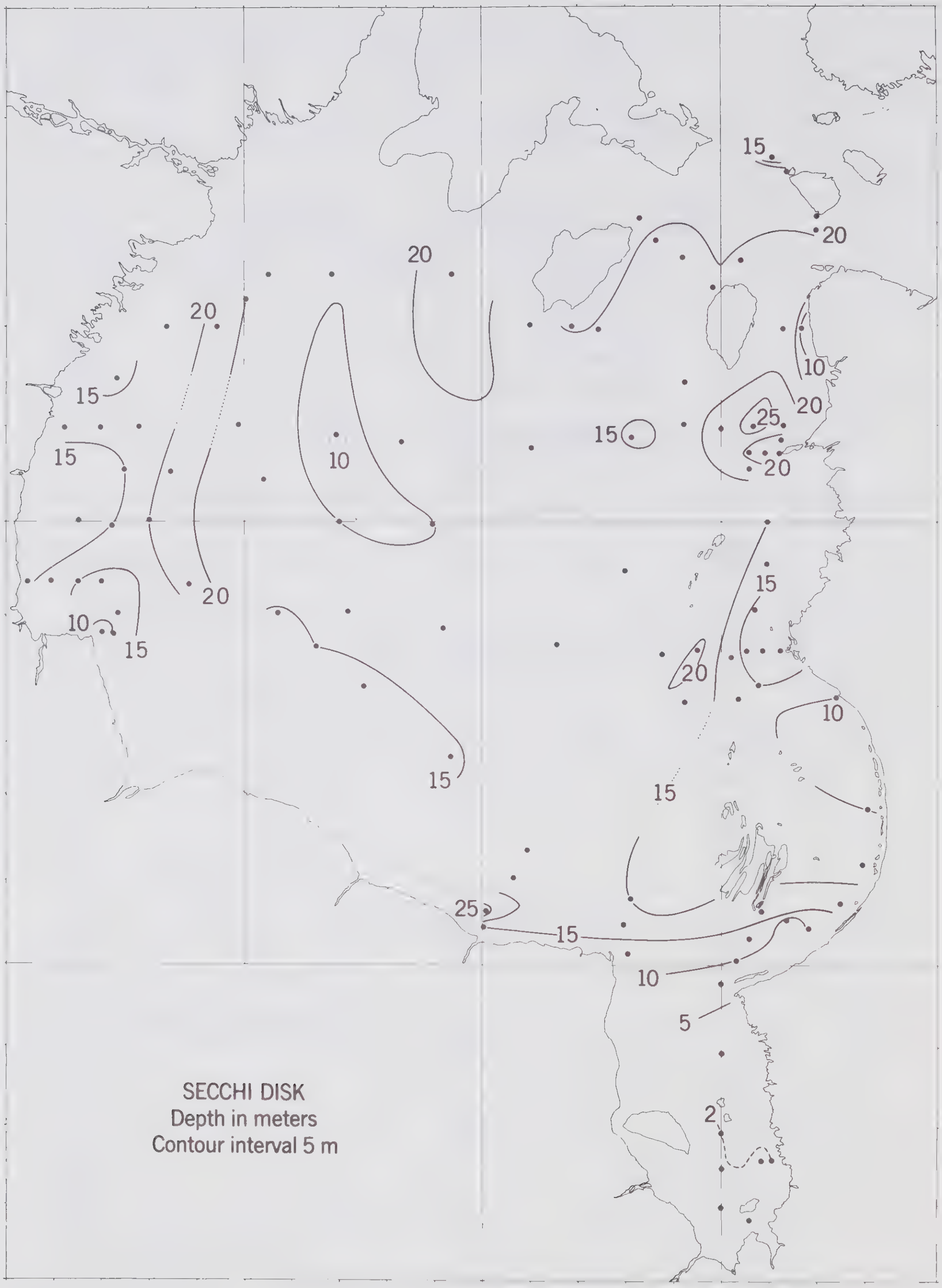

Figure 14. The distribution of Secchi disk observations in James Bay and Hudson Bay based on data observed in 1959 (Grainger, 1960) and in 1961 (Anon., 1964a). The solid circles indicate the location of the observations. A dashed contour indicates one additional to the regular contour interval; a dotted contour indicates a doubtful interpretation.

considered that as the sediment load of the Moose River appeared small in summer months it must be large during the spring freshet. The north-south gradient at the mouth of James Bay does not appear to be reflected in the distribution of either the surface salinity (Figure 4) or the freshwater content (Figure 11) as might be anticipated. A tentative conclusion is that the relatively low Secchi disk readings in James Bay represent the influence of that portion of the input of freshwater derived from runoff*, as opposed to that derived from the ice cover of the previous winter; the gradient across the mouth would represent the "front" of the seaward moving freshwater from runoff. This suggests that the stored volume of freshwater within James Bay derived from rivers entering there may be distinctly time-dependent. Secchi disk depth would then be time-dependent, and so might sediment deposition.

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*It was anticipated that the optical difference of the sea surface caused by the sediment laden runoff into James Bay during the spring and early summer might be visible in APT optical photography. While the available data are not sufficient to allow a study of this, it does seem that the contrast is so slight that a muddy water would not be recognized. On the other hand, Taggart *et al.*, (1965, p190) were able to detect tone variations in the APT photography of the bay in September which they related to the shallow water there.

The latter feature could lead to the existence of annual varves in the sub-bottom vertical sediment structure. Leslie (1965, p136) studied the sediment core obtained at "Theta" station 104 in the approach to James Bay but did not indicate the existence of varves. However, he did suggest that the bay is an important source of finer material in the bottom sediments of Hudson Bay. He remarked (p18), "North and east of James Bay the bottom sediments consist mainly of medium grey silty clay," and (p20), "James Bay is the source of medium grey sediment along the southeast coast and around the Belcher Islands. Rivers flowing into James Bay drain the region to the south which is underlain by soft Mesozoic shales and siltstones. Much of this fine detritus is carried into James Bay and thence into Hudson Bay". This distribution of sediment finer than 2mm in diameter as deduced by Leslie (1964) is shown in Figure 15. The area of silty clay, which apparently originates in James Bay, extends to the north and east and the westward in the northernmost part of its distribution. The distribution is compatible with present knowledge of the circulation; the main feature of which is believed to be an anticlockwise movement around the bay with relatively strong northerly currents along the east shore. The westward extension may reflect the

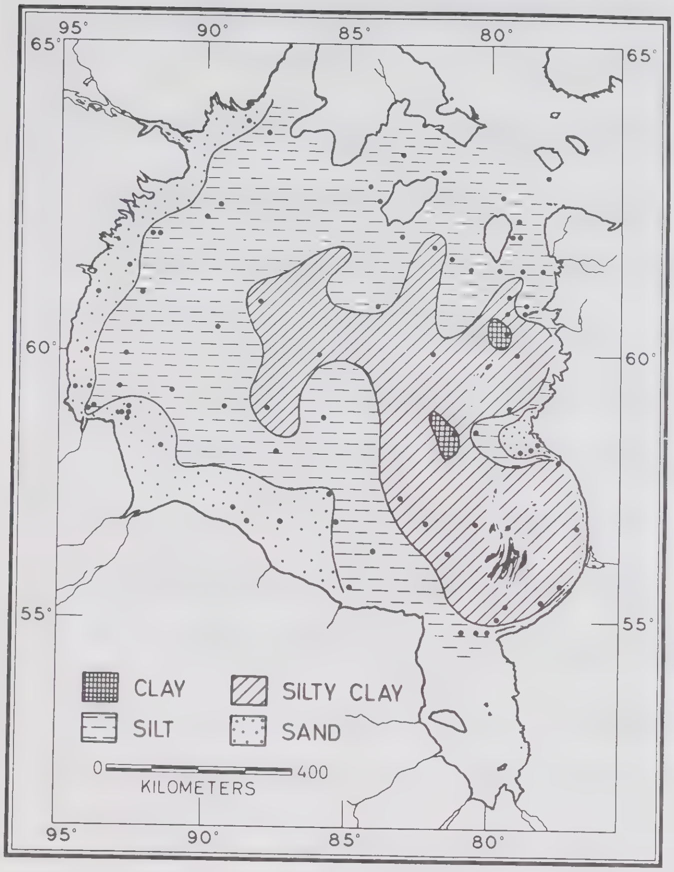

Figure 15. A re-presentation of Figure 8 of Leslie (1964) the caption to which read, "Distribution of bottom sediment finer than 2mm in diameter, or the predominately water-deposited material".

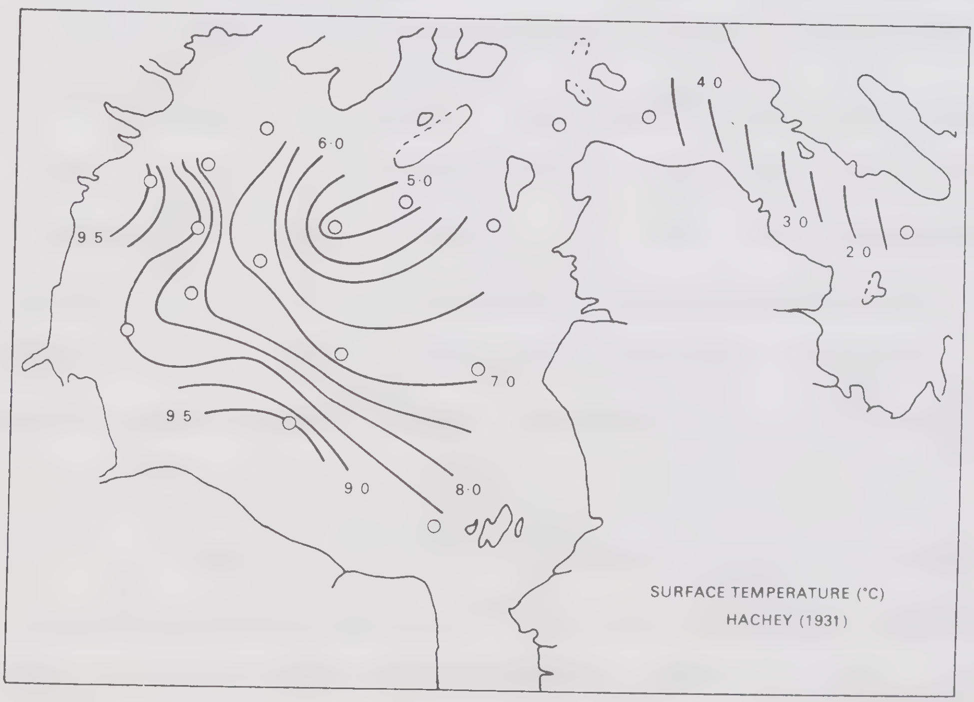

Figure 16. A reproduction of the distribution of surface temperature in Hudson Bay in 1930 (from Hachey, 1931b his Figure 8).

influence of a pattern of water re-circulation* within Hudson Bay.

2.3 Some recent observations within the system

Wendland and Bryson (1966; 1967) obtained surface temperature data in Hudson Bay using airborne radiation thermometry and concluded, "A signature of last ice of Hudson Bay apparently can be located throughout the remainder of the ice free season". They attributed the persistence of the feature to the "Stratification of the upper layer of the Bay"; a stratification in salinity due to melting ice. It seems that such stratification could have quite the opposite effect and also it seems that other factors could be important including the ice cover itself. Nevertheless, the existence of such a "signature" in the temperature distribution at the surface appears to be entirely possible as other summer data reflect such an influence and it is known that each year a characteristic pattern of ice dissipation, from north to south, can be expected (Markham, 1962, p6; Larnder, 1968, p335; Danielson, 1971). The 1930 data of Hachey (1931b) are an exception, for not only is a signature not apparent (Figure 16) but also the surface temperatures, particularly in the west and

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*A (drift) bottle set adrift in 1952 at a position off the northeast coast of Hudson Bay, 10 miles west of Povungnituk, was found in 1968 on the southwest shore, 4 miles northwest of the Kaskattama River (Richard H. Russell, personal communication).

in the south, are relatively high. Of the 1930 data Barber (1967, p7) remarked, "...There does not appear to be any effect due to a recent ice cover or accumulation of ice as in the 1961 season". He also remarked (p7) that 1930 may have been a light ice year, but this is quite speculative as the data (Anon., 1931) on the distribution of ice are few. Of these the second voyage of the tug "Ocean Eagle" during July 10 to 18, 1930 suggests a scattering of the ice (p14 and 15) eastward of Churchill rather than a concentration. If a subsequent movement did not concentrate* the ice in 1930 it is possible that a condition similar to the unusually open season of 1962 occurred. In this, the main portion of the bay seems to have been effectively clear of ice by August 1, (Anon., 1963) so that there would have existed sufficient time and open water for the surface waters to have been warmed by insolation in 1930 to the extent indicated by Hachey.

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*Ice was observed off Port Harrison on August 8 and 9 in SS "Nascopie" (Anon., 1931, p19) and "loose ice" was observed in SS "Ungava" (p21) on August 2 at a position (57°06', 82°58') west of the northernmost Belcher Islands and north of Cape Henrietta Maria on a voyage to Charlton Island. Both support the considerations that the ice condition in 1930 was similar to that of 1962 and not similar to either 1967 (Anon., 1969, Figure 16) or 1969 (Anon., 1971e, Figure 16).

It is a tentative conclusion that in some years the "signature of last ice" may not be apparent.

That 1967 would not likely be such a year was predicted from study of satellite imagery and of ice forecasts (Anon., 1967b) which indicated that an open water situation was well-defined in northwest Hudson Bay and close along the east coast by about May 11. Subsequent imagery indicated increasing open water in the northwest and little or no open water along the east coast where it had been earlier observed. The latter change suggested a movement or "pressure" from the west to east. As well, there was little evidence of open water in the eastern half of the bay prior to the end of June; indeed, some ice persisted in the area south of Coats and Mansel Islands to the end of July (Figure 17). An open water condition along the west coast toward the Bay of Gods Mercy was interpreted from the imagery of April 28; a condition which has been described (Dunbar and Greenaway, 1956, p418; Bowley, 1969, p13). Another feature was the persistence of an ice boundary in Roes Welcome Sound throughout the period April 28 to July 12 (Figure 20a). In 1961, aerial reconnaissance indicated open water throughout Roes Welcome Sound on July 13 (Archibald *et al.*, 1962, their Figure 8) and the relatively high (25 kg cal cm^{-2}) seasonal heat storage to about the latitude of Wager Bay (Barber, 1967) indicated that open water had occurred there early in the season and persisted up to the time of the temperature observations. Clearly, the heat storage in the area in

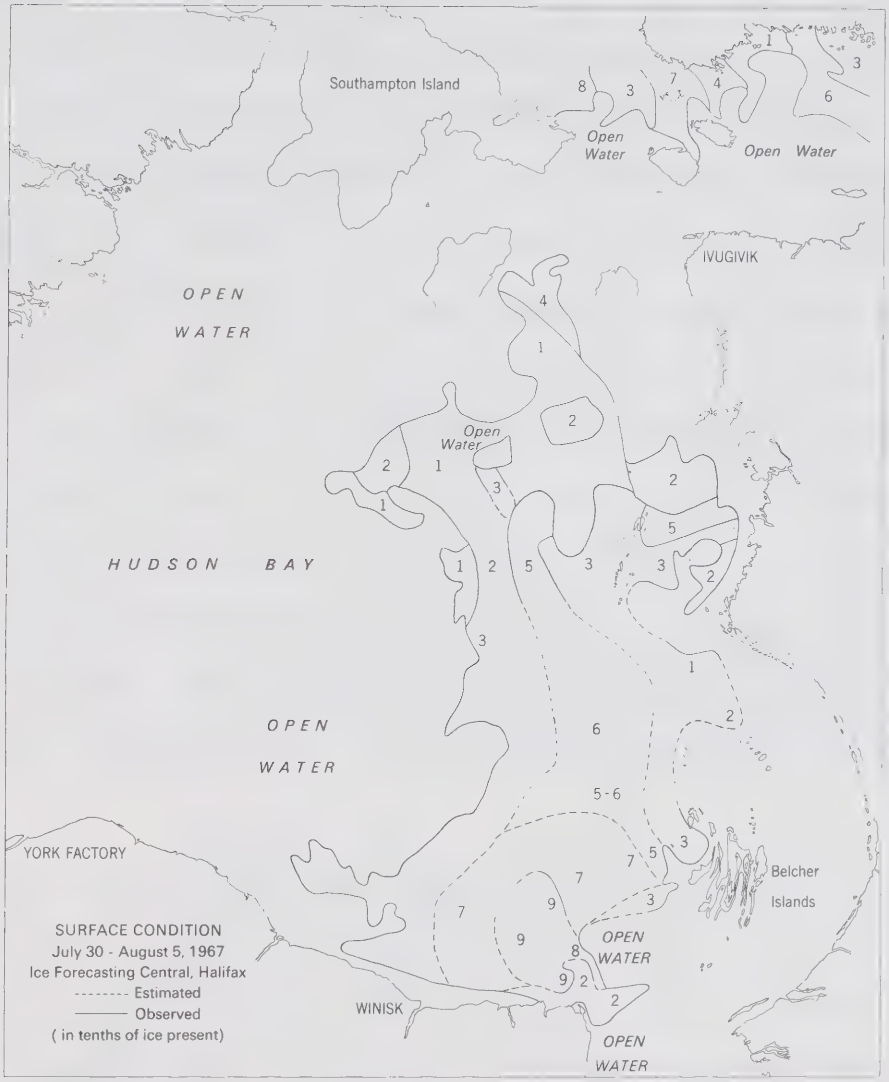

Figure 17. The distribution of open water in Hudson Bay about the end of July, 1967 from Ice Forecasting Central, Department of Transport, Halifax.

1967, if observed, would have proved much less than in 1961. Similarly the slow development of open water in the east suggested that the absorbed incoming shortwave radiation, and hence the surface temperature and seasonal heat storage of the water, would be much reduced. This prompted a request to the Department of Transport for bathythermograph observations in the northeast. These were obtained by CCGS "Labrador" (Anon., 1968) and revealed that the seasonal heat storage was low (Figure 18), certainly much lower than occurred at about the same time in 1961 (Barber, 1967). The subsequent airborne survey in late August (Wendland and Bryson, 1967, their Figure 3) indicated a relatively low surface temperature, which confirmed that in the eastern portion of Hudson Bay the peak of the seasonal heat storage was less than average. Hence their conclusion concerning the "signature of last ice".

On the other hand surface temperature data observed between the Bay of Gods Mercy and Churchill about mid-July in 1967 (Figure 19) indicated a level of temperature close to that observed about the end of July in 1961 (Barber and Glennie, 1964, their Figure 17). If it is assumed that the development of the surface layer was similar in the two years, then it is a tentative conclusion that the peak value of the seasonal heat storage there was at least as great in 1967 as in 1961.

More recently, the 1969 season in terms of development of open water also appears to have been

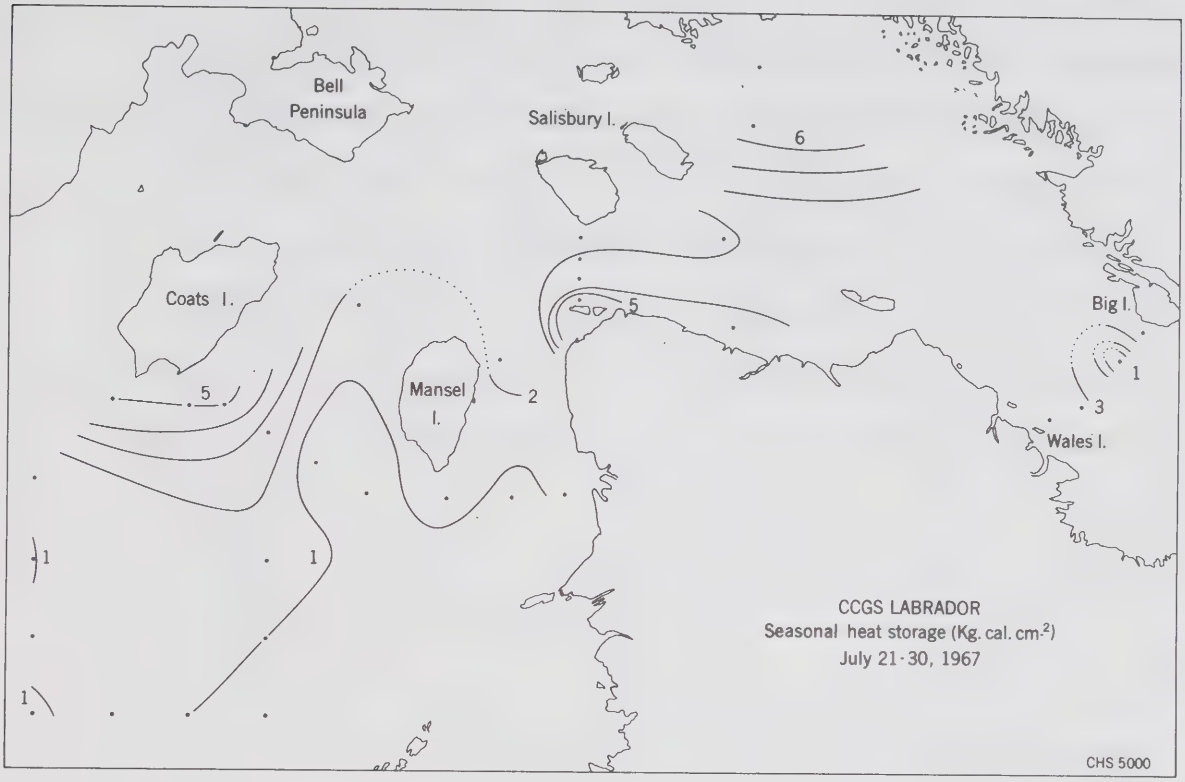

Figure 18. The distribution of the seasonal heat storage in late July, 1967 as interpreted from bathythermograms obtained in the C.C.G.S. "Labrador".

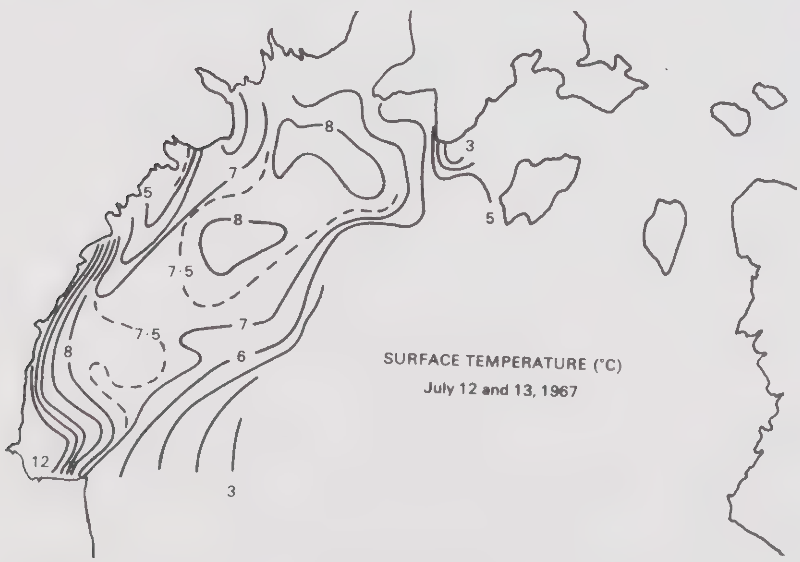

Figure 19. The distribution of sea surface temperature as observed July 12 and 13, 1967 using an airborne radiation thermometer (from Wendland and Bryson, 1967, their Figure 1).

anomalous for, "in southern Hudson Bay final clearing after mid-September was the latest on record" (Anon., 1971e, pl); the persistence was related to the occurrence of "some heavier than usual ice" (p42). At the same time the existence of, "Broad areas of open to close pack ice," west of Belcher Islands to August 20 (p42) was also considered unusual as was the, "Break-up and disintegration of ice which was earlier than normal in northern Hudson Bay..." (pl). The distribution of ice in May and June strongly suggested the influence of advection due to wind and examination of the data indicated that winds from the west through to north occurred 75 percent of the time at Chesterfield, i.e. about 25 percent greater than normal. This pattern of wind no doubt contributed to the earlier open water in the north and to the observed accumulation of ice toward the south and east. It is suggested that the accumulation here was such as to maintain a high average albedo well into the normal melting season and was a factor in the persistence of the ice cover there.

The deeper temperature data of 1967 (Figure 20a), while limited, suggest the existence of a slighly warmer water, to -0.9°C, in a tongue-like distribution southwest of Coats Island similar to that observed in September 1962 (Figure 20b). Such warming is thought to be due to downward mixing of heat absorbed at the surface in that season, but was not considered to occur as early as observed in 1967, i.e. as early as the end of July. It seems possible that the extensive open water in 1967 in the northwest

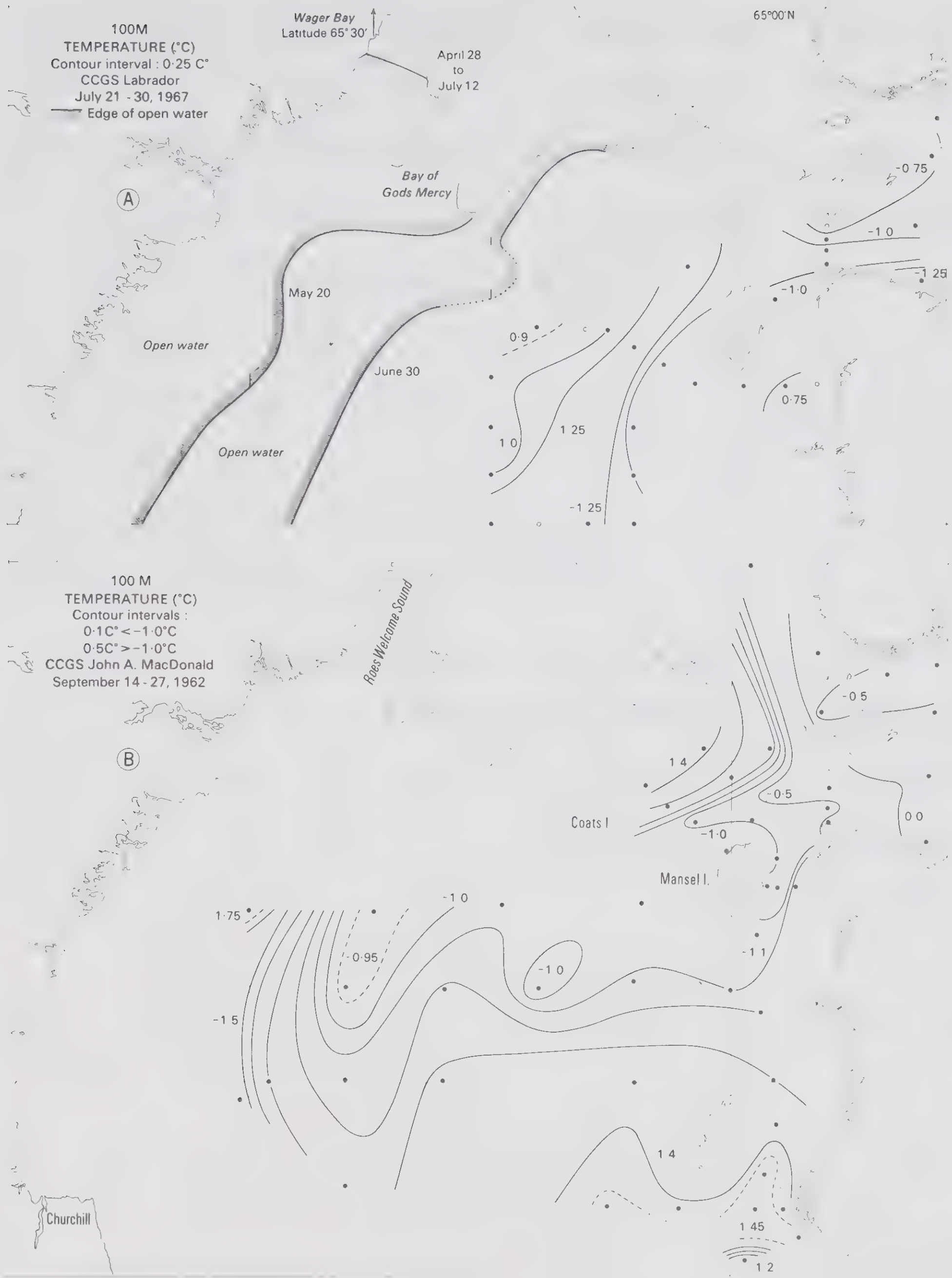

Figure 20. The temperature distribution at 100m depth in 1967 and 1962. (a) From the 1967 "Labrador" bathythermograms. The edges of open water at the dates indicated are an interpretation by the author of satellite (APT) imagery. (b) From the 1962 "John A. Macdonald" serial data (Anon., 1966).

may have contributed to this result which, due to the similarity in open water, probably occurred again in the 1969 season.

There is one major caveat here concerning the 1967 data and that is they are the result of but one instrument, i.e. one bathythermograph, so that both the values and pattern (Figure 20a) are suspect. The chances are, however, that the bathythermograph data are appropriately used here relative to the assessment of heat storage. It is realized that this may not be the case for all data observed within the system, for certain of the apparent differences are disconcerting. The most recent example is that of Pelletier *et al.* (1968) who, even though aware of earlier interpretations, could report for Hudson Bay, temperatures to -2.0°C (their Figure 4 and p565) and salinities in excess of 34.0 ‰ (their Figure 6* and p565) without particular comment, either to the precision of the observations or their relevance. Such extreme values were reported on earlier occasions, but at a time when data were relatively few. For example Campbell (1964, p49) reported temperatures to -1.97°C and salinities to 34.07 ‰ at unusually shallow depth in Foxe Basin.

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*Their figure 6 indicates a value of 35.56 ‰ which may be an error in the preparation of the illustration.

The existence of such extreme value generally leads to conjecture about the process which forms the water; a process which may be a major feature of the oceanography of the region. A consideration of all the data led to the definition of the probable winter surface salinity (Barber, 1967) and to a simple model (p54) of the Hudson Bay system. The model also served as a review of earlier concepts and emphasized that these were hampered through lack of winter data. Hachey (1931b) realized this and developed the view that, "the waters of Hudson Bay differ markedly from the waters of Hudson Strait and the waters of the open ocean". Of course he was not aware that conditions in Hudson Strait exhibit marked seasonal variation. He was led (Hachey, 1954) therefore to consider the evolution of Hudson Bay water alone and offered two conjectures. In one was visualized removal of the observed summer low salinity surface layer prior to freeze-up, so that water cooled at the surface would sink and become the deep water. In a variation, he visualized that water initially at the surface in winter in Hudson Strait moved into Hudson Bay replacing a less-dense seaward flowing water. In the other he suggested a movement "of deep, cold, saline water" into Hudson Bay from Foxe Basin; the origin of such water remained a question. Campbell (1964) expressed his accord with the latter hypothesis, describing observations made in 1955 and 1956 in support, and developed a theory wherein the cold high-salinity water in Foxe Basin was directly related to the production of winter ice in

the extensive inter-tidal zone (tide flats) of Foxe Basin. Apparently he considered the "exposure" of the tide flats the significant feature in the process and visualized the production of a "concentrated brine solution". No primary evidence which might lend credence either to the existence of such a brine solution, or to the main hypothesis, was presented. However, the described data are important as they demonstrate the existence below sill depth in Foxe Channel in both years of a high salinity water (about 33.7 $^{o}/oo$) close to the freezing point. This conditioning must have occurred at the surface in association with cooling during the winter months. The water so conditioned would sink and move away, if not depth limited, from the source area eastward into Hudson Strait, and into Hudson Bay, enhanced perhaps through increased heat losses in areas of open water, including tide flats. The extent of open water in the system at this time is not known; however, amounts likely significant in the formation of the water are believed to occur in northern Foxe Basin, throughout Hudson Strait, Foxe Channel, and Frozen Strait to Roes Welcome Sound.

Looking beyond the system, it is suggested that the process would be favoured by an extension of the Baffin Current into Hudson Strait along the north shore and into the area of Foxe Channel. The evidence for such a current during the winter is limited, but it is the hypothesis that it exists and in association with a process similar to that described by Mosby (1934) for the formation of

a cold, saline water on the Antarctic continental shelf, particularly in the Weddell Sea. In this, he elaborated the conclusion of Brennecke (1921), that the deep and bottom water of the Antarctic is derived in part from an extremely cold and moderately saline water formed in winter through cooling at the surface and freezing on the shallow shelf. Mosby emphasized the importance of horizontal movement whereby the surface water sinks and moves away from the source area, and off the shelf. The similarity of the situation in the Antarctic with that observed in the Foxe Basin - Hudson Bay system is heightened for according to Mosby, a portion of the winter-formed water is prevented from mixing into the Antarctic deep through the existence of a limiting depth. Thus, it retains its main characteristics, i.e. like the deep water in Foxe Channel a temperature very close to the surface freezing point. It was these data, specifically those at "Deutschland" station 125, that led Brennecke to the hypothesis concerning the formation of the winter shelf water, and its subsequent contribution to the deep water. Deacon (1937) and Fofonoff (1956) have contributed further to the understanding of the process.

In the situation for Hudson Bay it is thought that the observed cold and saline water at depth in Foxe Channel is the result of the cumulative effects of cooling and ice formation at the surface of the current throughout its movement along the east Baffin Island coast, into Hudson Strait, and into Foxe Channel and Foxe Basin. It

is not recognized as a distinct water outside the system in the Labrador Sea because it is too dense to remain at the surface and is not sufficiently dense to participate in the formation of a distinct bottom water. It must then contribute to an intermediate water there.

An assessment of this contribution will not be attempted here, although it is of major interest, for it seems that much more needs to be known of the influence of Hudson Strait, where intense mixing due to tides leads to a further modification of water characteristics. These subsequently became recognizable within Hudson Bay through an inward movement associated with the estuarine circulation, which in effect results in considerable recirculation, i.e. the coupling of the system to the Atlantic through the estuarine circulation is limited by mixing in Hudson Strait.

Assessments of plankton data by Grainger (1961) and Bursa (1961) showed that both Atlantic and Arctic plankton types were found in Hudson Bay, and Grainger (1962) made a similar observation with regard to Foxe Basin. In a study of the distribution of three species of copepod, Grainger (1963) showed that while two Arctic species were widespread in Hudson Bay, the Atlantic was not. Huntsman (1954) in a discussion of the production of life in Hudson Bay outlined a number of reasons for the apparent low productivity as compared to areas at similar or higher latitude and, while appearing to emphasize the "lack of heat," concluded that as yet the data are

too few "on which to base proper judgement". Other factors which might influence the "apparent low production" (Grainger, 1968, p357) include "the effects of long periods of ice cover" (p358) and consequences related thereto. Dunbar (1970) on the other hand apparently considers nutrient levels to be important. He suggested that the oceanographic regime of Hudson Bay would be altered if the "supply were cut off" of Arctic water moving eastward through Fury and Hecla Strait. He considered the significant alteration would occur in the stability which would become less and as a consequence of mixing processes, more of a nutrient of the deeper water would become available to the surface layer, where productivity would increase. Of a number of questions which might be raised, that concerning the influence of Hudson Strait seems the most important. It is visualized above that mixing processes in the strait determine to considerable degree the distributions within Hudson Bay, such that a change in a surface water characteristic, e.g. to a higher salinity, would be reflected in a change in the same direction in the deeper water. Thus, a decrease in the contribution to the system of low-salinity surface water brought about by damming Fury and Hecla Strait would lead to an increase in salinity throughout, perhaps with little change in stability.

It seems that Dunbar assumed that the nutrient level in the deeper water is high, but this may not be. Should the uncoupling and recirculation suggested here

be significant it is possible that the level of nutrient, or of a nutrient, may be generally low throughout the system (it may be that the limited occurrence of Atlantic copepod there may also be a partial result of this uncoupling).

Nevertheless, it would be of interest to demonstrate that Dunbar's secondary consideration is indeed possible, i.e. that productivity would increase in a region of annual ice cover were the level of nutrients increased. As noted, data are not available for Hudson Bay, but the evidence for some other areas with ice cover indicate that a depletion of at least one nutrient occurs by mid-summer (Apollonio, MS undated; McLaren, 1969). If this is so, it seems that an experiment (McLaren, 1969) in which nutrient is applied to a relatively isolated (uncoupled) body of water in order to avoid the development there of a period of nutrient depletion might provide a useful result. Omarolluk Sound is suggested as a suitable site should it be determined that the nutrient depletion occurs there.

A further consideration here relative to Hudson Bay is that of the dissolved oxygen. It seems that it is characteristic of the water in Hudson Strait to be near saturation levels, from which it follows that the moderate depletion of oxygen observed in the deeper water of Hudson Bay occurs entirely within the bay. If the consumption there were known, it would be possible to estimate an age for the oxygen depleted deep water. This is apparently

not known, but is likely about the 0.21 ml/l/year suggested for the North Atlantic by Riley (1951). Assuming about 80 percent is retained after leaving the surface and subsequent oxidation of surface nutrients (Redfield *et al.*, 1963), then the age is in the range 5 to 7 years.

2.4 Longer term change within the system

Bailey and Hachey (1951) recognized that the general level of salinity observed by Hachey (1931b) in 1930 in Hudson Bay was low. They compared the data to observations made in 1948 and suggested that the observed difference was due to an increased Atlantic influence. It was suggested (Barber, 1967, p55) that it is not possible through study of other data to reject their hypothesis. In the latter work and in Bailey and Hachey (1951) it appears to have been assumed that the tabulated depth and salinity values (Hachey, 1931b, p96) are without more than the usual error.

It is not difficult to accept the assumption as regards the salinity data for, although they are generally low, it is possible to see in the observed distributions similarities with more recent data. For example, the distribution in Hudson Bay at 50m (Hachey, 1931b, his Figure 4) can be interpreted so that the pattern is compatible with a 50m distribution shown here (Figure 4) based on much more data. However, there are two salinity values in Hudson Strait, each at 200m depth at stations 57 and 48 (Hachey, 1931b, p96), which on the basis of all the data are 0.5 $^{o}/oo$ lower than is to be expected at the

depth. At nearby station 58 the salinity values within the surface layer, 32.5 $^{o}/oo$, are appropriately high. Thus, the data in Hudson Strait and in Hudson Bay suggest that if a persistent error exists it is not in the salinity and could be in the depth.

Depth data presented in the report of the 1930 observations (Hachey, 1931b) are of two kinds. One is the depth to the bottom at each station (p95) which when plotted appear to fit current information. The other data are the tabulated value of the serial samples (p96). A feature of the tabulation is that each is at a "nominal" depth. This could, of course, be easily achieved, particularly in Hudson Bay where water movements are generally small. In Hudson Strait, however, strong water movements are known to exist so that extraordinary consideration would have had to be given in order to achieve a desired sampling depth, particularly in the circumstance that a cast comprised a lowering of only one reversing bottle (Hachey, 1931b, p95). It is suggested that this did not occur and consequently the depth data are liable to more than the usual error. Consideration of the validity of the hypothesis of Bailey and Hachey (1951) must therefore include an evaluation of the precision of the depth data.

It does appear therefore that there is little good physical evidence on which to base hypothesis for recent changes of marine climate, i.e. beginning about 1930. It seems that such hypotheses were of much interest in about the late 1940's and early 1950's, e.g. Dunbar's

(1951) extensive work, so that even limited data in rather complicated fiords were used to suggest the existence of change (Nutt and Coachman, 1952); more recently such data were said to "document" a change in the water of the fiord (Coachman, 1969, p215).

3. Discussion

The region is ice and snow covered for much of the year so the extent that the relative proportion of land and water areas are altered by the project would not have a major influence during the winter because the ice and snow largely serve to uncouple the underlying ground or water and the atmosphere. This uncoupling and the fact that the winter climate is due to air mass movement of global scale suggests that if the project is to have an influence on the water it would occur at other seasons. We know that the world ocean is largely buffered against change and that the James Bay Project would not have significant impact were the project not located in a system so uncoupled from the world ocean. The limitation to the coupling is due largely to processes within the system including tidal mixing in Hudson Strait. In James Bay the coupling to Hudson Bay is also limited, but through a weak convective circulation. It seems that the project could influence this convective circulation.

It is characteristic that the freshwater from runoff entering James Bay would lead to the observed layer

of low salinity water in the surface and to a distinctive pattern of circulation. The freshwater moves seaward in this surface layer, entraining salt from below and eventually leaves the bay as salt water. The outward movement of salt is balanced, during some time interval, through a sub-surface inflow of water of relatively high salinity so that:

$$T_i \; S_i = T_o \; S_o;$$

As the freshwater moves seaward at a rate equal to the supply then:

$$T_o = T_i + R.$$

If these be applied in the section across the entrance where S_i and S_o may be 29 °/oo and 22 °/oo respectively, then the inflow is three times the runoff; thus the net transport is a significant part of the total circulation.

Present evidence indicates that the strength of the outflow is strongly time-dependent, presumably in association with seasonal changes in the volume of runoff. The outflow would probably be a maximum at some time after the peak in runoff in early summer. Earlier, in late winter, a minimum in the runoff is indicated about which time a minimum in the outflow would occur. As well, the runoff would occur to a region covered with ice and may not entrain the deeper seawater to the same extent as when not ice covered. The energy for mixing is believed input to the system from tides, wind and surface processes of heating and cooling. An ice cover would reduce the influence of wind considerably as well as of the other

surface processes, so that mixing would be less under an ice cover and the volume of the inflow would be even closer to that of the runoff than it is in the late summer.

The significance of the foregoing is that the runoff does not provide a strong coupling to the water of Hudson Bay, i.e. the estuarine circulation is relatively weak, so that changes in the pattern of runoff may not change the coupling significantly. Should changes in the extent of the ice cover occur, say to decreasing cover, then an increase in the coupling could be expected and a greater forced circulation would result. However, the return flow comprises a relatively cold water, -1.4° to -1.0°C, so that an increased coupling during the summer (period of heating) would tend to a colder surface layer. Conversely, uncoupling would tend toward a warmer surface layer.

For example, if James Bay were completely uncoupled from Hudson Bay through a physical barrier across the entrance, except that an outflow occurred to balance the inflow as in a lake, then a direct influence of Hudson Bay water would not occur. Eventually the bay would likely contain freshwater only, which would be warmer than now, probably close of 4°C in the deeper water, and would likely undergo a wider range of temperature in the surface as a greater storage of seasonal heat would take place. An ice cover would still form and would likely be similar to that which occurs now. The sum of the radiative and turbulent flux terms in the annual heat budget would be

close to zero, as it is now believed to be (assuming the average temperature of the inflow is the same as the outflow), but the average temperature would be higher throughout.

If the coupling were somehow steadily increased the water in the bay would tend toward that of Hudson Bay, in particular toward that comprising the inflow, i.e. toward a water of salinity 29 ‰ and temperature about -1.4° to -1.0°C, but eventually without an ice cover. Of course this situation would not likely be achieved, for processes within James Bay would, at some stage, begin to influence distributions within Hudson Bay, i.e. the radiative and turbulent flux terms of the heat budget would show a large deficit which could only be balanced by advection of water from Hudson Bay. It seems, therefore, than an increased coupling to Hudson Bay would tend to a decrease of water temperature, while a decrease in the coupling would have the opposite result. An increase in the runoff to James Bay, for example by diversion from Grande Rivière de la Baleine, would increase the coupling, while the smoothing of the runoff so that a greater portion occurs under an ice cover would tend to reduce the coupling. It does not seem possible now to provide a quantitative estimate of the results which may be anticipated, except that the influence of the smoothed runoff would be greater. This would decrease the coupling in summer so that the water in summer would tend to be warmer, or at least would tend to store more heat. Most of this heat would be given

off during the period of net heat loss prior to ice formation. During the latter part of this period rather massive losses occur, perhaps as large as 500 - 600 g cal cm^{-2} day^{-1}, so that in order for the time of formation of first ice to be influenced, the stored heat would have to be appropriately large. An estimate of the difference has not been possible. Neither has it been possible to determine the changes which might be brought about in the average pattern of ice cover.

In this simple analysis it is assumed that changes in the surface condition in Hudson Bay, mainly that of the ice and snow cover, will not occur. There is, however, the possibility, described in earlier section (2.2.1), that the present distribution of ice in early summer there may reflect the inferred strong time-dependence in the outflow at the surface from James Bay. Some conclusions might be possible were we to achieve an understanding of the factors which determine the present average situation.

Winds, which are generally from the northwest and north early in the breakup, have generally become light and variable by mid-July and continue so into August. Thus, after creating an accumulation of ice toward the south of Hudson Bay by about the end of July, the wind has little or no influence so that no subsequent ice movement occurs due to wind. Furthermore, there is evidence that during this period of stagnation the frequency of both cloud cover and fog is increased so that less insolation reaches the ice surface and melting is retarded;

complete clearance of the ice may not occur till about the end of August on the average.

In conclusion, it seems likely that advection is not a significant factor in the total heat balance of the system and although a real understanding of the relative influence of inflows and outflows and of processes is not achieved, the system within Hudson Strait appears to be closed rather than open. This tentative result emphasizes the importance of the radiative and turbulent flux terms, i.e. the mainly climatic factors, which in turn reflect the influence of a process global in scale, at least in winter through to early summer, which determines the nature of that important variable the surface condition, at this time either ice or snow. Within the system the water of James Bay is not strongly coupled to that of Hudson Bay but as similar climatic conditions prevail so the surface condition is one of ice or snow for a significant period. It does not seem likely that the project could measureably influence the coupling of the system to the world ocean. It could, however, influence the coupling of James Bay to Hudson Bay to the extent that a greater storage of heat may occur within James Bay during summer after breakup and influence the distribution of ice in the approach to the bay during breakup. Thus the impact of the James Bay Project on the system will be relatively small and it may be necessary, in order to assess the impact, to acquire a particularly extensive body of data; a significant portion of which should be obtained during the period of ice cover.

At other times it may be possible to utilize remote sensing techniques to considerable advantage; although the fact that the peak of the present runoff occurs when there is still considerable ice cover emphasizes the difficulty of the field problem. Of particular value now would be information on the water structure over a period of a year at one postion within the bay, although data in sections seaward of Rupert Bay, Fort-George and Poste de la Baleine and eventually at Inoucdjouac would be preferred. Part of the field programme could be based on relatively simple oceanographic instruments with a frequency of observations about once a week and perhaps sustained by residents there, at least during the period of ice cover, as occurred at Tuktoyaktuk (Kelly, 1967, p8).

4. Acknowledgements

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6. List of figure captions

7. Appendix

Figures 21, 22, 23 and 24 based mainly on the "Calanus" 1959 data (Grainger, 1960), but including some of the "Theta" data (Anon., 1964a).

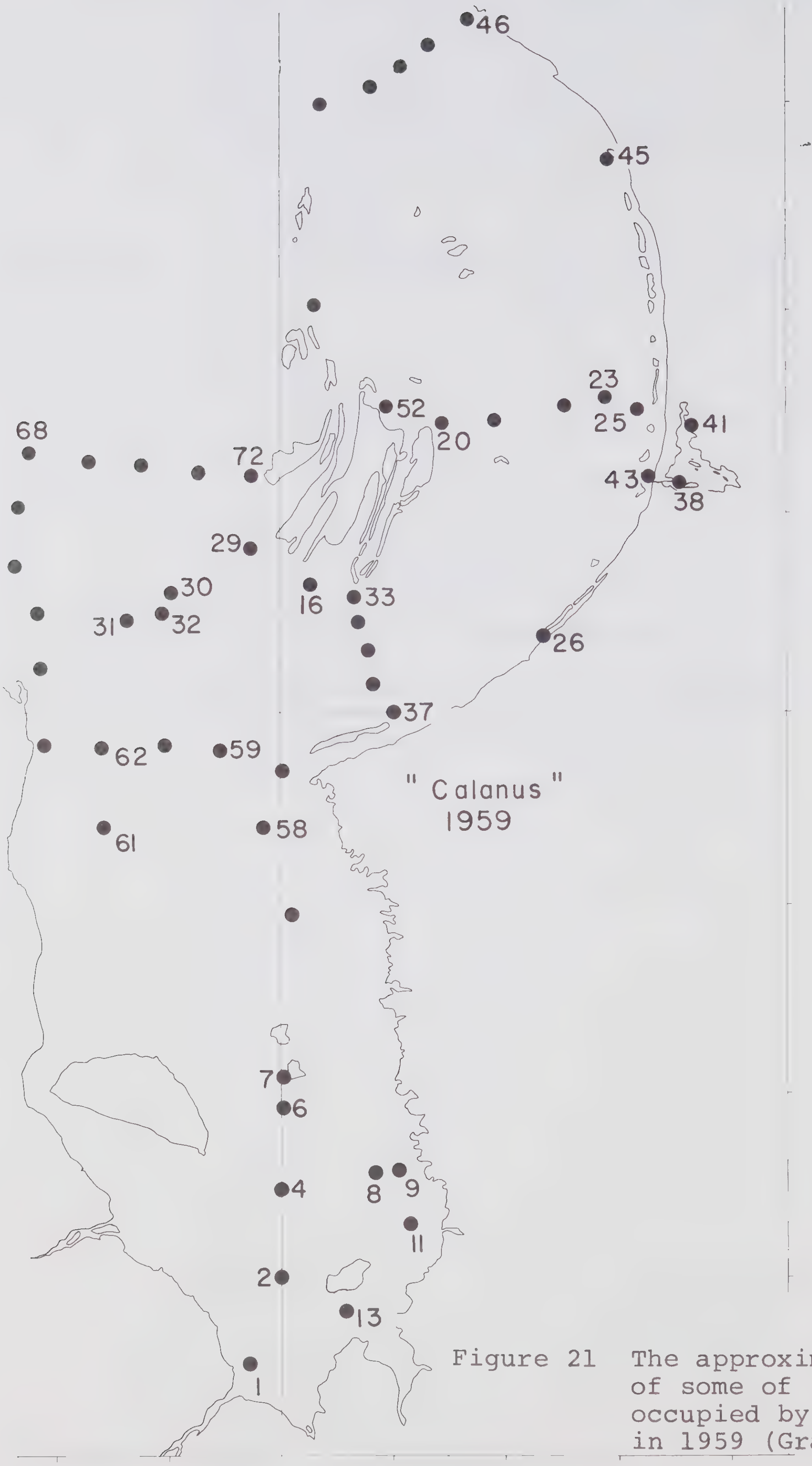


Figure 21 The approximate location of some of the stations occupied by "Calanus" in 1959 (Grainger, 1960).

Figure 22. Distribution at various depths of salinity (o/oo) and temperature (°C) as interpreted from the "Calanus" data of 1959. The positions of the stations are shown in Figure 21 and the date of sampling in Table 2 where it may be seen that the interval between occupations was relatively long.
(a) Surface. (b) 10m. (c) 25m.
(d) 30m. (e) 50m. (f) Deepest.


24
23
24
20
11
SURFACE
SALINITY (‰)
5
5
5
7
7
9
2
0
10
4
10
SURFACE
TEMP. (°C)

Figure 22(a)

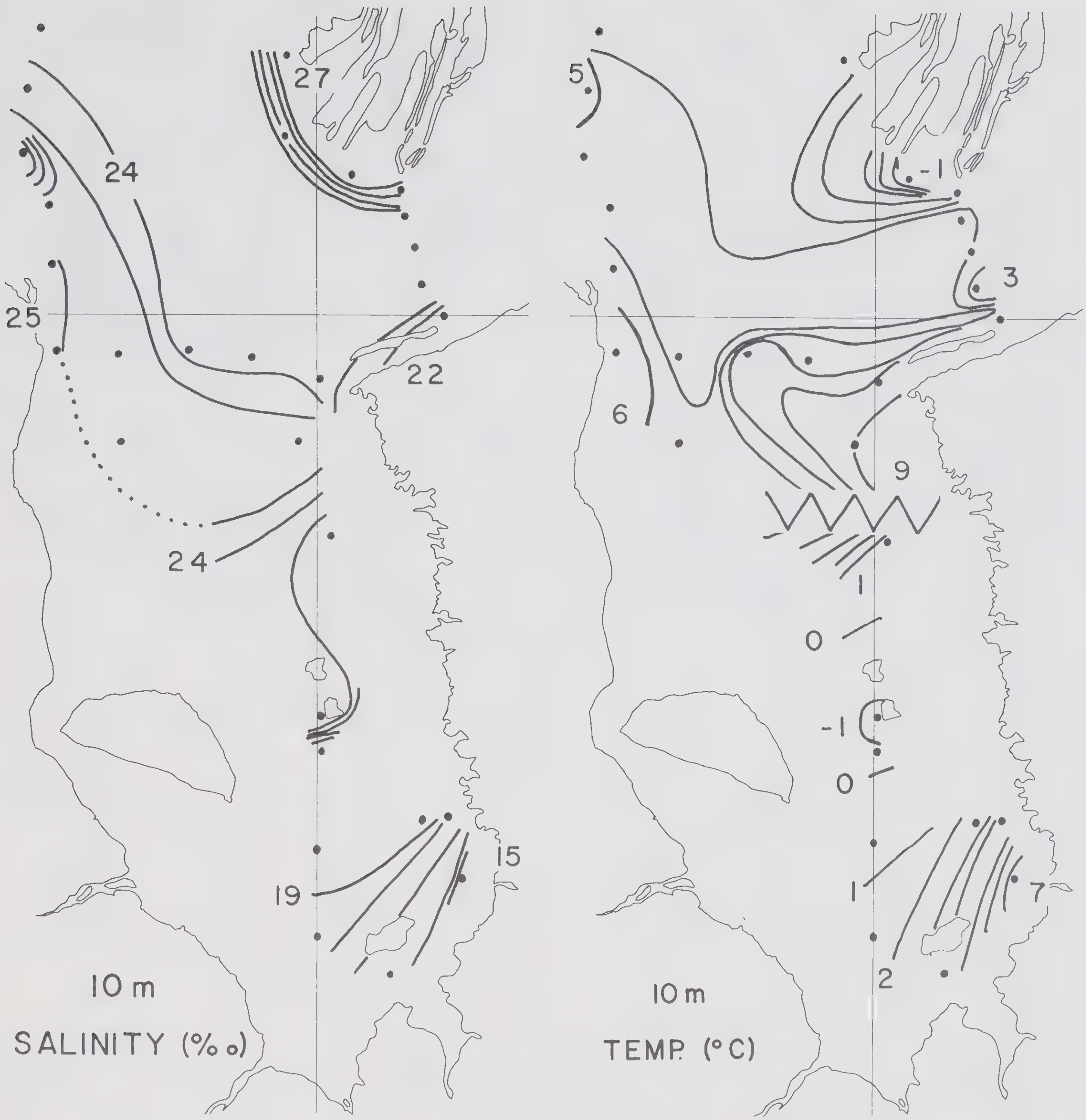


Figure 22(b)

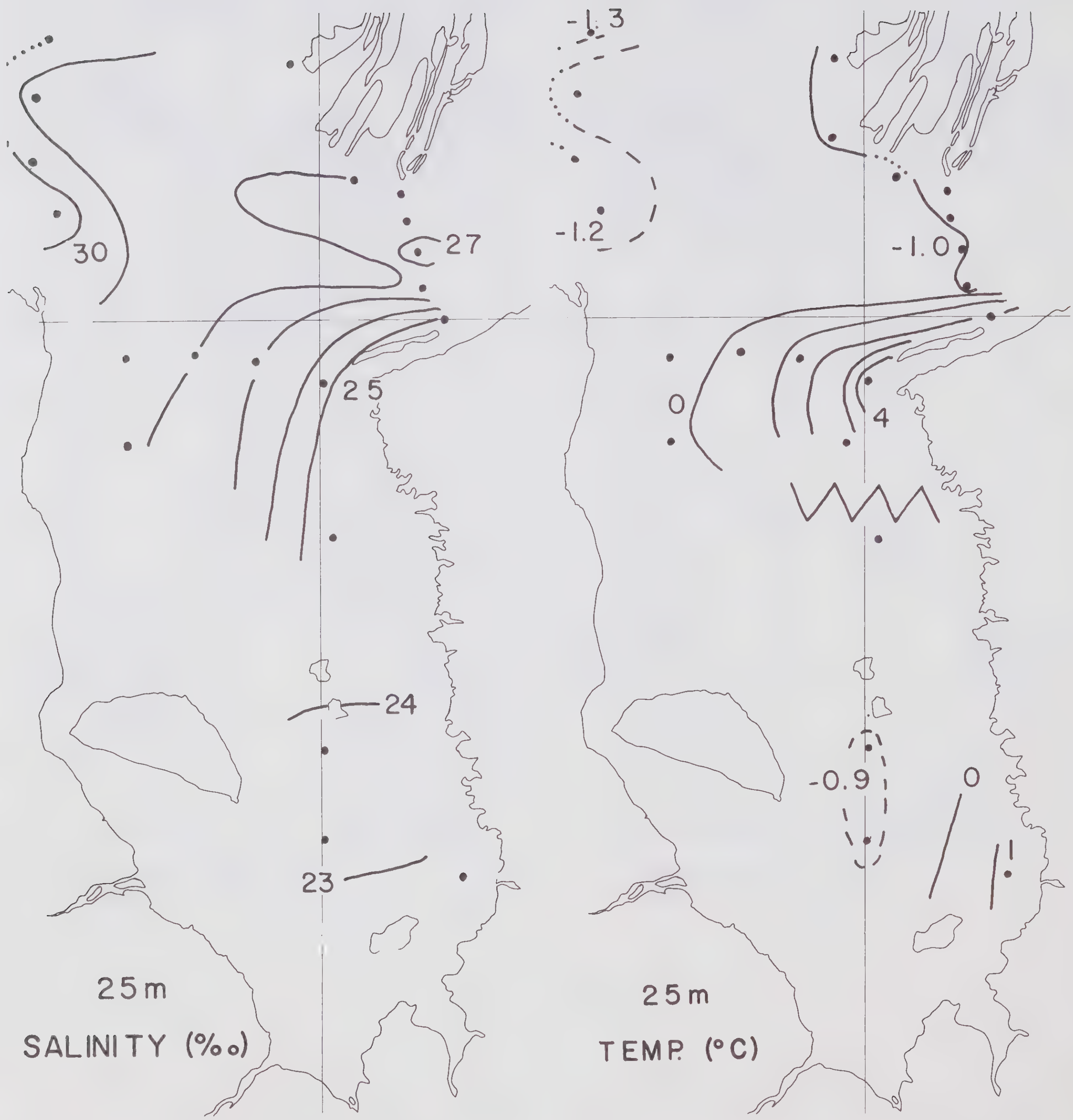


Figure 22(c)

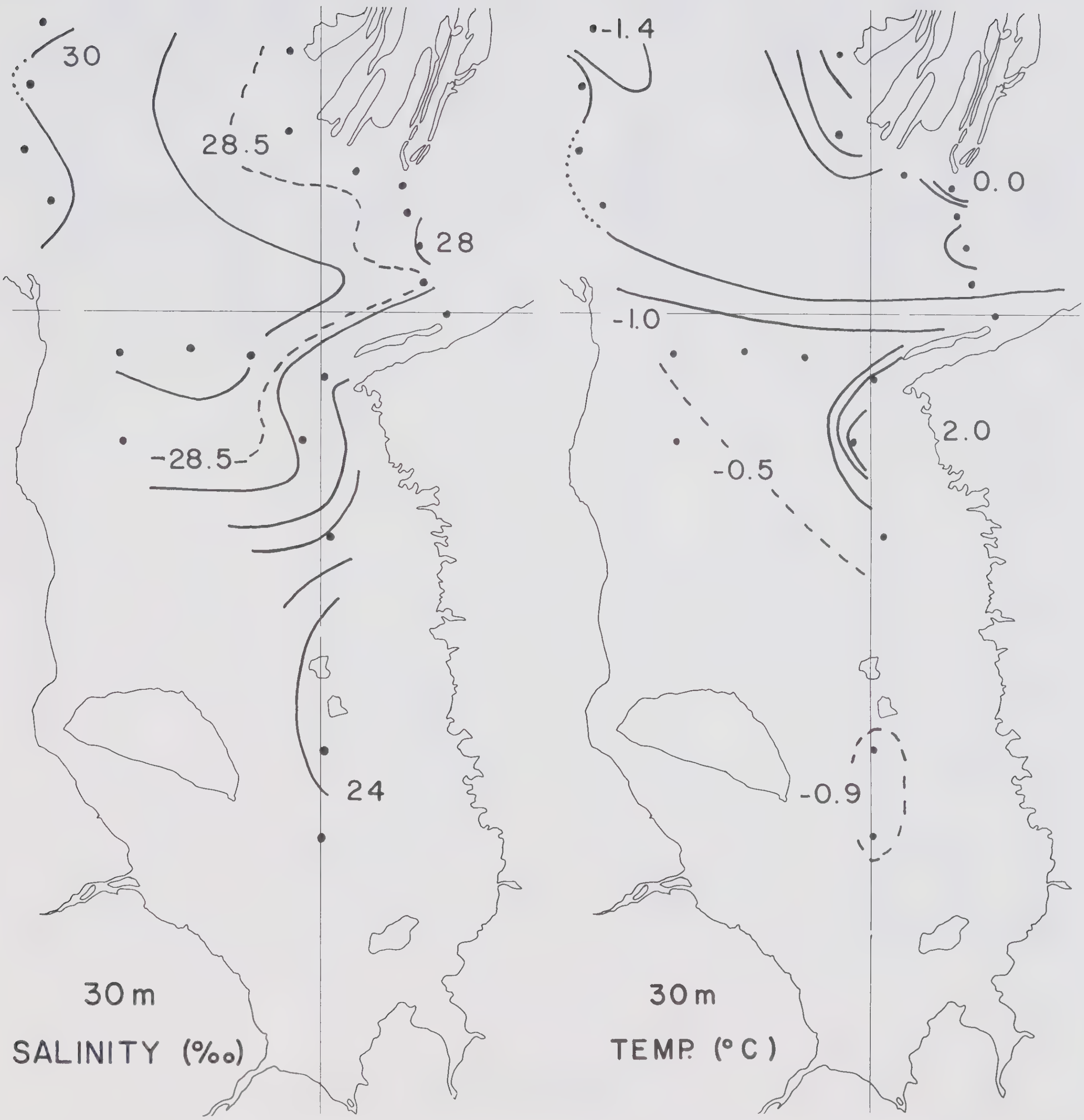


Figure 22(d)

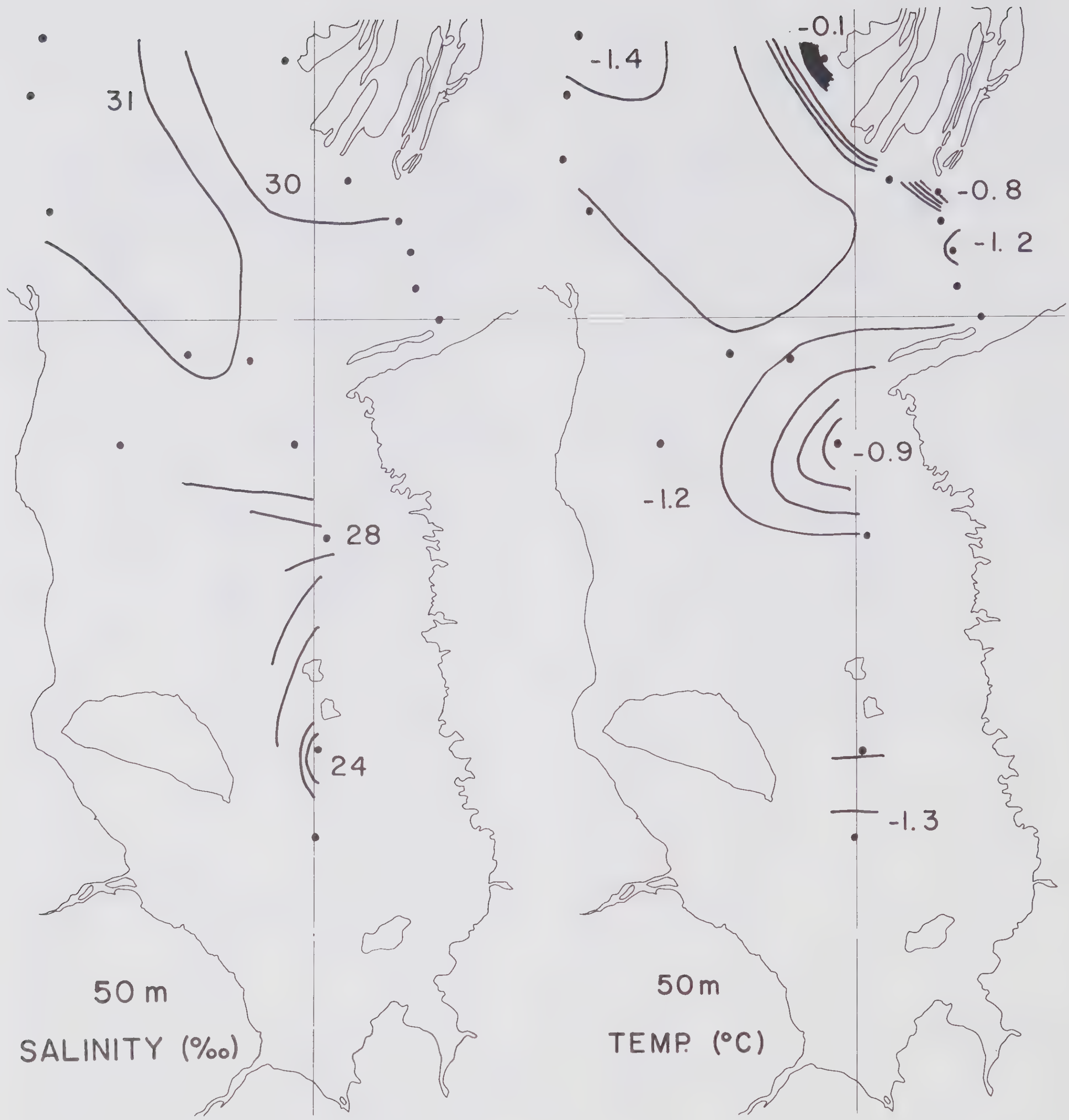


Figure 22(e)

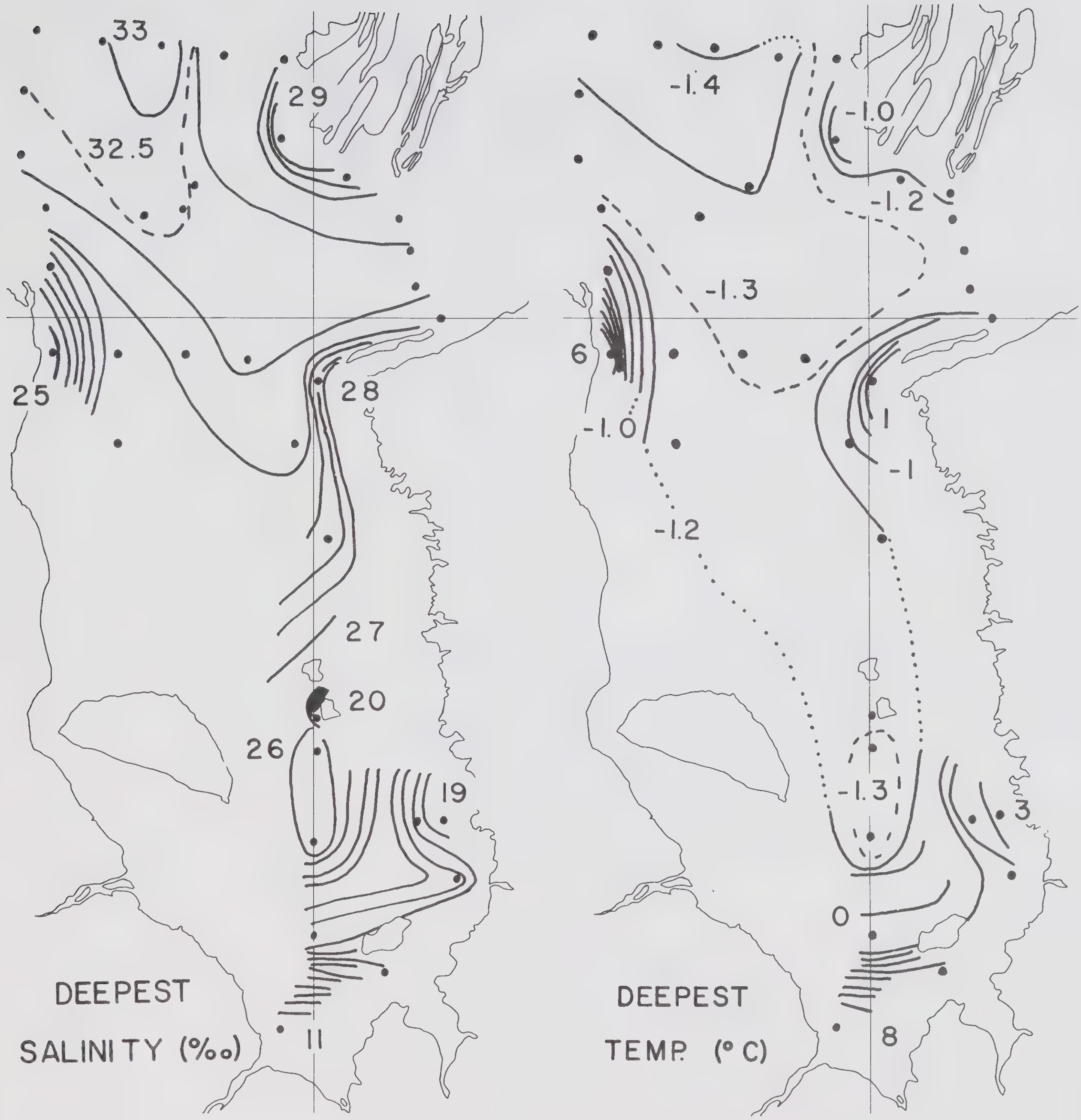

33
32.5
29
25
28
27
20
26
19
11
DEEPEST
SALINITY (‰)
-1.4
-1.0
-1.2
-1.3
6
-1.0
1
-1
-1.2
-1.3
3
0
8
DEEPEST
TEMP. (°C)

Figure 22(f)

Figure 23. The distribution of oxyty (ml/l) from the "Calanus" data of 1959 at the depth of deepest observation.

Figure 24. The distribution of temperature, salinity, dissolved oxygen and σ_t in a section across the mouth of James Bay. (a) From "Theta" stations 102 to 196. (b) From "Calanus" stations 57, 59, 60, 62 and 63. (c) From "Theta" stations 246 to 250.

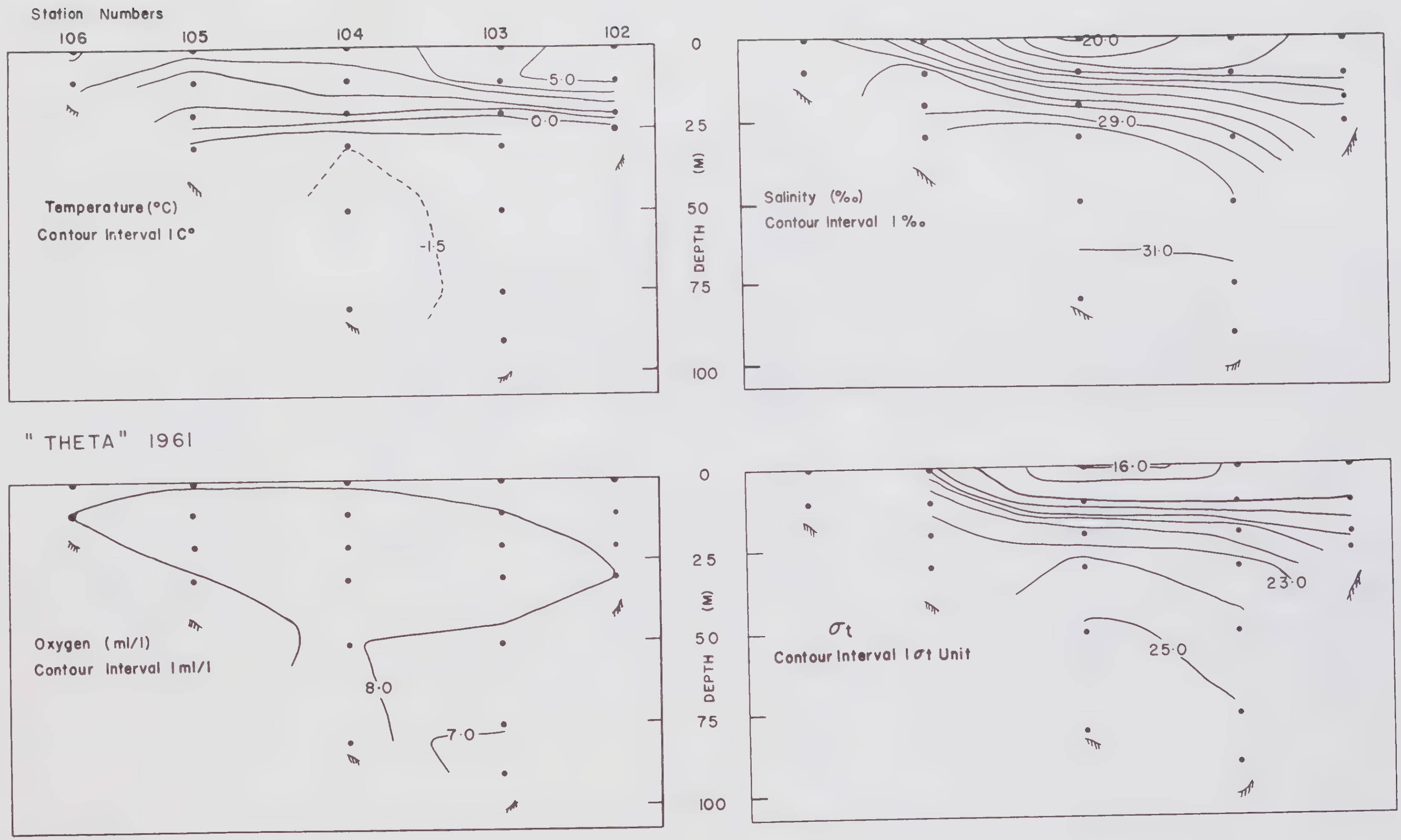

Station Numbers
106
105
104
103
102
5·0
0·0
-1·5
Temperature (°C)
Contour Interval 1 C°
0
25
50
75
100
DEPTH (M)
20·0
29·0
31·0
Salinity (‰)
Contour Interval 1 ‰
"THETA" 1961
8·0
7·0
Oxygen (ml/l)
Contour Interval 1 ml/l
16·0
23·0
25·0
σ_t
Contour Interval 1 σ_t Unit

Figure 24(a)

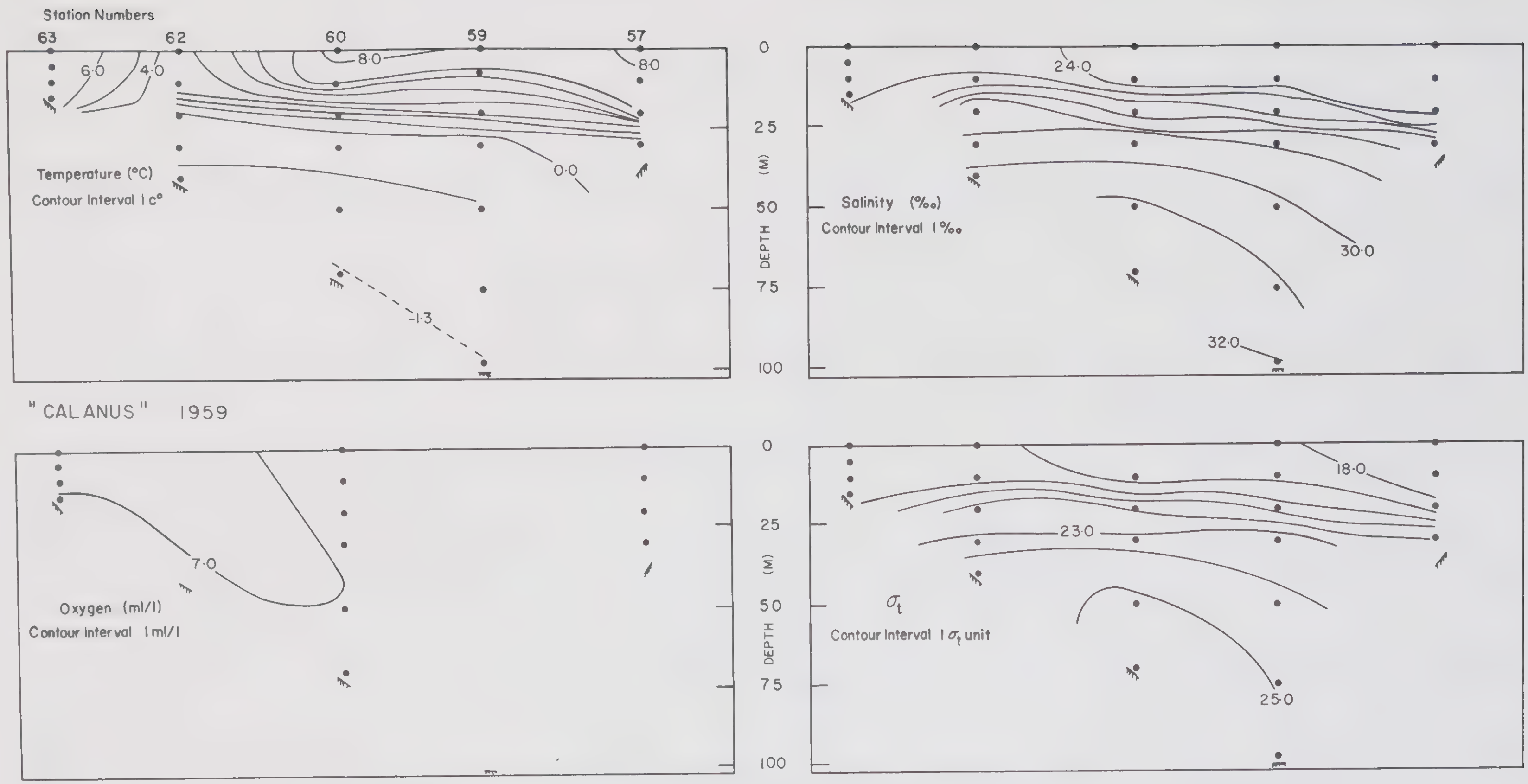

Station Numbers
63
62
60
59
57
6·0
4·0
8·0
8·0
0·0
-1·3
Temperature (°C)
Contour Interval 1 c°
"CALANUS" 1959
Oxygen (ml/l)
Contour Interval 1 ml/l
7·0
0
25
50
75
100
DEPTH (M)
Salinity (‰)
Contour Interval 1 ‰
24·0
30·0
32·0
σt
Contour Interval 1 σt unit
18·0
23·0
25·0

Figure 24(b)

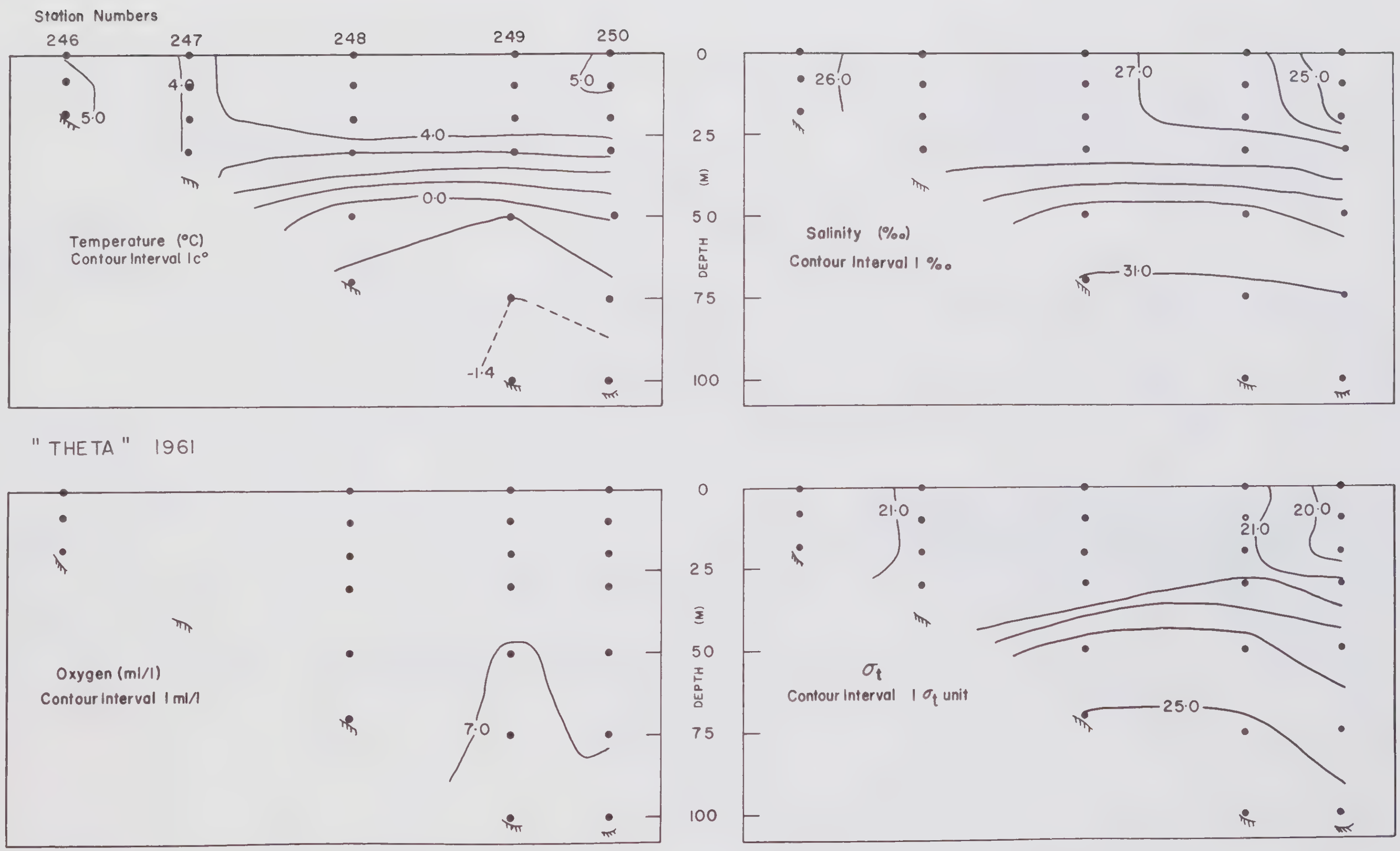

Station Numbers
246
247
248
249
250
5·0
4·0
4·0
5·0
0·0
-1·4
Temperature (°C)
Contour Interval 1c°
0
25
50
75
100
DEPTH (M)
26·0
27·0
25·0
31·0
Salinity (‰)
Contour Interval 1 ‰
"THETA" 1961
Oxygen (ml/l)
Contour Interval 1 ml/l
7·0
21·0
21·0
20·0
25·0
σ_t
Contour Interval 1 σ_t unit

Figure 24(c)

The tides in James Bay

by

Gabriel Godin

Contents

1. The data available

James Bay (frontispiece) because of its subarctic climate and its inhospitable shores has not attracted a large permanent population around its rim. Scientific information about its geography, bathymetry and oceanographic characteristics has been acquired in a succession of hesitant steps. Its extensive flats, its hundreds of islands and reefs make most of it virtually inaccessible to any hydrographic vessel. This severe limitation on ship traffic inhibits any motivation for acquiring more extensive soundings with the help of launches, an expensive and time-consuming project. As a consequence the bathymetric information about the western and eastern sides of James Bay consists mostly of blanks. The extent of soundings in the central portion of the bay however, suffice to delineate a relatively deep channel of depths exceeding 25 m, extending from its mouth down to the west side of Charlton Island, while the remainder of the bay remains very shallow.

Temporary tide gauges have been installed at odd stations at irregular time intervals and records of short duration have been accumulated. The majority of stations were established on islands such as Bear, Strutton, Charlton and Stag Islands, as well as inside sand and sediment choked rivers such as La Grande-Rivière, Eastmain, Moose and Albany. None of these observations can be considered of good quality with the exception of those carried out in the Moose River (Langford, 1963). As a matter of fact, the careful observations in the Moose River, which monitored currents and water levels from well outside the

estuary up to the Government Wharf in Moosonee at the foot of rapids, help assess the representativeness of the observations gathered at the other stations. For instance the tide takes 2 1/2 hours to travel the 13 nautical miles from Sand Head to Moosonee; therefore, one must expect a lag of the same order of magnitude between the tide recorded at other trading posts located at the head of rivers and the tide present in their respective estuaries, which makes such observations quite unrepresentative of what is happening in James Bay proper. The observations gathered at the island stations should represent the actual tide in James Bay more adequately. Even at that, there exists a peculiar phase difference between the M_2 tide observed at Charlton Island and Strutton Island which might be accounted for by the very sheltered position of Charlton Depot.

2. Cotidal charts using the gauge observations and a one dimensional model

With the help of these rather sparse observations and the use of a simplified one dimensional model of James Bay, it is still possible to gain a fair idea of the progression of the tide inside the bay and of its change in amplitude.

The tidal wave which progresses along the southern rim of Hudson Bay is strongly refracted around Cape Henrietta Maria and enters James Bay as a damped progressive wave. Its advance appears to be considerably retarded in the vicinity of Akimiski Island and it eventually reaches Hannah Bay and Rupert Bay seven hours after it has rounded Cape Henrietta Maria. Its amplitude is larger on the western and southern portions of the bay due to the presence of a degenerate node of the semidiurnal tide on the

eastern side of the bay. The mean amplitude of the tide varies from over 90 cm in the western and southern portions to less than 40 cm in the eastern portions near Akimiski Island. Table 1 lists the amplitude and phase of the major constituents of the tide observed at various gauge stations established in James Bay; we include Poste de la Baleine and Winisk to relate these observations to Hudson Bay proper. M_2 is the major lunar semidiurnal constituent; it represents the mean tide. S_2, the solar semidiurnal constituent, adds or subtracts its contribution to that of M_2, creating spring and neap tides. N_2 is of lunar origin and reflects the variable distance of the moon from the earth. We have kept it in parentheses because most likely it has not been properly separated from the other semidiurnal constituents. K_1 and O_1 are diurnal constituents and are related to the declination of the orbits of the moon and of the earth. Their amplitude and phase are irregular and O_1 appears to be twice as small as K_1 throughout the bay. Normally O_1 should be about two thirds of K_1. If there were no friction in James Bay, both of these constituents should have a node just at the mouth of the bay, the node of K_1 being positioned a little further inside the bay. In practice the node is degenerate and somehow O_1 enters the bay considerably weakened compared to K_1. All in all, the diurnal constituents are very much smaller than the semidiurnal constituents thus indicating that the tide throughout James Bay is truly semidiurnal at all times.

Table 1 Amplitude and phase of various tidal constituents observed at thirteen stations around James Bay.

Station	M_2 amp cm	M_2 phase deg	S_2 amp cm	S_2 phase deg	(N_2) amp cm	(N_2) phase deg	K_1 amp cm	K_1 phase deg	O_1 amp cm	O_1 phase deg
Poste de la Baleine	63	232	15	302	9	184	4	25	0	334
Pte. Louis XIV	64	222	20	281	20	191	6	27	1	311
Fort George	66	238	15	329	10	211	8	116	5	51
Eastmain	34	49	6	134	5	349	10	145	2	110
Strutton	55	14	5	124	5	355	10	141	6	90
Charlton	63	48	16	128	12	3	12	128	3	78
Stag	91	80	20	164	16	38	12	145	9	110
Sand Head	94	44	22	126	18	357	16	128	7	63
Ship Sands	71	71	16	155	12	31	15	152	3	64
Moosonee	62	111	14	200	11	68	12	171	2	60
Fort Albany	88	40	22	120	18	0	15	139	4	90
Bear Island	97	219	19	302	19	182	8	62	3	349
Winisk	109	83	29	158	19	45	5	322	3	280

We may use the data presented in Table 1 to draw cotidal charts of the constituents over James Bay. In these charts the lines of equal phase may be considered as delineating the front of the tidal wave as it progresses up James Bay. A phase difference of 60° between two such lines represents about two hours for a semidiurnal tide and four hours for a diurnal tide. The lines of equal amplitudes denoted by dotted lines define areas within which the amplitude exceeds or is less than the value indicated on the rim.

Such isopleths may be drawn in an almost infinite number of ways because of the scarcity and problematic value of the actual observations. It seems wise at this stage to solve the equations of hydrodynamics over a simplified one dimensional model of James Bay. In this way we may predict the mean vertical tide and tidal currents at various sections of the bay assuming that the tidal wave is perfectly reflected at the head. The results of such calculations may be combined with the coastal observations to draw the most plausible system of cotidal and coamplitude lines, besides yielding values of the tidal currents which have not been observed in the bay except around the Moose River. The equations of hydrodynamics in one dimension are

$$\frac{\partial}{\partial x}(Au) + B\frac{\partial Z}{\partial t} = 0 \tag{1}$$

$$\frac{1}{g}\frac{\partial u}{\partial t} + \frac{|u|}{C^2H}u + \frac{\partial Z}{\partial x} = 0 \tag{2}$$

where x is the distance along the bay moving northward,

A is the area of a vertical section of the bay drawn across its width,

B is the width of such a section,

H is the mean depth of such a section,

Z is the amplitude of the tide and

u is the tidal current,

while C stands for the Chézy coefficient which measures the intensity of friction. Since the friction all over the bay must be rather high we take the value of C as $C = 45\ m^{\frac{1}{2}}/sec$.

Because of the friction term $(|u|/C^2H)u$, the phase difference between Z and u varies from section to section and for a sinusoidal oscillation of frequency σ (a given tidal constituent), we may write a solution to (1) and (2) in the form

$$Z=Z_1(x)\ \cos \sigma t + Z_2(x)\ \sin \sigma t \tag{3}$$

$$u=u_1(x)\sin\sigma t - u_2(x)\cos\sigma t, \tag{4}$$

so that we may write for the x dependence of Z and u:

$$\frac{\partial}{\partial x}(Au_1) = B\sigma Z_1 \tag{5}$$

$$\frac{\partial}{\partial x}(Au_2) = B\sigma Z_2 \tag{6}$$

$$\frac{\partial Z_1}{\partial x} = - \left[\frac{\sigma}{g}\ u_1 - \frac{|u|}{C^2H}\ u_2\right] \tag{7}$$

$$\frac{\partial Z_2}{\partial x} = - \left[\frac{|u|}{C^2 H} u_1 + \frac{\sigma}{g} u_2 \right] \tag{8}$$

u_1, u_2, Z_1, Z_2 are inextricably linked because of the friction. (5) to (8) become useful to us in difference form:

$$(Au_1)_{j+2} = (Au_1)_j + \Delta x\sigma\ B_{j+1}(Z_1)_{j+1} \tag{9}$$

$$(Au_2)_{j+2} = (Au_2)_j + \Delta x\sigma\ B_{j+1}(Z_2)_{j+1} \tag{10}$$

$$(Z_1)_{j+1} = (Z_1)_{j-1} - \Delta x \left[(\sigma/g)(u_1)_j - (|u|_j/C^2 H_j)(u_2)_j \right] \tag{11}$$

$$(Z_2)_{j+1} = (Z_2)_{j-1} - \Delta x \left[(|u|_j/C^2 H_j)(u_1)_j + (\sigma/g)(u_2)_j \right] \tag{12}$$

By dividing the bay into segments of length Δx, (9) to (12) allow us to evaluate Z and u at section $2\Delta x$ apart from the values set at the boundaries. We have in fact, subdivided James Bay into 23 segments of length Δx=10 nautical miles extending from 51°10'N to 55°00'N. The integration has to be performed for each constituent. We restrict ourselves to M_2 and K_1 only, since calculating S_2, N_2 and O_1 would simply be repetitious. We use for M_2 (σ=28.98°/hour) as boundary conditions

$$u=0 \quad \text{at} \quad j=0 \quad (51°10'\text{N}) \tag{13}$$

$$Z=91(\cos \sigma t - 40°) \quad \text{cm} \quad \text{at} \quad j=1 \ (51°20'\text{N}),$$

which is equivalent to taking

$$Z_1=70 \text{ cm and } Z_2=59 \text{ cm at } j=1 \tag{14}$$

For K_1 (σ=15.04°/hour) we take

$$u=0 \quad \text{at} \quad j=0 \tag{15}$$

$$Z=15\cos(\sigma t-125°) \text{ cm} \quad \text{at} \quad j=1$$

$$\text{or} \quad Z_1=-9 \text{ cm} \quad Z_2=12 \text{ cm} \quad \text{at} \quad j=1 \tag{16}$$

Table 2 lists values of B, A and H which have been derived from our schematization of the bay into the 23 subsections; the actual profiles are presented in the appendix. Table 3 contains the results of the integration of (9) to (12) using (13) to (16). These values cannot describe any two dimensional features of the tidal motion and represent averages over a whole section. They are plotted at the points indicated by x and ⊙ in the cotidal charts and they supply considerable assistance in the drawing of plausible and consistent coamplitudes and cotidal lines. We draw similar charts for S_2, N_2 and O_1 using the features that had been delineated for M_2 and K_1 using the one dimensional model. All these charts are shown in Figures 1 to 5.

A side product of the numerical calculations is that they yield values of the mean current at the various sections; these have been noted on the charts of M_2 and K_1. Actual

Table 2 A one dimensional schematization of James Bay indicating the section j, (equation 13) and the width B, the area A and the depth H (equations 1 and 2).

Section j	Width B (km)	Vertical Area A ($x10^6 m^2$)	Depth H (m)
0			
1	38		
2		.68	11
3	109		
4		2.07	16
5	118		
6		2.91	19
7	203		
8		3.68	18
9	126		
10		3.09	24
11	125		
12		3.50	44
13	195		
14		3.42	33
15	200		
16		5.80	29
17	175		
18		7.21	37
19	194		
20		9.31	51
21	170		
22		11.44	69
23	245		

Table 3 Values of M_2 and K_1 deduced from the one dimensional model of James Bay.

j	M_2 u amp cm/sec 0	M_2 u phase deg	M_2 z amp cm	M_2 z phase deg	K_1 u amp cm/sec 0	K_1 u phase deg	K_1 z amp cm	K_1 z phase deg
1			91	40			15	125
2	26	310			2	35		
3			78	31			15	124
4	31	304			3	34		
5			64	22			14	123
6	37	299			5	33		
7			49	5			13	22
8	45	291			6	33		
9			39	332			11	120
10	61	285			8	33		
11			49	285			9	116
12	56	274			8	32		
13			61	258			7	112
14	58	252			9	31		
15			75	238			4	100
16	36	224			6	29		
17			78	226			3	87
18	32	200			5	27		
19			76	216			2	62
20	30	180			4	25		
21			71	207			2	29
22	28	167			3	24		
23			65	199			2	359

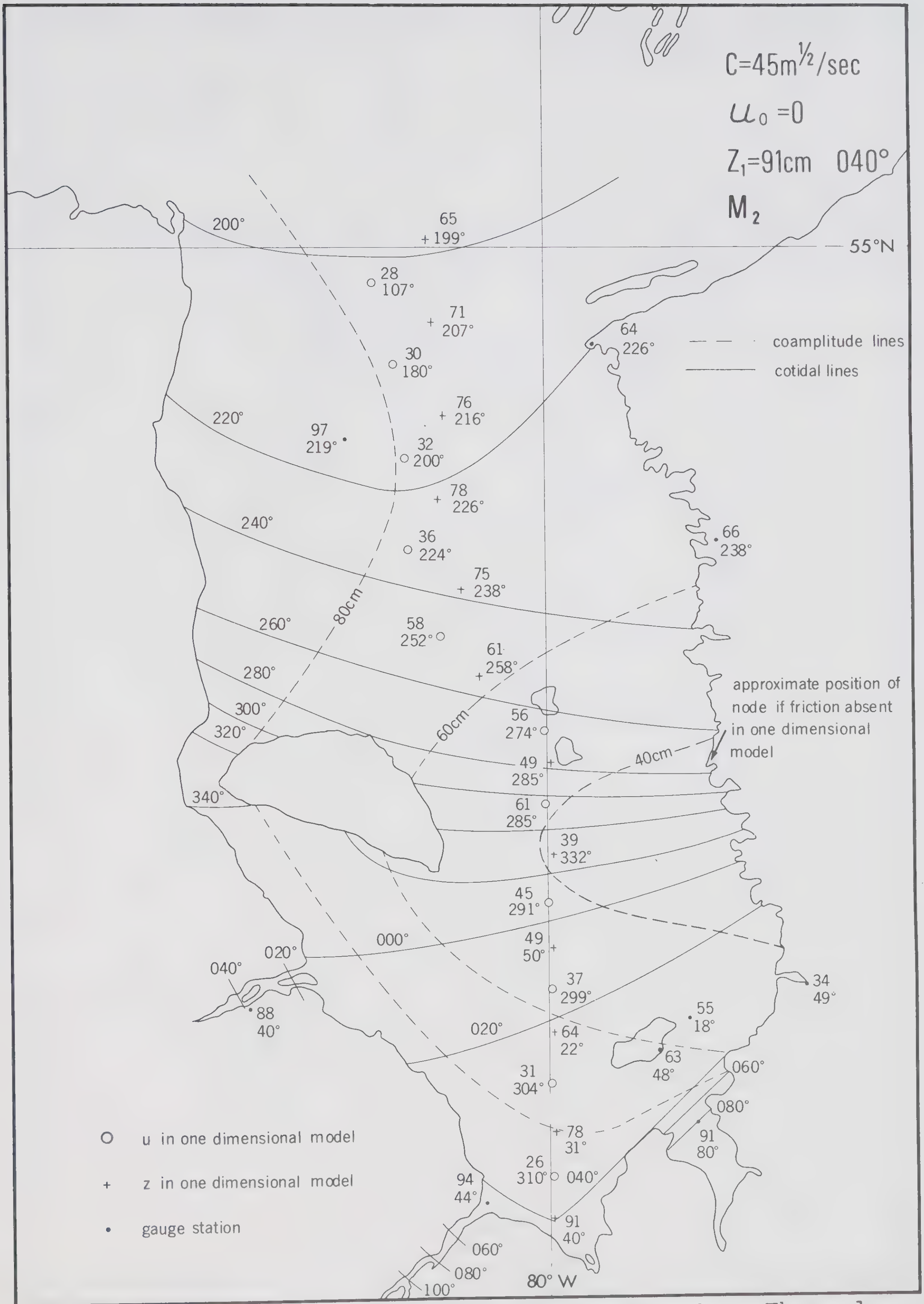


Figure 1 Cotidal and coamplitude lines for M_2. The values observed are squared. x and ⊙ indicate the values of Z and n deduced in the one dimensional model.

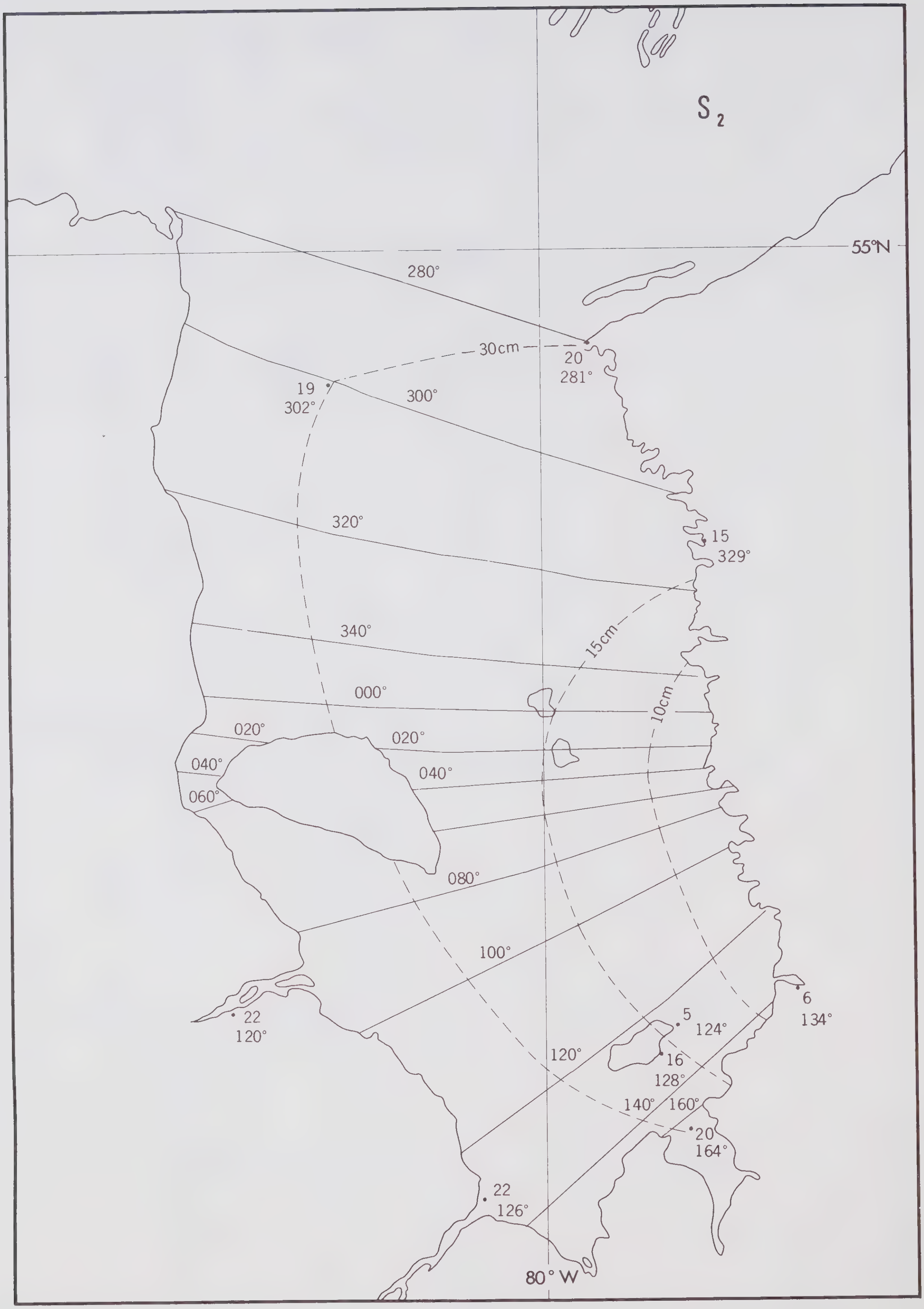


Figure 2 Cotidal and coamplitude lines for S_2.

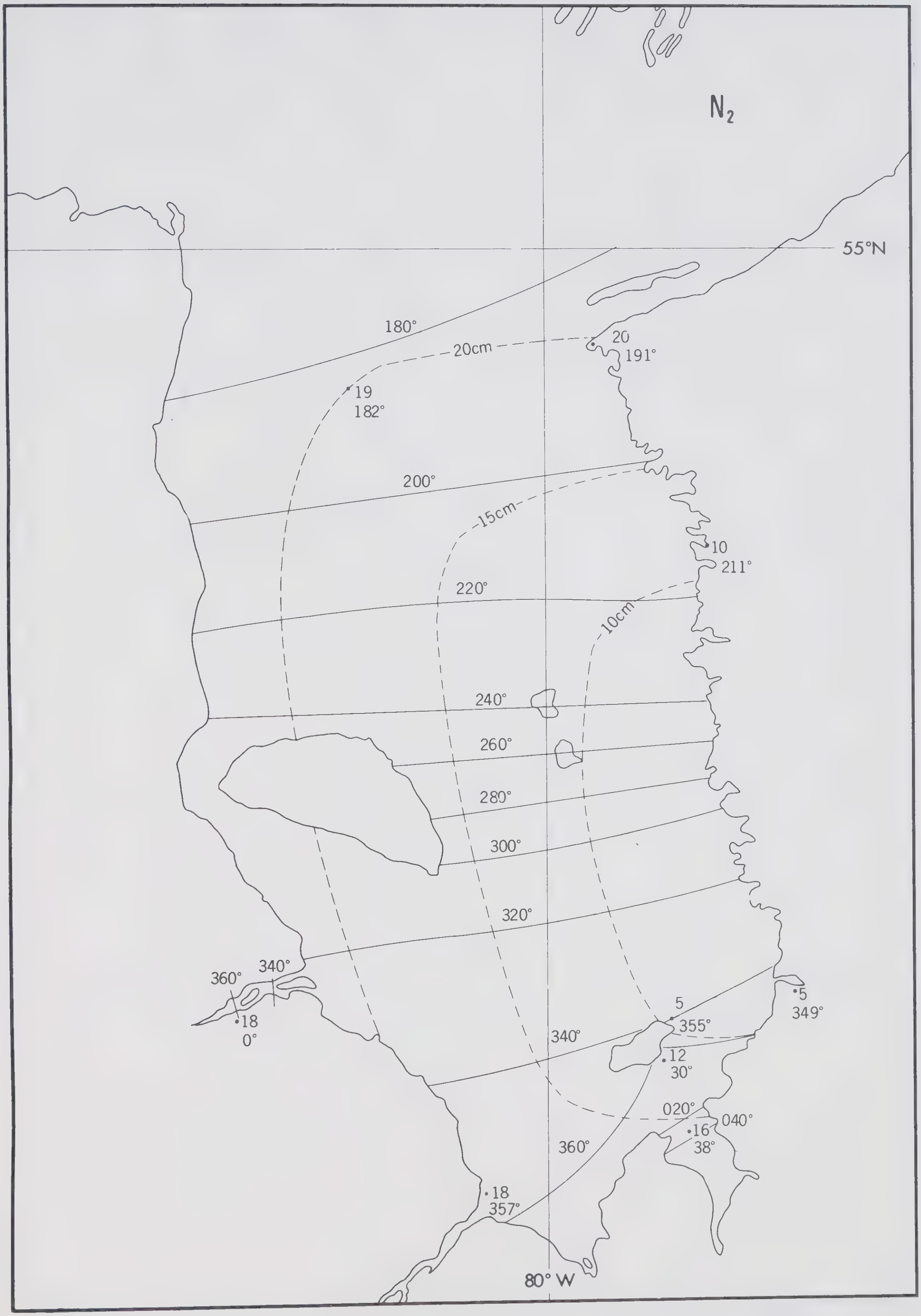


Figure 3 Cotidal and coamplitude lines for N_2.

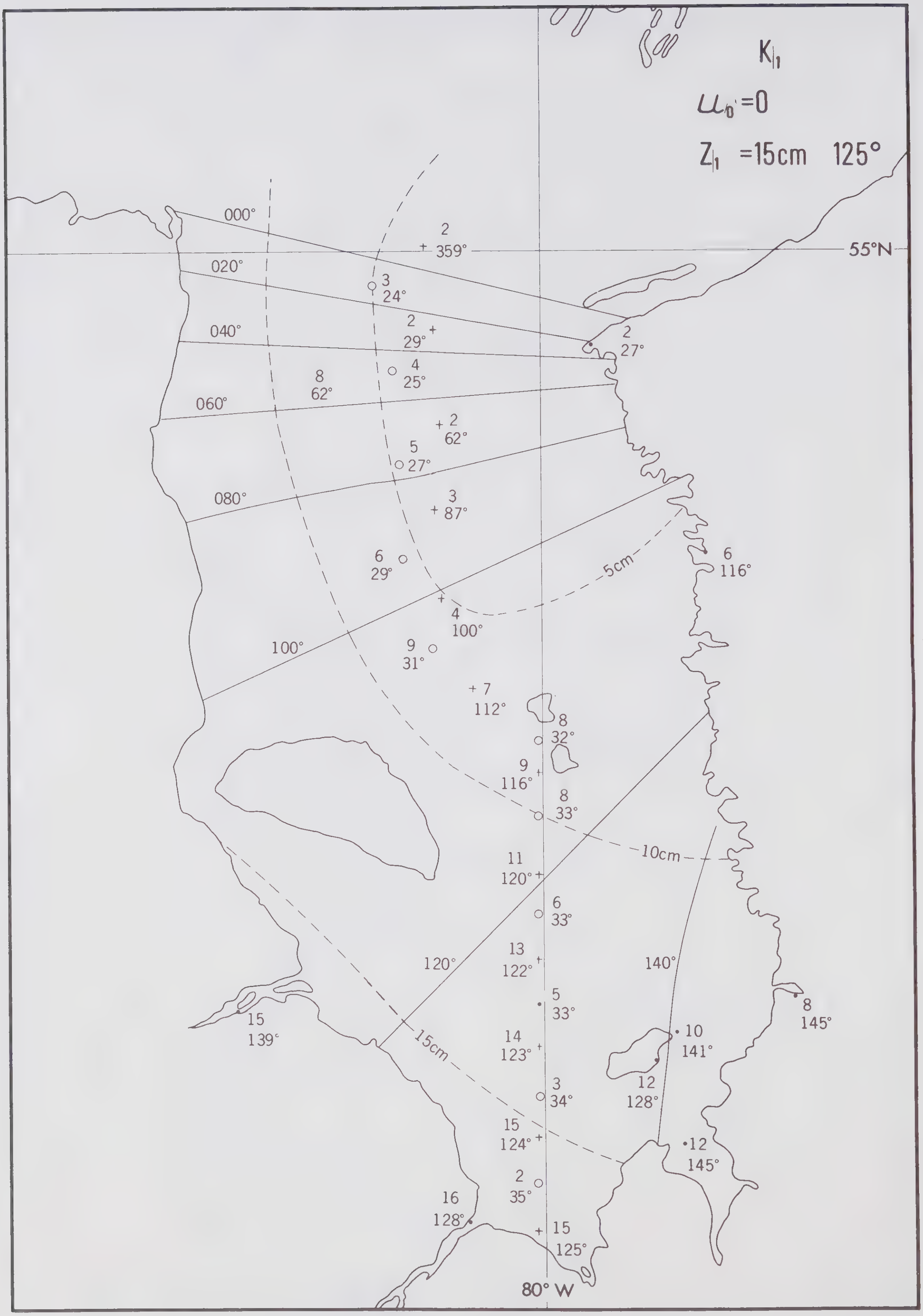


Figure 4 Cotidal and coamplitude lines for K_1.

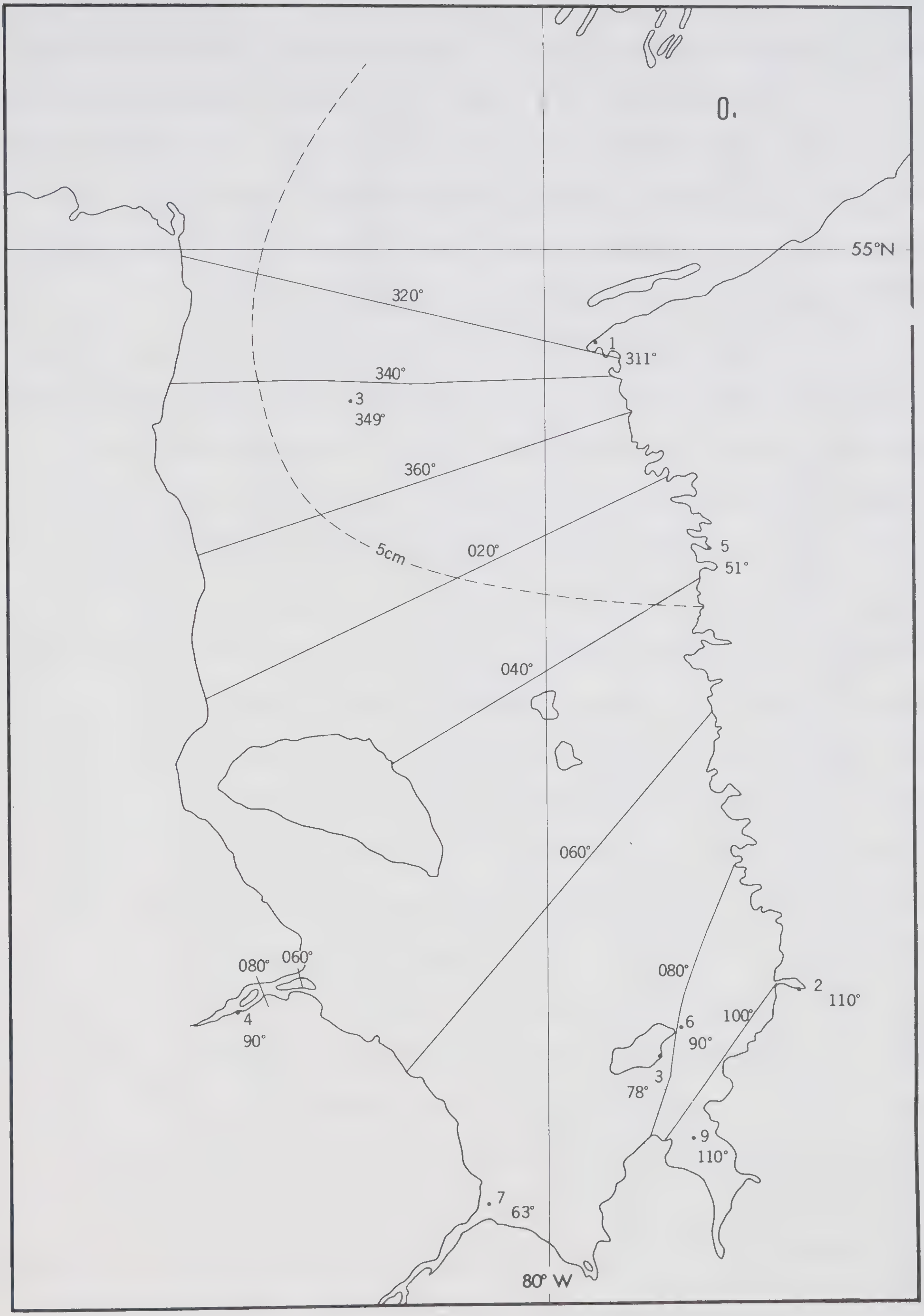


Figure 5 Cotidal and coamplitude lines for O_1.

observations of the currents in James Bay are not available but some currents were measured in the estuary of the Moose River.

Our solution to (1) and (2) depends on our assumption of perfect reflection at the head of the bay; in practice the reflection is rather diffuse and we cannot expect that the current observed in that area should lead the vertical tide by 90° of phase as our calculations suggest.

In the vicinity of Sand Head in the estuary of the Moose River, the vertical tide was observed on the tidal flat while the current was monitored in two neighbouring channels. At station B (Figure 6) the current was exactly 90° of phase ahead of the vertical tide as predicted by the model, but at Station E the current was more like 45° of phase ahead; the current at E is more representative of the current in the bay proper because the channel there is deeper and the flow is much less influenced by friction as it is at Station B. The observed currents are also larger than those predicted by the model, but it must be kept in mind that the model predicts the average current over a whole section while the current observed is present at a given point which happens to be more shallow than the remainder of the section. Moving away from the boundary the currents predicted from the model should be more representative and we notice that they reach their maximum of 61 cm/sec off Akimiski Island for M_2 and 9 cm/sec for K_1. Taking the contribution of S_2 and N_2 into account, the currents caused by the semidiurnal tide can therefore reach a magnitude of 80 cm/sec at spring tide when the moon is in perigee. This gives an

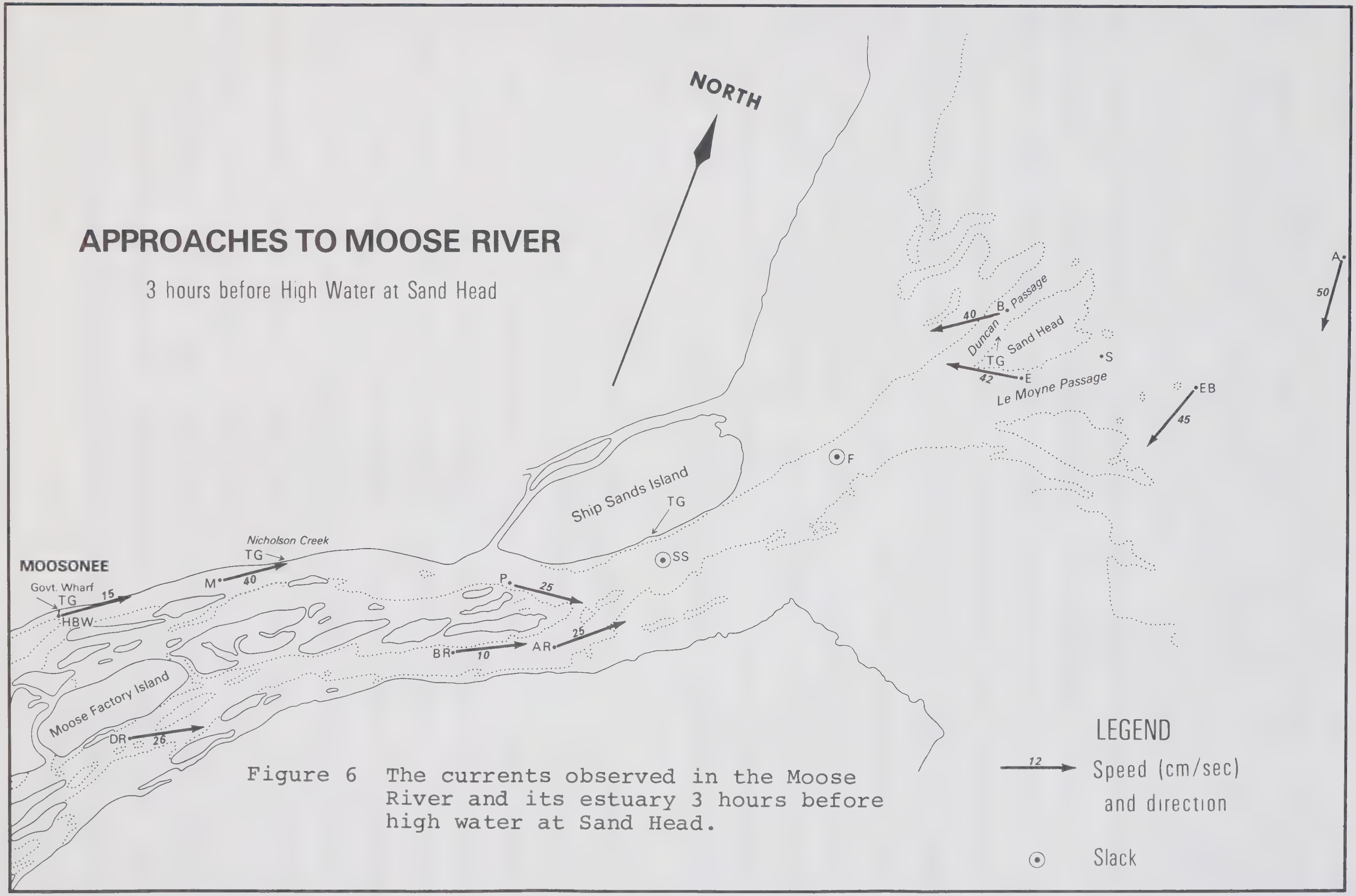


Figure 6 The currents observed in the Moose River and its estuary 3 hours before high water at Sand Head.

indication that the tidal currents over the body of James Bay in the vicinity of Akimiski Island can reach quite a respectable intensity.

3. A detailed view of the flow caused by the tide in Moose River

Up to now we have struggled with a handful of data and a coarse model in order to obtain some idea of the tidal motion in James Bay. It comes as a relief to this bleak situation to have in our possession the results of the detailed survey carried out in the Moose River and its estuary by the Canadian Hydrographic Service (Langford, 1963). In this fashion we can actually witness the flow of water in and out of a tributary of James Bay which most likely is typical of what is happening inside the other rivers emptying into James Bay.

The bed of the Moose River is quite undefined, shallow and cursed with numerous islands and drying flats of mud, sand and boulders. Sand flats nearly block its estuary and only the pressure of its impounded water manages to keep a gully open towards the sea (the Le Moyne Passage). The shores of the Moose River consist in many places of soft materials which are undermined by the ice and rushing waters during the spring freshet; later in the season, these mined areas collapse. Rapids effectively block the river upstream. The Moose River appears as rather typical of the other rivers around the bay and a study of the intricacies of the water movements inside it during a tidal cycle will help give an idea of the actual complexity of the patterns of flow in the vicinity of any of the other tributaries.

In Figures 6 to 9, the solid line delineates the shores and the islands and the dotted line the flats that dry at lower low water. The tide gauge stations indicated by "T.G." were set up at four locations. The first station at Sand Head measures with little distortion the tide that comes in from the main body of the bay. The distance between the first and last gauge amounts to approximately 13 nautical miles. Current meter stations were established at points indicated by the origin of the arrows; they are labelled by letters. We have shown 11 such stations. A circled point at one station indicates that at the moment of observation, no detectable current was noticed (slack water, turn). The arrow shows the orientation and the velocity in cm/sec is indicated. The origin of time is chosen as the moment when it is high water at Sand Head, the station located furthest out in the estuary.

In Figure 6, three hours before high water at Sand Head, which corresponds to nearly 1/4 period of the semidiurnal tide (and therefore approximately to mean water), flood has been established in the estuary while the water is still ebbing in the upper regions of the river; as a consequence we find a region of no motion in the vicinity of Ship Sands Island. This front moves gradually upstream as the tide rises and by the time it is high water at Sand Head (Figure 7) the river is in flood as far as Moosonee. We may notice that the directions of flow in individual channels depends very sensitively on the bottom configuration and that it may change rapidly at times. In the estuary, slack water is already reached in Duncan Passage and in the shallower portions

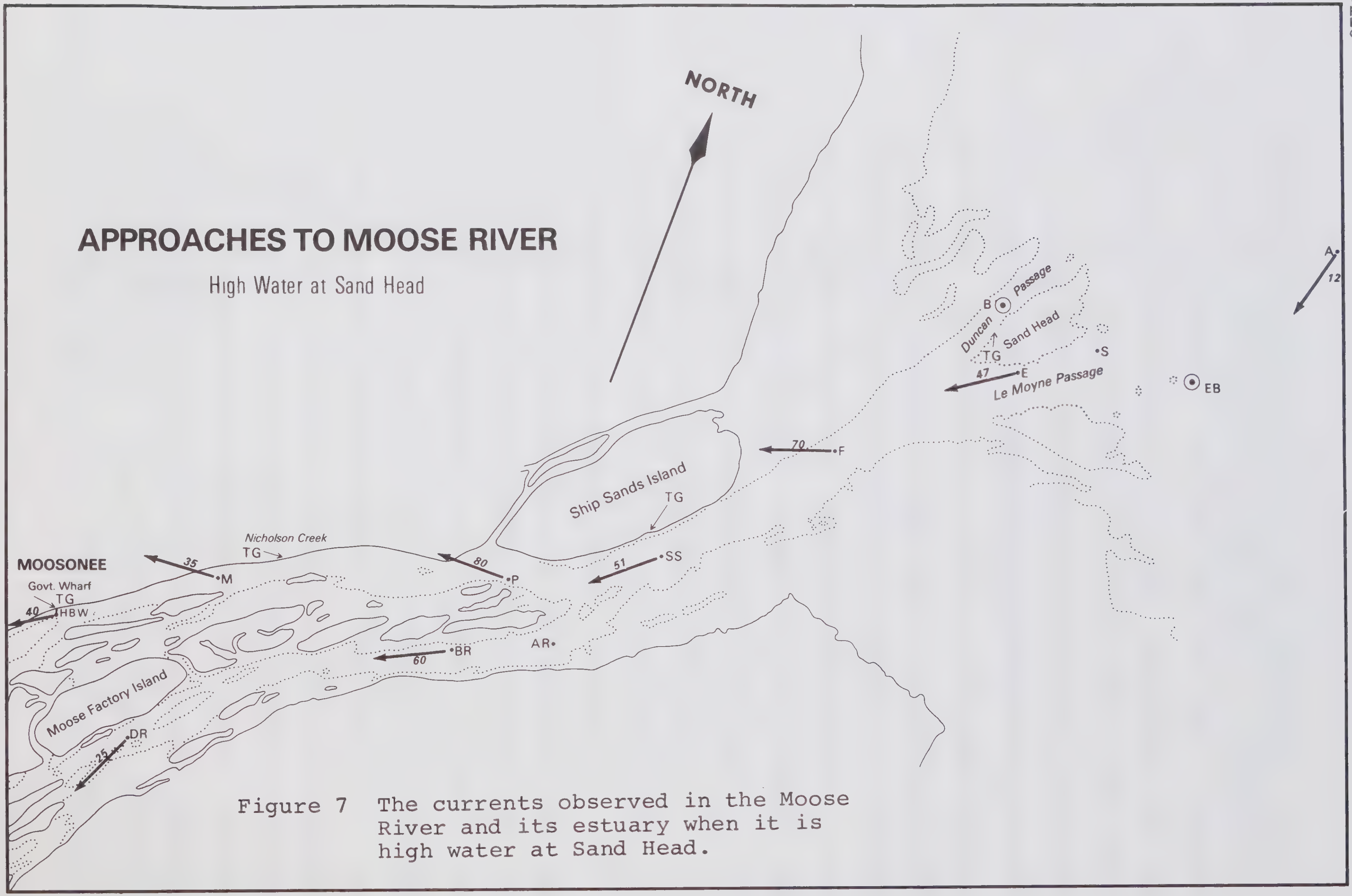


Figure 7 The currents observed in the Moose River and its estuary when it is high water at Sand Head.

of the estuary while it is still flood over the deeper portions. Slack water occurs earlier in shallow areas because of the friction and it will take a further 1 1/2 hours before the current turns in the deeper portions of the estuary. The currents turn clockwise there, while in Le Moyne Passage it slackens suddenly and turns to flood 1 1/2 hours after high water. This zone of slack water progresses upstream as the water level starts lowering, reaching the northern portions of the river first and the southern portions later. Three hours after high water (Figure 8) the river is in ebb. The water level is decreasing, high water having occurred at Ship Sands one hour after Sand Head while the water was still in flood. High water occurred at Nicholson Creek two hours after Sand Head and then it was slack water in its vicinity. Finally high water at Moosonee occurred 2 1/2 hours after Sand Head while the ebb started to take place in its vicinity. Six hours after high water at Sand Head (Figure 9) it is low water at Sand Head and the entire river continues ebbing.

Figure 10 shows the relationship between the water level at the four gauge stations and the current in their immediate vicinity. The time interval between high water and slack is noted since it gives some idea of the phase difference between the current and the vertical tide. Because of the net outflow from the river, the actual time difference between flood and high water is actually longer than the time noted on the graphs.

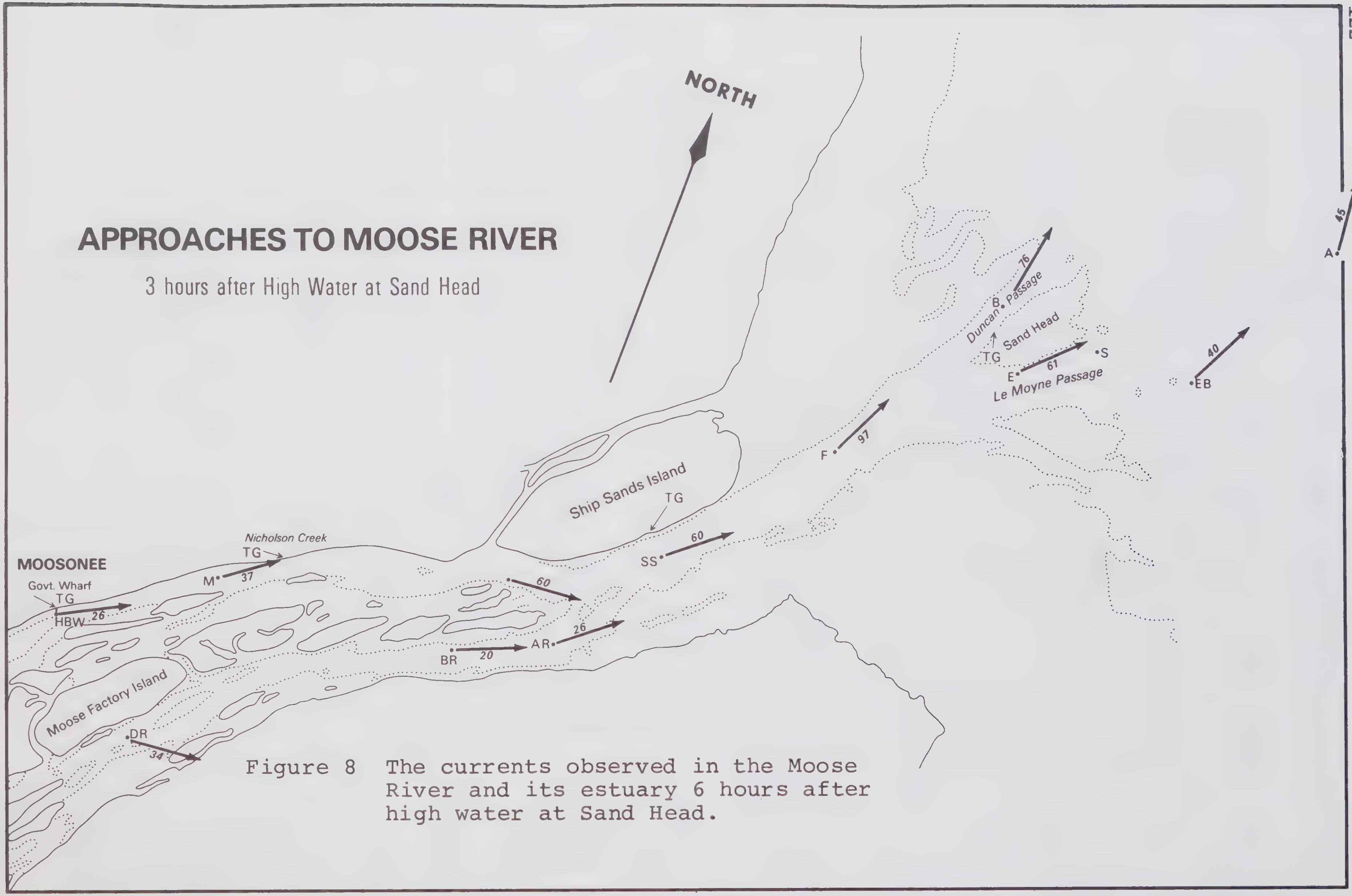


Figure 8 The currents observed in the Moose River and its estuary 6 hours after high water at Sand Head.

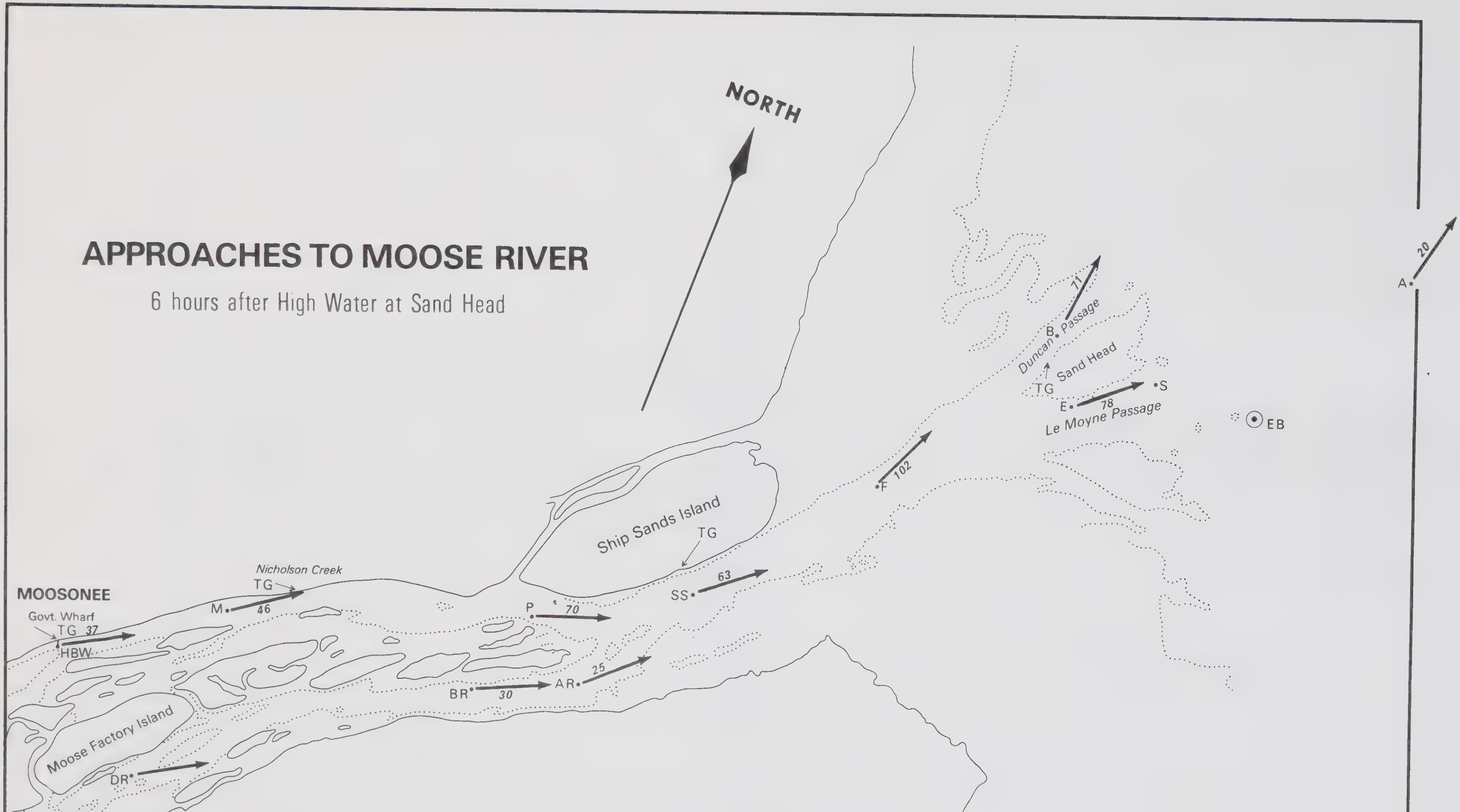


Figure 9 The currents observed in the Moose River and its estuary 6 hours after high water at Sand Head.

Figure 10 The relationship between the vertical tide and the current observed in the vicinity of the observing station.

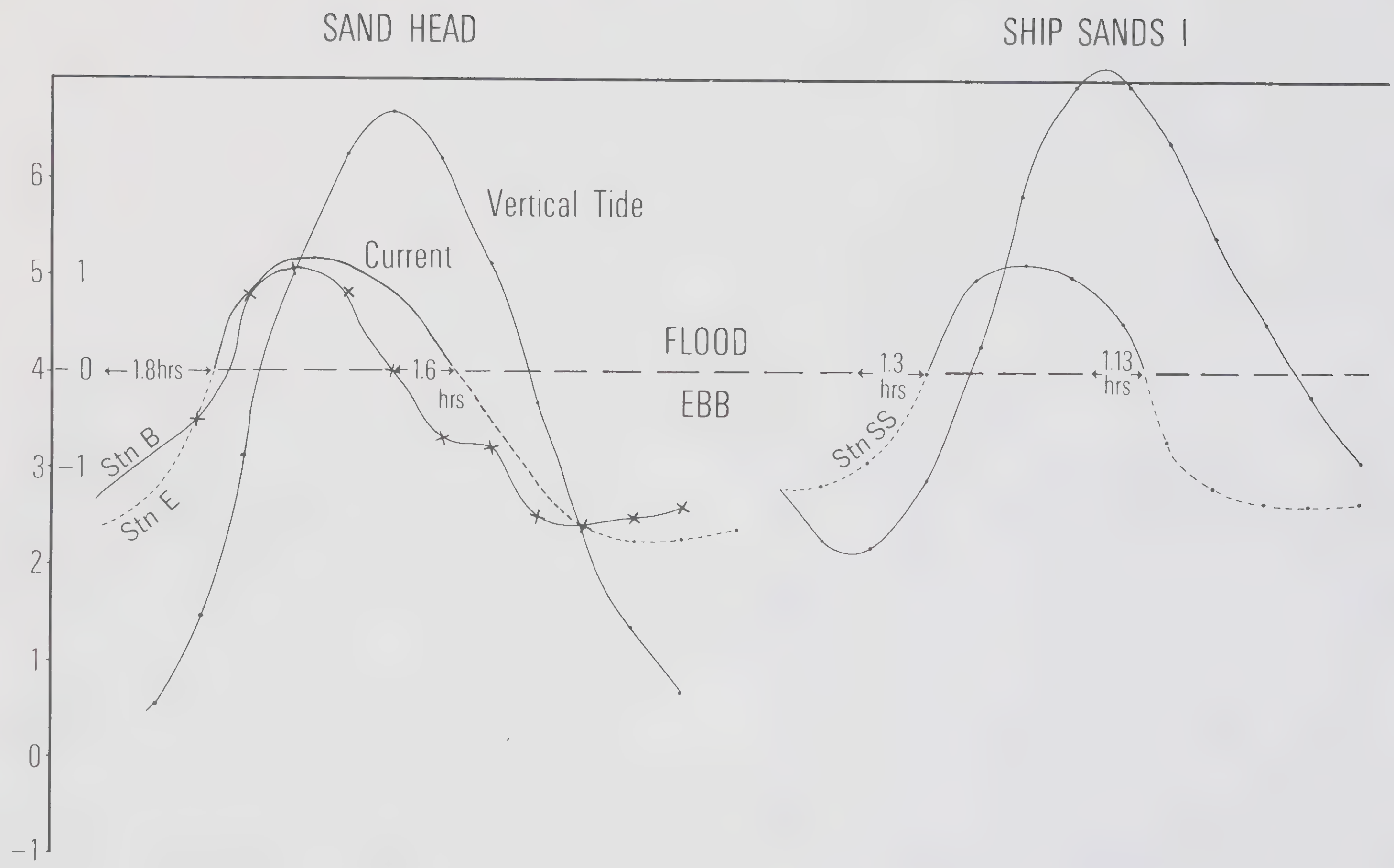

SAND HEAD
SHIP SANDS I
Vertical Tide
Current
FLOOD
EBB
1.8hrs
1.6 hrs
1.3 hrs
1.13 hrs
Stn B
Stn E
Stn SS
6
5
4
3
2
1
0
-1
1
0
-1

Figure 10(a)

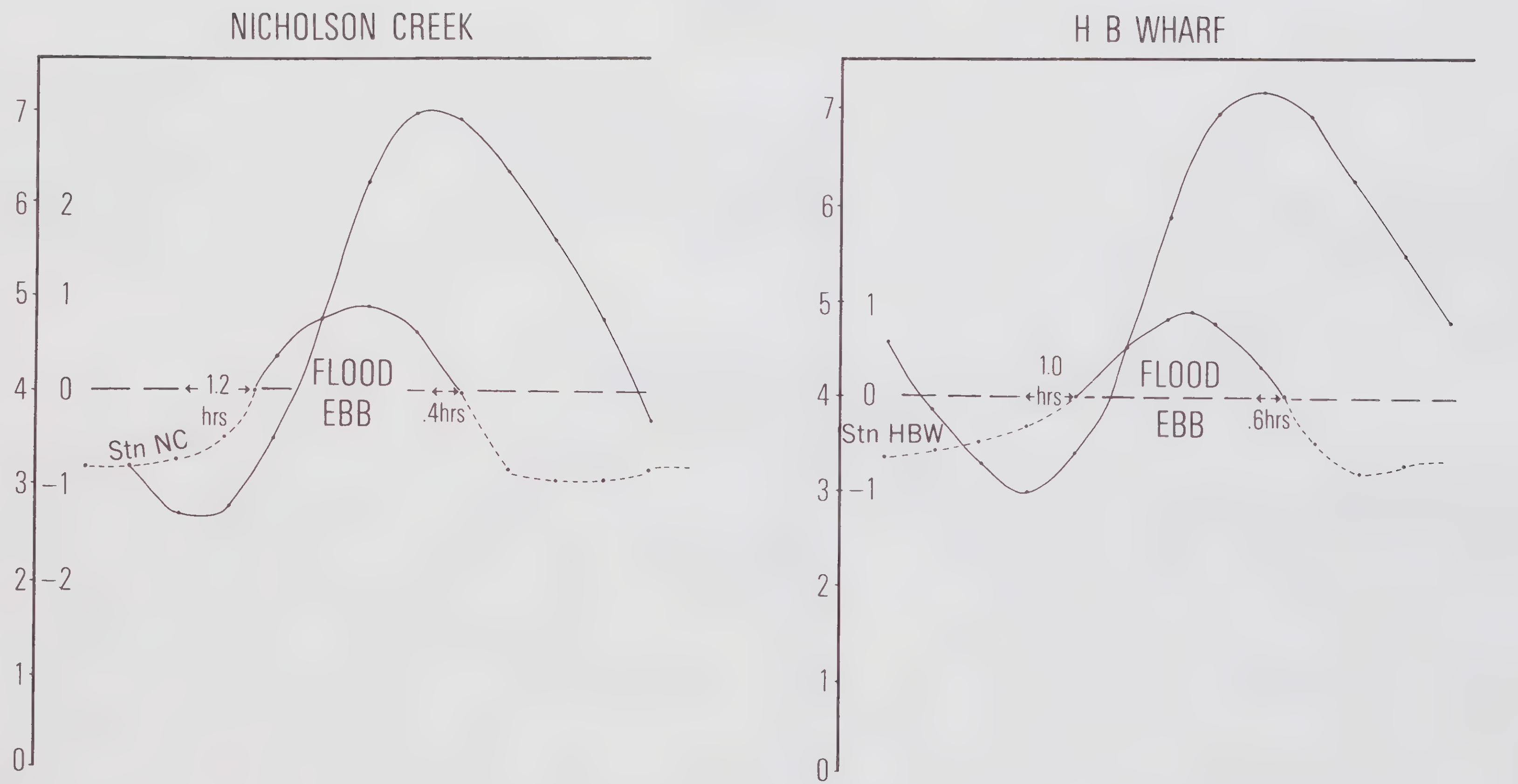

NICHOLSON CREEK
H B WHARF
7
6
5
4
3
2
1
0
2
1
0
-1
-2
FLOOD
EBB
1.2 hrs
.4hrs
Stn NC
1.0 hrs
.6hrs
Stn HBW

Figure 10(b)

4. Irregularities in the tidal regime

Two factors may affect the regularity of the tides: the spring freshet and weather disturbances. During April-May ice breaks up in the rivers surrounding James Bay and their discharge increases abruptly. An increase in discharge inhibits the penetration of the tide into the river and a wall of fresh water may extend a fair distance into James Bay where the tide will be reduced in range and where the mixing of salt and fresh water will take place at an intensified rate. This damping of the tide by a strong current is indicated by the solution of the equations of hydrodynamics (1) and (2) if we represent the current by

$$u = u_o + u_1 \cos(\sigma t - \alpha), \tag{17}$$

where u_o represents the steady current due to the discharge of fresh water and the second term represents the oscillatory term contributed by the tide. Assuming that the tidal current is never strong enough to cause any flood current,

$$|u|u = u^2 \simeq u_o^2 + \tfrac{1}{2}u_1^2 + 2u_o u_1 \cos(\sigma t - \alpha) \tag{18}$$

for which (1) and (2) have a solution of the form

$$u,h \sim \exp i\sigma t \exp \pm \frac{i\sigma}{\sqrt{gD}} \sqrt{1 - \frac{2igu_o}{C^2 \sigma D}}\, x \tag{19}$$

There is a damping term in x of the form

$$\exp - \left[\frac{\sqrt{2u_o\sigma}}{CD} \sin\left(\tfrac{1}{2} \arc \tan \frac{2u_o g}{C^2\sigma D} \right) \right] x \qquad (20)$$

which would vanish if u_o were zero.

This situation must occur with various degrees of intensity depending on the strength of the freshet. At that time the water will be fresh in the rivers, the overall salinity of James Bay in the vicinity of their estuaries will be significantly reduced and processes of mixing will be intensified but will take place further away from the shores. The stronger currents will carry sediments into the bay at an accelerated rate, particles of a larger size will be dragged along and most probably during this time the major portion of the nutrients fed into James Bay over the course of one year will be carried into it. Observations on the Moose River support this conjecture.

Weather disturbances must also have a marked effect on the motion of water on account of the general shallowness of the area and the presence of modes of resonance in the bay of a period of only a few hours. James Bay might respond dramatically to the passage of some of the cold fronts which are frequent and most severe during early summer and the fall and to the passage of depressions. Rupert Bay, Hannah Bay and the portion of James Bay south of Akimiski Island have minimodes of their own which could be excited by specific patterns of wind and pressure. During the course of a weather surge, the level of water will undergo irregular oscillation and transient currents will be

created. Masses of salt water could invade the rivers. The non-linear interaction of the surge with the tide may enhance or decrease the water level depending on fortuitous combinations of pressure gradients and wind stress. Surges are most likely to occur during the storm seasons, namely between September to December and April to June. A surge, a mild one, was observed on the Moose River while tidal observations were being carried out; it occurred between October 12 and October 14, 1963 and had a height of 120 cm with a period of 30 hours. The accidental detection of such a surge indicates that these phenomena are probably quite frequent in the James Bay area (Figure 11).

5. Changes caused by the regulation of the Nottaway, Broadback and Rupert Rivers

Once the NBR project is completed, the tide in Rupert Bay will not be inhibited during the spring freshet as it normally is at that time of the year; this implies that a more regular tidal regime will tend to prevail in and around Rupert Bay throughout the year. The zone of mixing of fresh water and salt water which must wander appreciably inside Rupert Bay at the spring freshet will no longer suffer such fluctuations and will be restricted to an area defined by the intensity of spring and neap tides and the mean value of the regulated outflow from the NBR complex. Consequently, there will be fewer localized short-range fluctuations in density, salt and sediments. The net amount of nutrients brought into Rupert Bay will be reduced since the larger currents necessary for their intensified transport will no longer have any occasion to prevail: the migratory birds

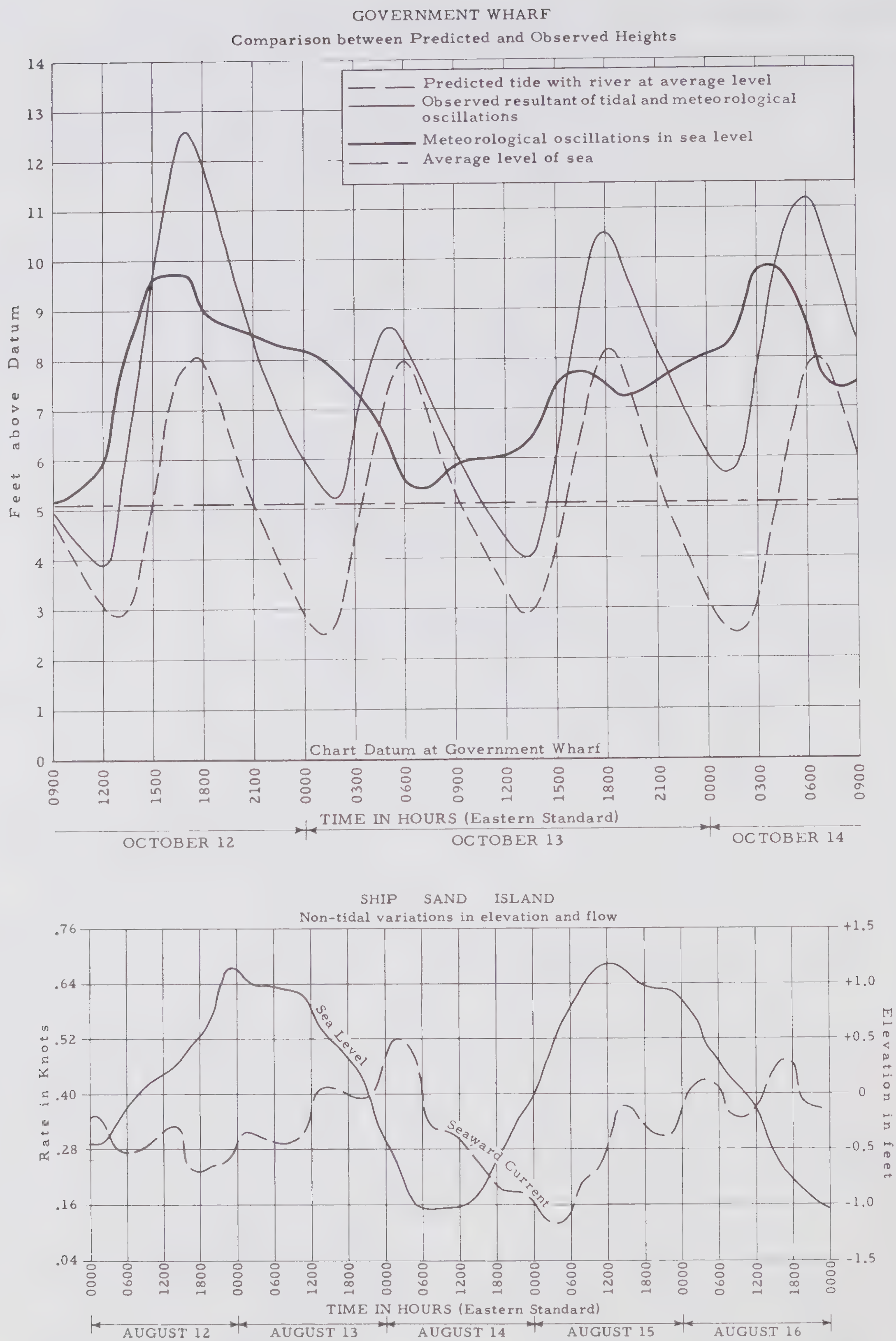


Figure 11 A surge observed in the Moose River during October, 1963.

which feed in the bay for a short time might choose to move to better feeding grounds while the local sea life which is already at a low level will be further reduced.

The probability of occurrence of surges will in no way be affected by the more regular outflow from the NBR complex. The primary factors controlling surges are wind strength, pressure patterns and pressure fluctuations; changes in the density of the air and of the water affect them too but imperceptibly compared to the other factors.

6. Other sources of energy in Hudson Bay and Hudson Strait

Besides the obvious sources of hydroelectric power which it is now intended to tap around James Bay, another potential source of energy could be exploited in the New Quebec with a minimum of ecological upheaval: the very large tide in Ungava Bay. It is a little known fact that the tidal range in the vicinity of Leaf Basin in Ungava Bay is as large as in Minas Basin in the Bay of Fundy and that at some times of the year the tide in Ungava Bay is larger than in the Bay of Fundy. In Ungava Bay the tidal range increases southwestward and reaches its maximum in the vicinity of Leaf Basin and the Koksoak River. Not only is the tide very large but numerous fjords and basins exist in that area which would be most suitable as reservoirs for tidal energy. Not only do they have an appreciable storage capacity but the width of their mouth is relatively small so that construction of a dam across them would not be at all prohibitive. As well, the depths in the entrances are smaller than those which would have to be faced in the Bay of Fundy.

The exploitation of the tidal energy in these basins would not preclude in any way the conventional exploitation of the upper waters of the rivers emptying into some of them for the production of hydroelectric power. As a matter of fact the two modes of exploitation would rather complement each other since the construction of hydroelectric plants upstream would reduce the flow of sediments into the tidal basin and this would prolong the useful life of the tidal plant. Such an arrangement is not possible in James Bay because of the reduced range of the tide there.

The tidal power which can be extracted from some basins in Ungava Bay may be estimated using the formula (Godin, 1969)

$$P = \frac{\rho g S_o \, [(2h_o)^2 + (1/3)e^2]}{T} \tag{21}$$

where ρ = density of the water, S_o = the area of the basin, h_o = the amplitude of M_2, $1/2e$ = the amplitude of $N_2 + S_2 + K_2$ and T = the tidal period, 12.4 hours. The equation (21) is an upper limit of the value of the power; in practice a tidal plant can extract at most 30 percent of this power. Table 4 lists the potential power output of some basins in Ungava Bay.

Table 4

Potential tidal power for some sites in Ungava Bay.

Basin	Surface area S_o	M_2	$S_2+N_2+K_2$	30 percent of potential tidal power
	$10^7 m^2$	m	m	Megawatts
Payne River	11.4	4.12	2.20	617
Leaf Basin	45.2	4.33	2.28	2450
Koksoak River	5.7	4.09	2.12	274
George River	8.0	3.37	1.68	258
Ablóviak Fjord	4.4	2.14	1.73	57

The total assessed power amounts to 3650 MW and compares with the whole power potential of the Grande-Rivière. In addition, the tidal power output is highly reliable on a yearly basis (Bernshtein, 1965) while fluctuating from day-to-day and its yearly regularity could be used to compensate the yearly variabilit of the output of the rivers exploited around James Bay. This would involve integrating the network of the Ungava Bay project with the NBR, Grande-Rivière and Eastmain River projects; this would be most natural to do in any case, simply to take advantage of the networks already established.

7. References

Bernshtein, L.B. 1965. Tidal energy for tidal power plants. Program for Scientific Translations. 378 p. Jerusalem, Israel.

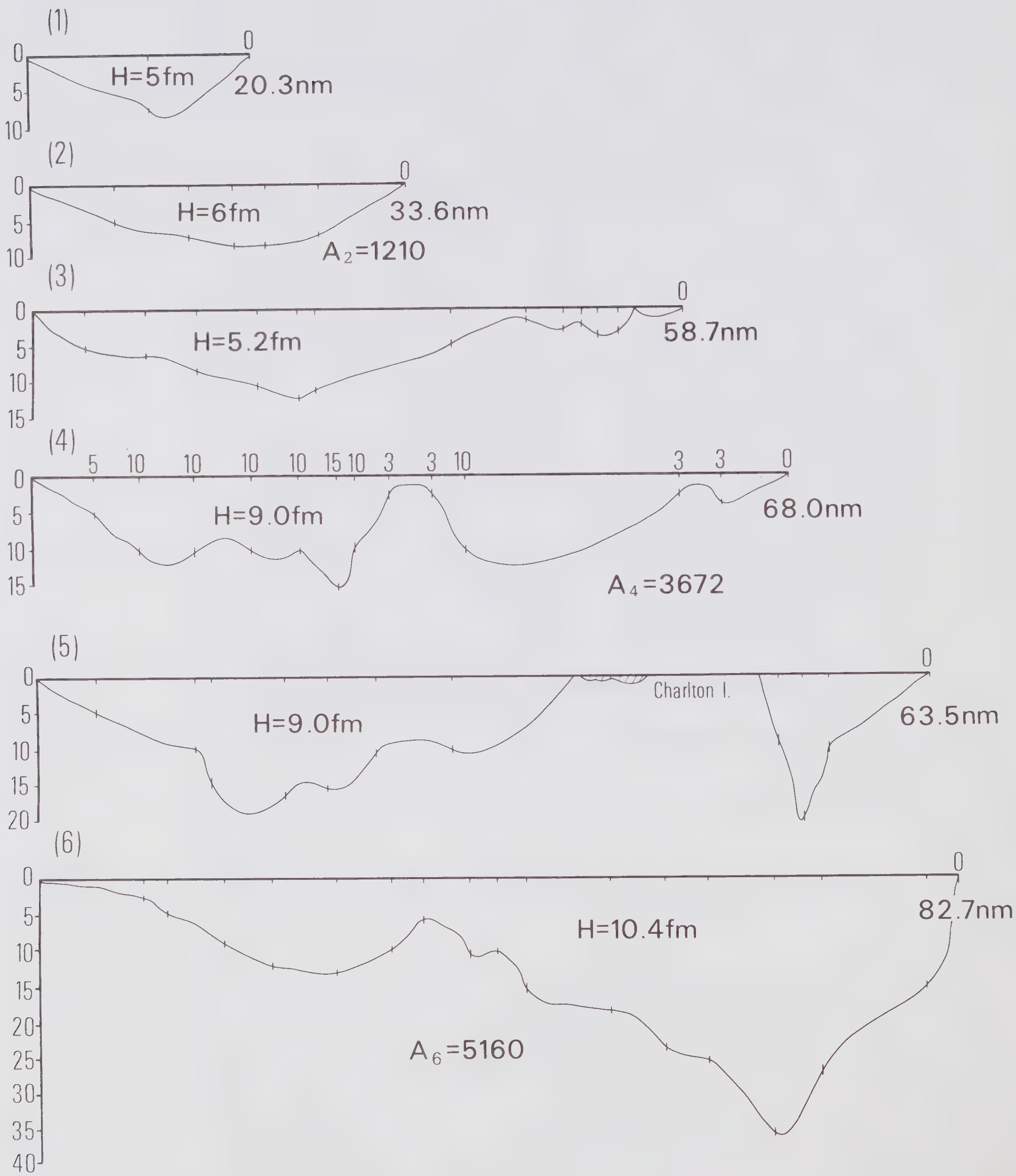
Godin, G. 1969. Theory of the exploitation of tidal energy and its application to the Bay of Fundy. J. Fish. Res. Bd. Canada 26:2887-2957.

Langford, C.J. 1963. Moose River and approaches. Survey of tides, currents, density and silt. Canadian Hydrographic Service. Ottawa, Canada.

8. List of figures and tables

9. Appendix

Pictorial presentation of the profiles derived from the schematization of the bay (Table 2).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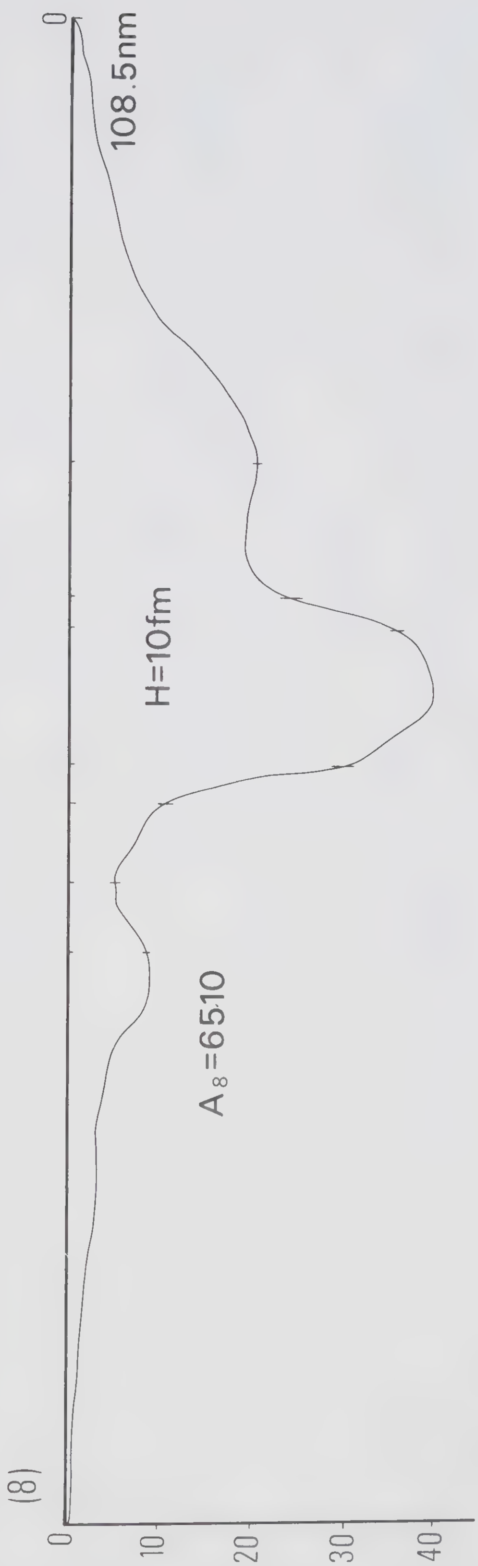
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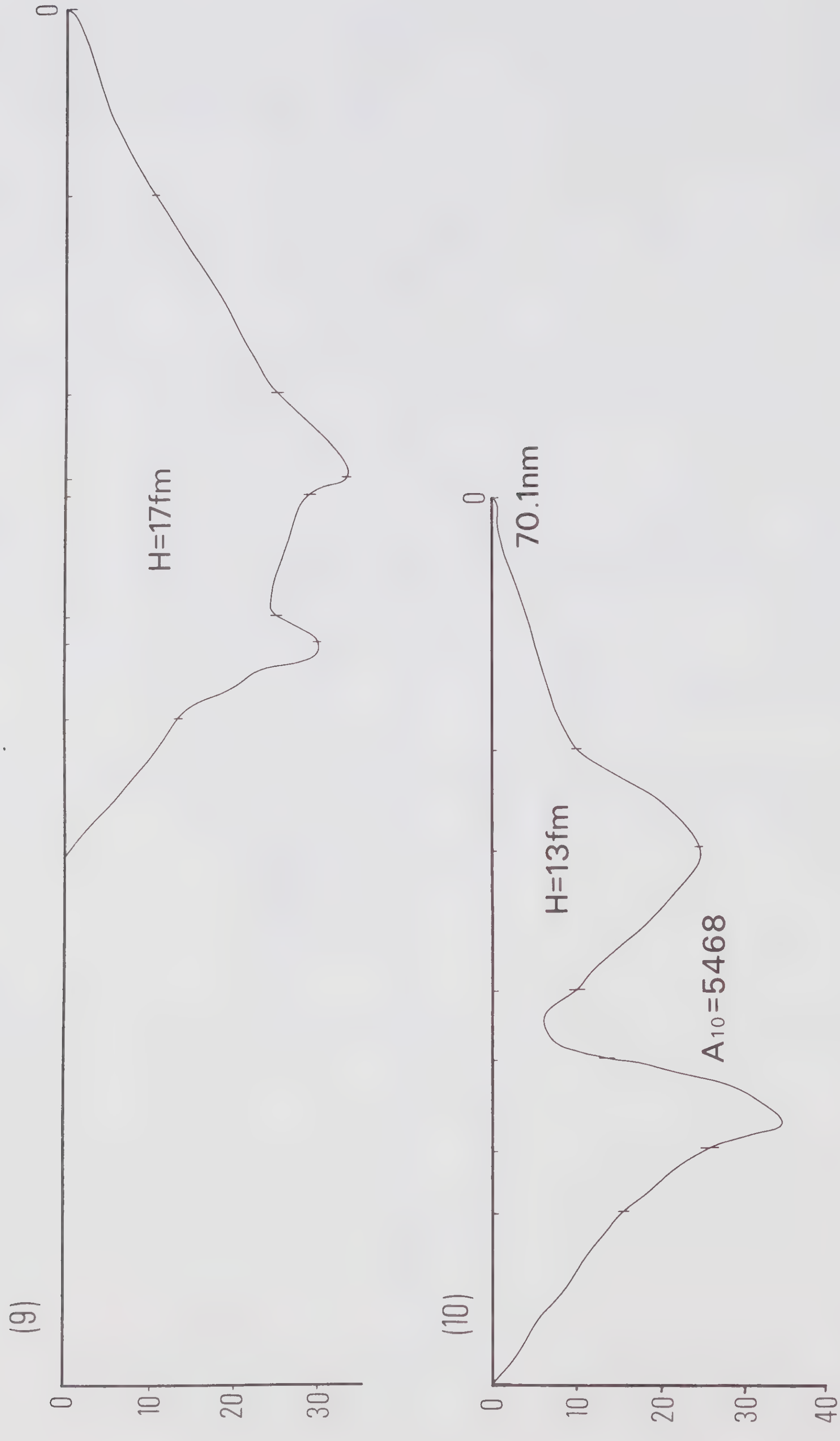
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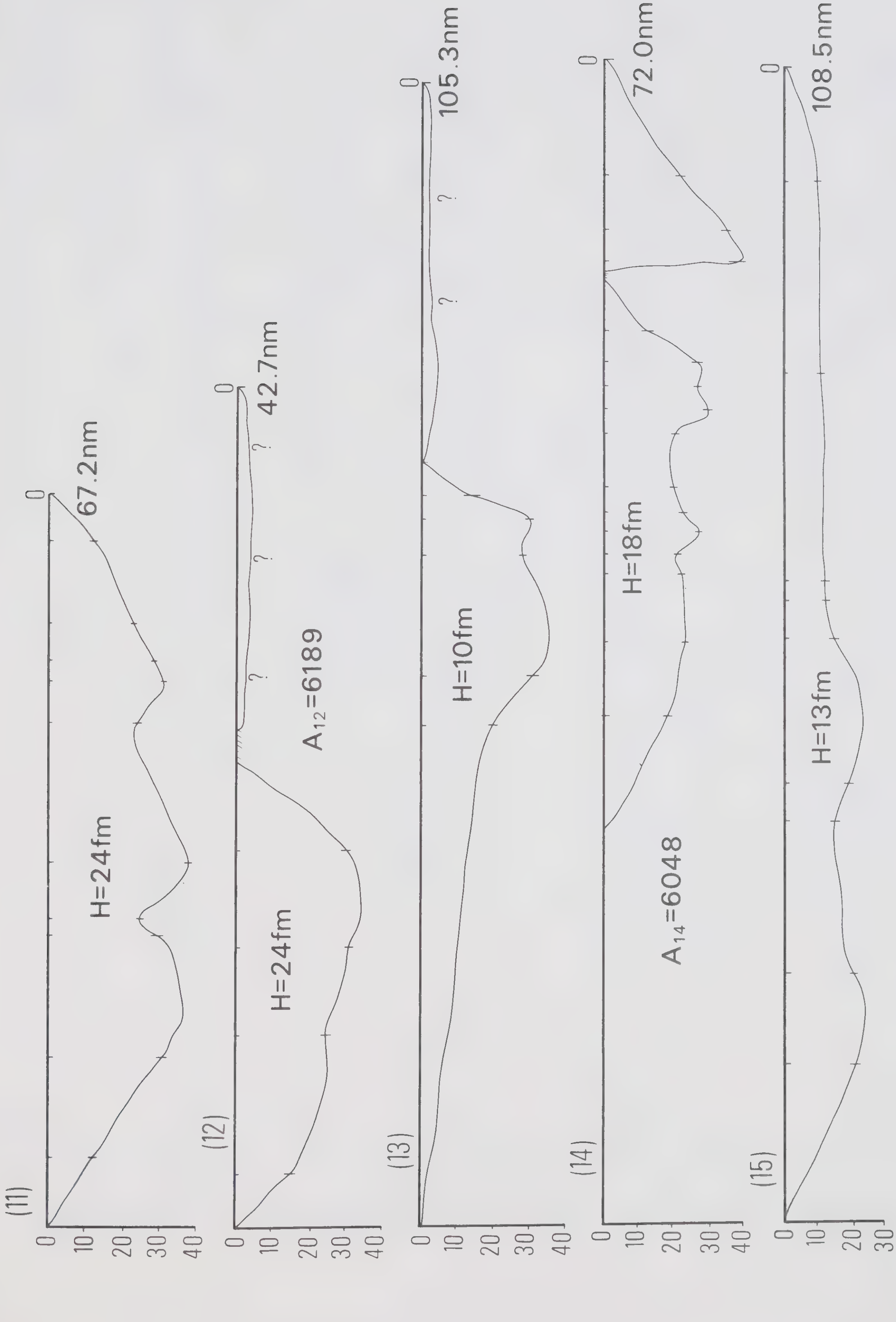
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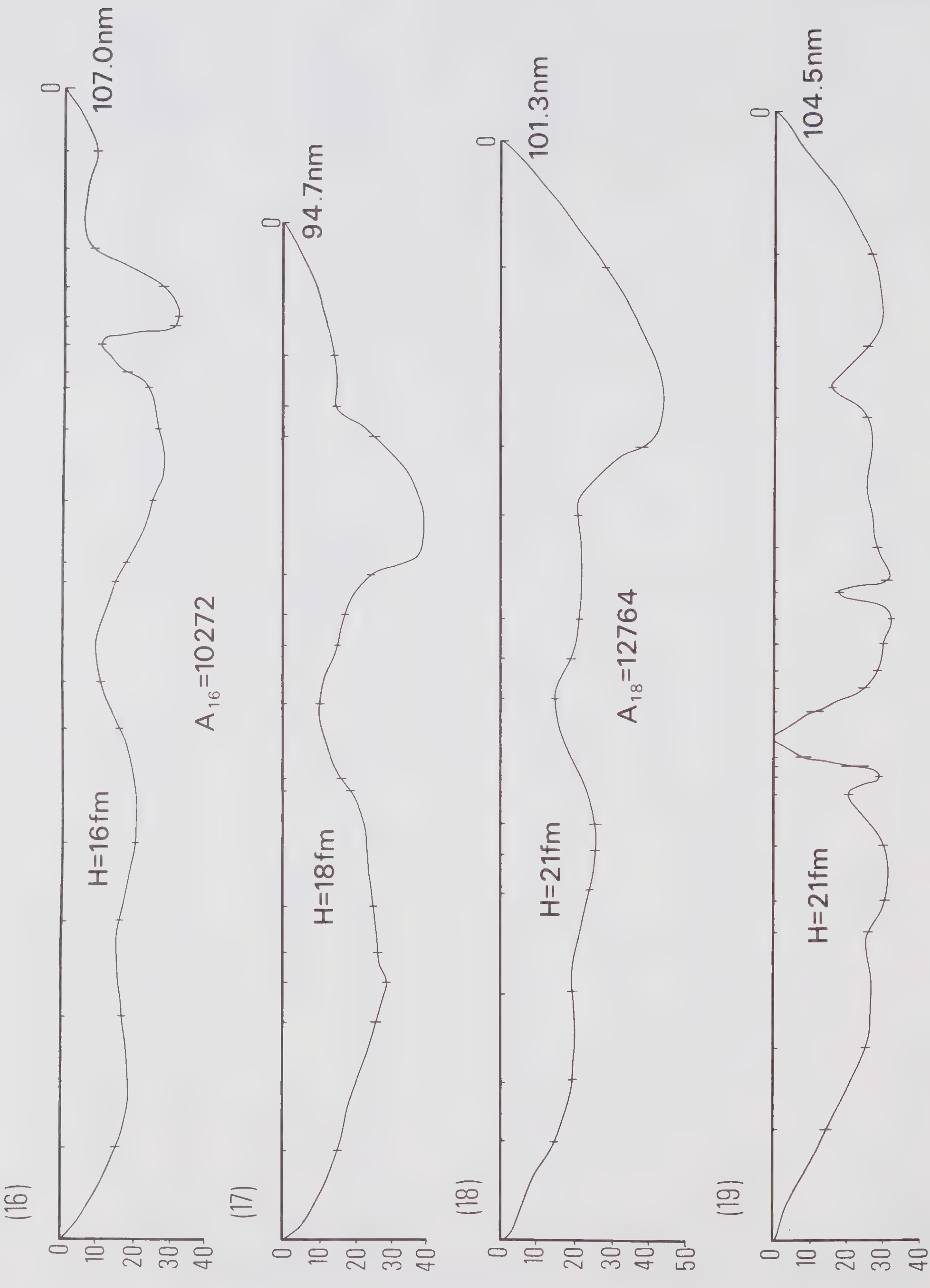

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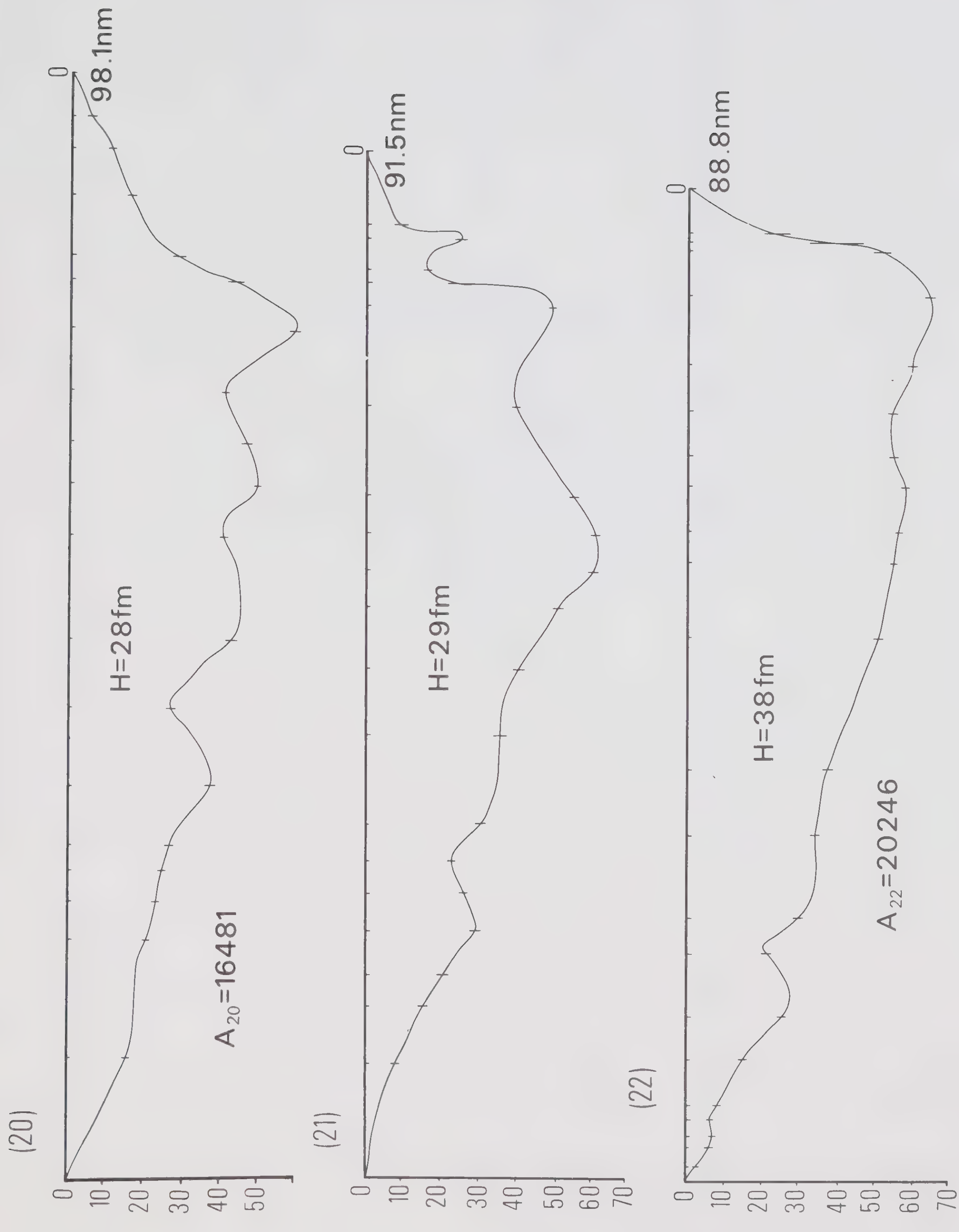

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A_{22}=20246
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(23)

Circulation in James Bay

by

T.S. Murty

Marine Sciences Branch

Department of the Environment

Ottawa, Ontario

Contents

0. Abstract

This is a preliminary study on some physical oceanographic problems of James Bay. The interest in this study arose in connection with the proposed hydroelectric power project on some rivers draining into James Bay. The normal modes of response of James Bay are calculated using a one dimensional topographic model. Using the method of characteristics it has been shown that at the southern shores of James Bay the storm surge amplitude could reach up to 19 feet. It was shown that the thermohaline circulation in James Bay could be as intense as the wind generated circulation and this may explain some of the anomalous ice drift patterns. Although no barotropic coastal jets appear to be possible in James Bay, baroclinic coastal jets with a width of 7 kilometers (km) can occur with the core of the jet being at about 7 km seaward from the position where the thermocline intersects the bottom. The circulation in some rivers draining into James Bay is examined theoretically using the concepts of the three modes of estuarine circulation, namely the river-discharge mode, wind-stress mode and gravitational-convection mode. The question of atmospheric water balance over these river basins is briefly examined and some possible consequences of the man-made changes have been speculated.

1. Introduction

This is a preliminary study of some physical oceanographic problems in James Bay. Because of the deadline imposed on the time available for this study no rigorous numerical modelling has been attempted. The present interest in this study arose from the recent announcement by the government of Quebec to divert the waters of certain rivers flowing into James Bay for hydroelectric power purposes.

Barber (1971) summarized some of the physical oceanographic problems in James Bay and its connected waters, namely Hudson Bay. He also pointed out the considerations that led to this present study. Godin (1971) studied the propagation of the tide into James Bay using a one dimensional topographic model. The topographic data used in the normal mode study has been kindly supplied to me by G. Godin and the input data on the estimates for the circulation problem has been supplied by F.G. Barber.

2. The normal modes of James Bay

A knowledge of the normal modes of response of James Bay is useful in understanding the phenomena of tides, storm surges and circulation and for this reason these have been computed using a one dimensional topographic model. The linearized momentum and continuity equations for flow in a water body of varying cross-section are given by the so-called channel equations (Rao, 1968),

$$\frac{\partial \hat{M}}{\partial t} = -gA \frac{\partial \hat{\eta}}{\partial x} \tag{1}$$

$$\frac{\partial \hat{\eta}}{\partial t} = - \frac{1}{B} \frac{\partial \hat{M}}{\partial x} \tag{2}$$

where the x axis is locally tangent to the principal axis of the channel, t is time and g is gravity. The other parameters have the following meanings:

$\hat{M}(x,t)$ = volume transport through a vertical section,

$\hat{\eta}(x,t)$ = water level deviation from the undisturbed level,

$A(x)$ = area of cross-section and

$B(x)$ = surface width of the section.

In the above equations, friction and terms due to the earth's rotation are ignored. Later we will estimate the possible effects of these terms on the frequencies of the normal modes. For free oscillations of the systems assume,

$$\hat{M}(x,t) = M(x)\cdot\sin(\sigma t) \text{ and} \tag{3}$$

$$\hat{\eta}(x,t) = \eta(x)\cdot\cos(\sigma t)$$

Here $M(x)$ and $\eta(x)$ are the space dependent normal mode functions of James Bay and σ is the frequency. Substitution of (3) into (1) and (2) gives:

$$\sigma M = -gA \frac{d\eta}{dx} \tag{4}$$

$$\sigma\eta = \frac{1}{B}\frac{dM}{dx} \tag{5}$$

The boundary conditions are the following:

M is arbitrary and $\eta=0$ at the mouth of James Bay, (6)

$M=0$ at the head of James Bay. (7)

Equations (4) to (7) form an eigenvalue problem of the frequencies σ.

Define, following Rao (1968)

$$C_i \equiv \frac{\Delta x}{gA_i} \quad \text{and} \quad D_i \equiv -B_i\Delta x \tag{8}$$

Then equations (4) and (5) become:

$$\eta_{i+1} = \eta_{i-1} - \sigma C_i M_i, \quad i=2(2)22 \tag{9}$$

$$M_{i+1} = M_{i-1} - \sigma D_i \eta_i, \quad i=3(2)23 \tag{10}$$

These finite difference forms are written with respect to a staggered grid in which M and η are computed at alternate points. The James Bay topography used here is the same as the one used by Godin (1971) in his calculation of the propagation of the tide. The grid distance Δx is 20 nautical miles, this being the distance either between two successive M sections or two successive η

sections. In our model the grid numbering is such that, section one is the mouth and section twenty-four is the head of James Bay, opposite to the scheme of Godin (1971).

The equations (9) and (10) were solved by prescribing $\eta_1=0$ and $M_2=10^8$ cm^3 sec^{-1} and alternately solving these equations. Since these are linear equations, any arbitrary value can be assigned to M_2. A trial value is provided for σ for the first mode and if M_{24} is not very close to zero, the value of σ is changed slightly and the computation is repeated. This is continued until that value of σ is found which makes M_{24} equal to zero for practical purposes. This value of σ gives the frequency of the fundamental mode. The same procedure can be used to calculate the frequencies of the higher modes.

The trial values for σ for each mode can be calculated from the Merian formula:

$$T_m = \frac{4L}{m\sqrt{gH}} \tag{11}$$

where T_m is the period of the m^{th} longitudinal mode, L is the total length of James Bay and H is the average depth of James Bay. The values used were L=230 nautical miles and H=32 metres. Table 1 shows the periods of the normal modes of James Bay calculated both from the Merian formula and from the topographic model.

Table 1 Periods of the normal modes of James Bay.

Modal Number	Period Calculated from Merian Formula Hours	Min	Period Calculated from Topographic Model Hours	Min
1	26	45	22	42
2	13	23	8	54
3	8	55	6	00
4	6	41	4	24
5	5	21	3	48
6	4	28	3	06

Figures 1(a) to 1(f) show the structures of these six longitudinal modes in terms of the modal functions M(x) and $\eta(x)$. In each, the abscissa denotes the grid point number with 1 denoting the mouth and 24 the head. The ordinate scale on the left side shows η and on the right side shows M. Figure 2 shows the positions of the nodes in James Bay computed from the topographic model.

In the above calculation the effect of the earth's rotation is ignored, which is a serious drawback considering the size of James Bay. The inclusion of rotation not only changes the frequency of the mode but also destroys the standing nature of the oscillation by introducing amphidromic systems into the modal structures. James Bay may be treated as a rectangular bay with length about three times the average width for purposes of estimating the effect of rotation on the frequency of a given mode. Rao (1968) mentions that Van Dantzig and Lauwerier (1960) gave the following formula for a rectangular bay with length twice the width,

$$\sigma = \sigma_o + 0.504 \frac{f^2 L}{2\pi C} + O(f^4) \tag{12}$$

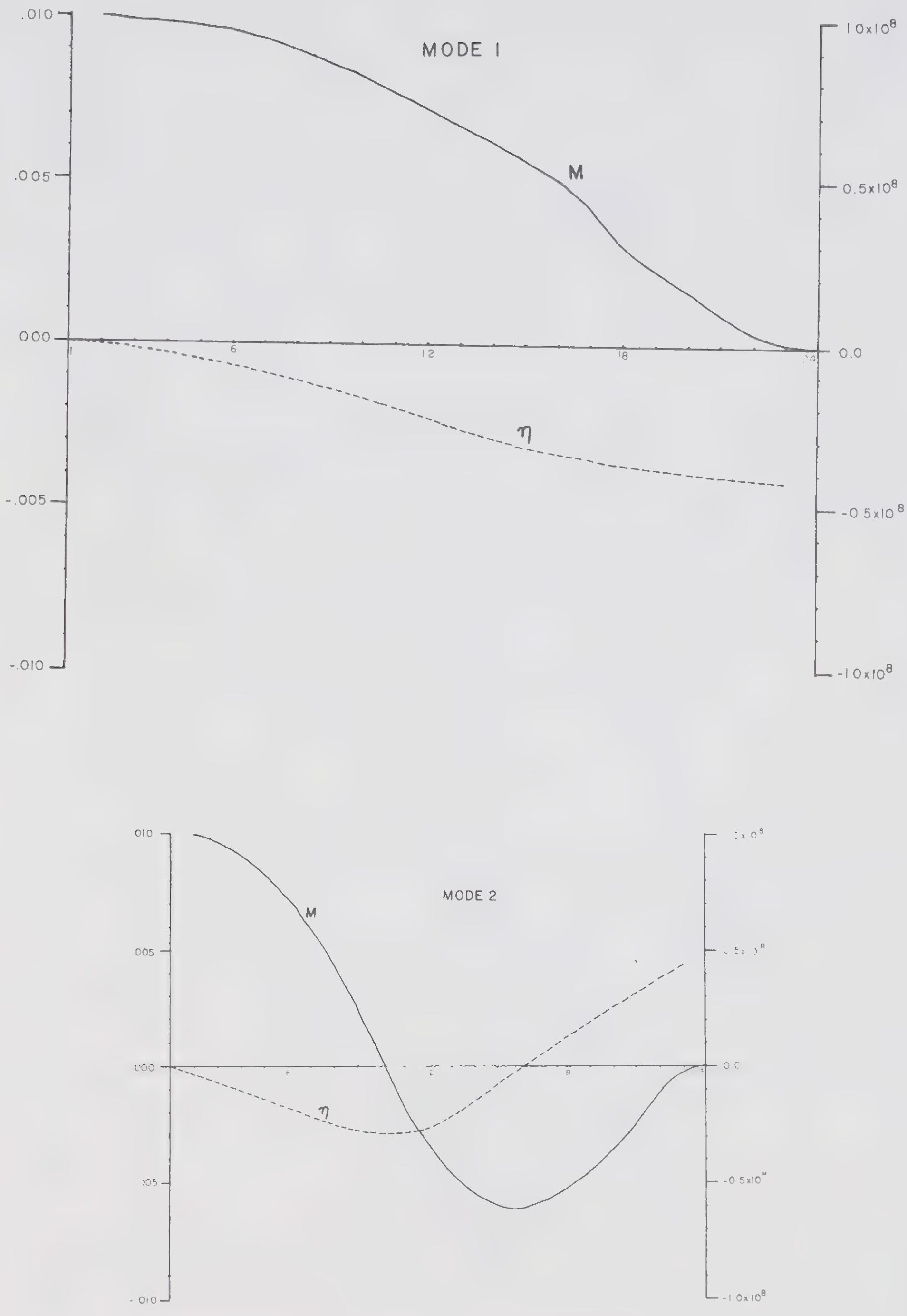
where σ_o is the lowest nonrotating frequency, given by,

$$\sigma_o = \frac{\pi C}{2L} \quad \text{and}$$

$$C = \sqrt{gH} \tag{13}$$

and where the other symbols have the same meaning as before.

Figure 1 Structure of the first six longitudinal modes of James Bay. The abscissa shows the grid number (1 is the mouth and 24 is the head). The ordinate scales on the left and right sides are for the water level η and the volume transport M respectively. Since this is a linear problem the actual units are arbitrary.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M
η
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.005
0.00
-.005
-.010
1.0x10^8
0.5x10^8
0.0
-0.5x10^8
-1.0x10^8
6
12
18
MODE 2
M
η
-0.5x10^8
-1.0x10^8

Figure 1

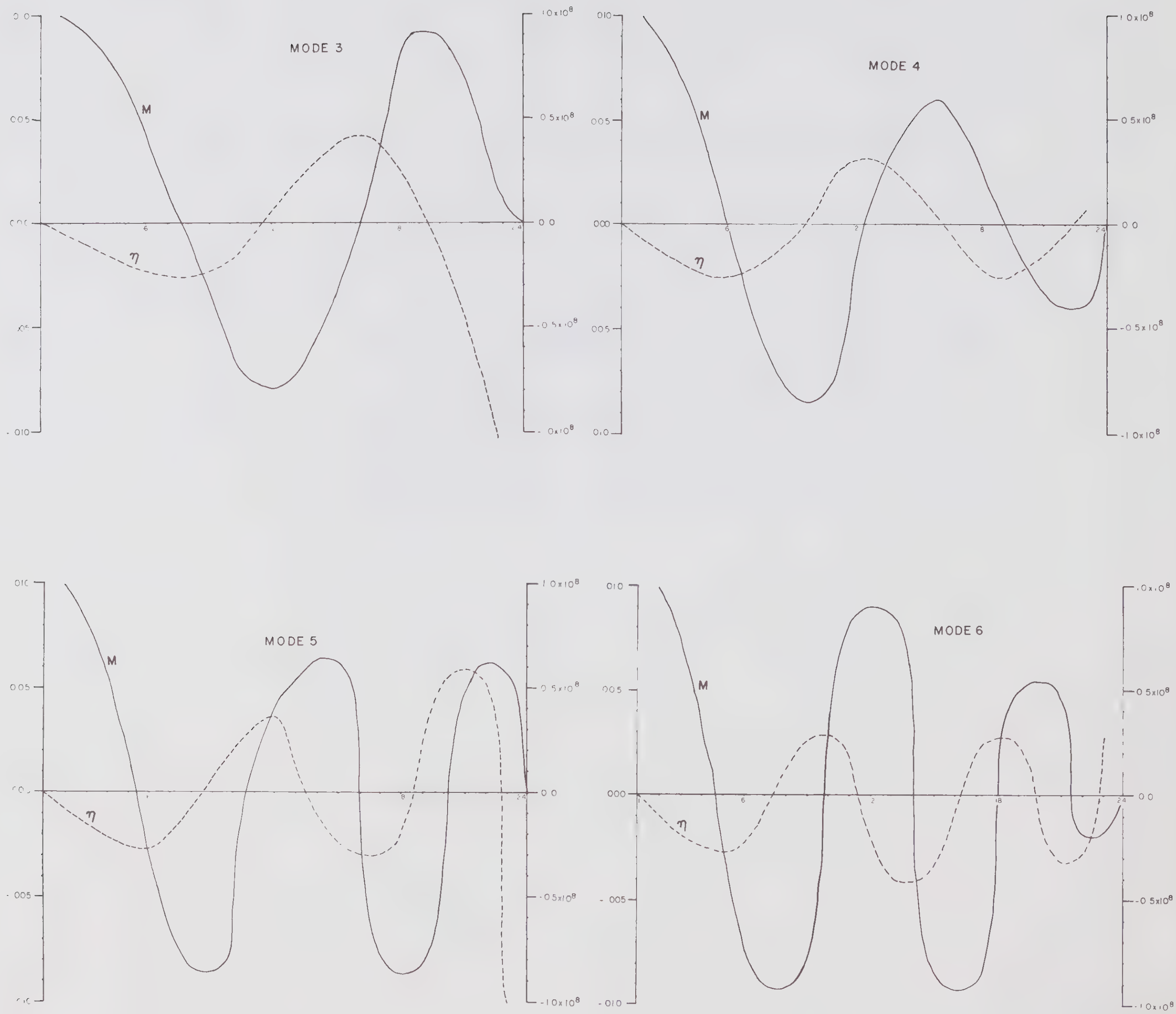


Figure 1 (con't)

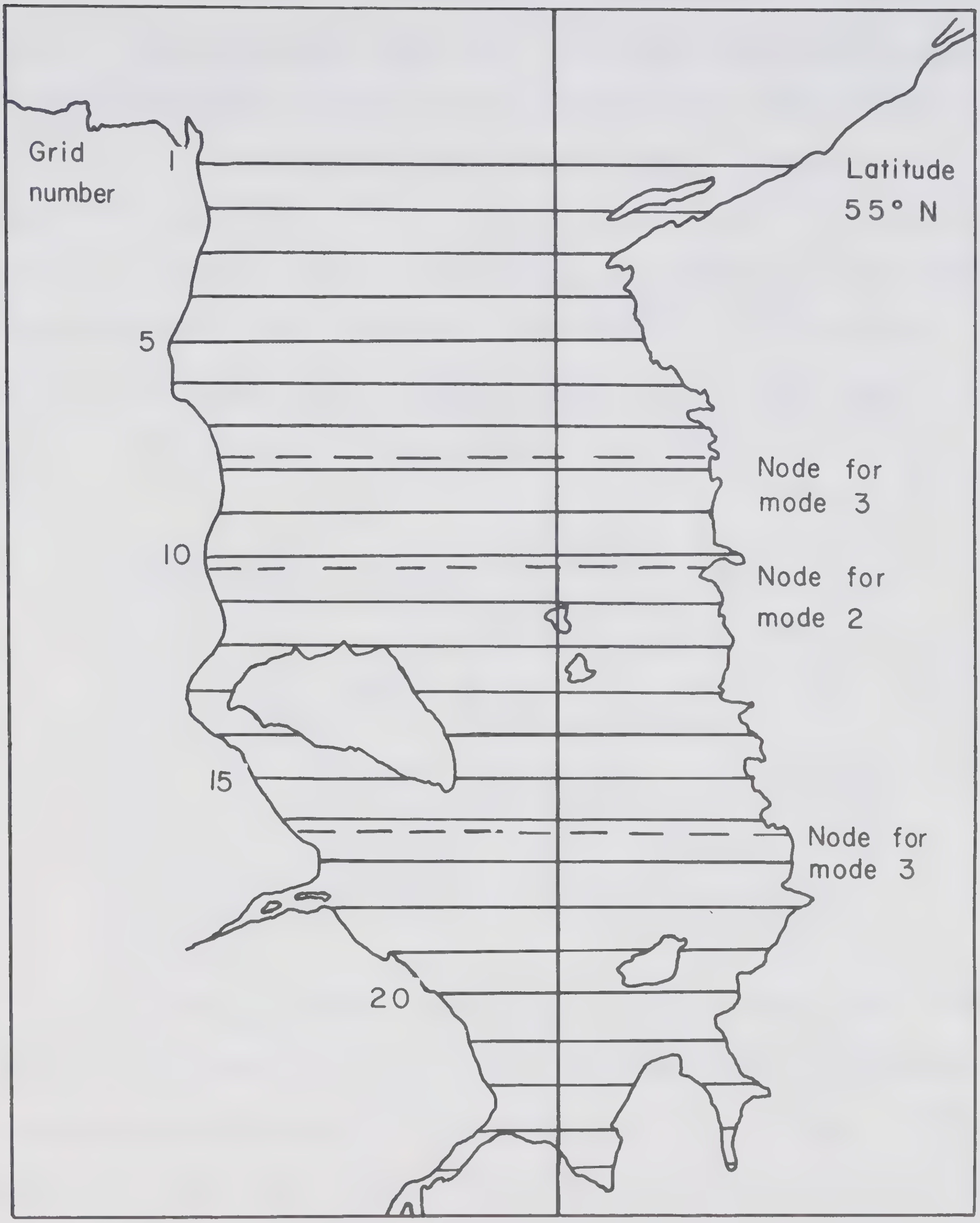


Figure 2 Nodal positions in James Bay calculated from the topographic model.

The equation gives a result that is exactly opposite to what happens in a completely closed basin. That is, in the case of a rectangular bay, rotation increases the frequency of the lowest mode. Since in our case (Table 1) the period of the fundamental mode is 22 hours 42 minutes then,

$$\sigma_o = \frac{2\pi}{22.7 \times 3600} \sec^{-1} \tag{14}$$

$$\text{Since } \frac{L}{C} = \frac{\pi}{2\sigma_o} = \frac{22.7 \times 3600}{4}$$

$$= 20{,}430 \text{ sec}$$

From (12), (13)

$$\frac{\sigma L}{C} = \sigma_o \frac{L}{C} + 0.504 \frac{f^2}{2\pi} \left(\frac{L}{C}\right)^2$$

$$= \frac{\pi}{2} + 0.335$$

The nondimensional period, $T = 2\pi/(\sigma L/C)$ is therefore 3.3, while in the nonrotating case it is 4. Thus rotation can reduce the period by 17.5 percent for a rectangular bay with length and fundamental mode period of James Bay and with width equal to half the length of James Bay. Since, in reality, James Bay is about three times as long as it is wide, the reduction of the period due to rotation would be less than 17.5 percent.

3. Storm surge estimation for James Bay

Thompson (1968) summarized the climate of Hudson Bay. He stated that the upper air circulation over Hudson Bay is the persistent counter-clockwise air flow around a low pressure vortex over northern Baffin Island in winter. This gives rise to a general transport of cold air in a northwest to southeast direction in winter. Thompson further stated that since many of the storm centers lack sufficient moisture to cause heavy precipitation, their principal effect is in inducing strong north winds over Hudson Bay as they travel across it. He further stated:

> "In contrast to the broad expanses of Arctic tundra that surround Hudson Bay the sub-Arctic lands bordering James Bay are partially forested and thus protected from strong winds. As a result two of the most distinguishing features of Hudson Bay winter climate - wind chill and blowing snow - are not nearly as evident near James Bay. With this important exception and the fact that James Bay is several degrees of latitude farther south,the factors that influence the climate of James Bay are essentially the same as those outlined for Hudson Bay".

Archibald (1969) studied the storm tracks over Hudson Bay and eastern Canada. As can be seen from his diagrams the storms move from different directions over James Bay. Since my present interest is in the maximum possible amplitudes of storm surges in James Bay, I will concern myself mainly with storms moving from north to south over James Bay thereby piling up water on the southern shore and causing large surges. Since the magnitude of the surge is strongly dependent upon the nature of

the coast of James Bay I will briefly summarize some relevant facts from the article by Robinson (1968).

It can be seen from Figure 1 of Robinson's article that the eastern half of James Bay falls into the classification of "East Coast Uplands", while the western half falls into the category "South Coast Lowlands". Regarding the south coast lowland, I quote from Robinson (1968):

> "Along the whole coast there is a flat strip five to ten miles wide, with the widest parts generally being to the north. The coastal zone is treeless, but grass or marshes are common. Storm beaches, a few feet high, are the only topographic features. Tidal flats may be exposed for one to six miles; even at high tide shallow water extends far offshore. ...Deeper water may be found at the drowned river mouths, but shifting sand bars and minor deltaic deposits are navigation hazards".

Regarding the east coast upland I again quote Robinson (1968):

> "The east coast and adjoining interior have a combination of landform features which are different from those west of Hudson and James bays, but in some characteristics they have regional similarities. The central and northern sections of the east coast are high and rugged, with rocky hills and drift-covered uplands inland, as is characteristic of the Northwest Hills; the southern section along James Bay is a poorly-drained lowland, although not as wide as that on the west side."

Because of the time limitation of this study, no numerical modelling with topography taken into account has been attempted. Instead, use was made of the analytical solutions developed by Rao (1969) for a rectangular bay of uniform depth, making use of the method of characteristics. In this study

James Bay will be treated as a rectangular bay of length 230 nautical miles, of width up to 100 nautical miles and of average depth 32 meters, connected to Hudson Bay at the mouth. James Bay can be considered narrow because its length is about three times its average width and a one dimensional model is used here and the effect of earth's rotation is suppressed. Since we are interested in the transient response of the bay to a time and space-dependent wind field, the bottom friction is not included. It can be shown by order of magnitude considerations that the nonlinear term in the momentum equation can be omitted. Hydrostatic assumption has been made for the pressure field. For convenience, the relevant equations and the solutions will be summarized here.

Let $\bar{x}$, $\bar{z}$ represent a cartesian coordinate system such that the $\bar{x}$-axis is oriented along the length of James Bay with $\bar{x}=0$ at the mouth and $\bar{x}=\bar{L}$ at the southern shore. The $\bar{z}$-axis is positive upwards and is measured from the undisturbed position of the water surface. Then the bottom is at $\bar{z}=-\bar{h}$. Here, superior bar denotes dimensional quantities. The vertically integrated forms of the momentum and continuity equations are:

$$\frac{\partial \bar{M}}{\partial \bar{t}} + \bar{g}\ \bar{h}\ \frac{\partial \bar{\eta}}{\partial \bar{x}} = \bar{R} \tag{15}$$

$$\frac{\partial \bar{\eta}}{\partial \bar{t}} + \frac{\partial \bar{M}}{\partial \bar{x}} = 0 \tag{16}$$

where

$$\bar{M} = \int_{-\bar{h}}^{0} \bar{U}\, d\,\bar{z} \tag{17}$$

is the volume transport through a vertical section, $\bar{U}$ being the horizontal velocity in the $\bar{x}$ direction and $\bar{\eta}$ is the deviation of the water level from its mean position, $\bar{g}$ is gravity, $\bar{t}$ is time and $\bar{R}$ is the force due to wind stress given by:

$$\bar{R} \equiv \frac{\bar{\tau}}{\bar{\rho}} \tag{18}$$

The boundary conditions are the following:

$\bar{M}=0$ at the head of James Bay, i.e. at $\bar{x}=\bar{L}$, (19)

$\bar{\eta}=0$ at the mouth, i.e. at $\bar{x}=0$. (20)

Initially, $\bar{M}=0$, $\bar{\eta}=0$ at $\bar{t}=0$ for all $\bar{x}$. (21)

Rao (1969) introduced the following nondimensionalization:

$$x \equiv \frac{\bar{x}}{\bar{L}} \qquad \eta \equiv \frac{\bar{c}^2}{\bar{R}_0 \bar{L}}\,\bar{\eta} \qquad R \equiv \frac{\bar{R}}{\bar{R}_0} \tag{22}$$

$$t \equiv \bar{t}\,\frac{\bar{c}}{\bar{L}} \qquad M \equiv \frac{\bar{c}}{\bar{R}_0 \bar{L}}\,\bar{M}$$

where $\bar{c} \equiv \sqrt{\bar{g}\ \bar{h}}$ (23)

and $\bar{R}_0$ is a scale value of the wind stress. Using this scheme, equations (15) and (16) can be nondimensionalized. From addition and subtraction of these we get:

$$\frac{d}{dt}(M \pm \eta) = R \text{ for } \frac{dx}{dt} = \pm 1 \tag{24}$$

This states that the quantity $M \pm \eta$ is conserved along the positive and negative characteristics $dx/dt = \pm 1$, respectively. Since the bay is assumed to have a uniform depth $\bar{h}$, the positive and negative characteristics are straight lines given by $x = \pm t + \text{const}$.

I will consider two different types of stress bands and assume that the wind stress force R is in the form of a step function and moves with a constant speed $\bar{V}$ starting from $x=0$ at $t=0$. The nondimensional form of this translational speed is

$$V \equiv \frac{\bar{V}}{\bar{c}} \tag{25}$$

The value of R at any given point in the bay is given by

$$R = \begin{cases} 1 \text{ for } t \geq \frac{x}{V} \\ 0 \text{ for } t \leq \frac{x}{V} \end{cases} \tag{26}$$

This stress band crosses the head of the bay ($x=1$) at $t = 1/B$ and in the case of the semi-infinite band, for $t > 1/V$ the entire bay is under the influence of the wind stress. In the case of the

finite stress band we assume a pulse of a square wave shape with zero forcing ahead and behind the pulse and the width of the pulse being $V(t-t') = VT = X$ where T is the time taken by the pulse to travel past a given point in the bay. The forcing function for the finite stress band can be obtained by superimposing on (26) another semi-infinite band $R'(x',t')$ where

$$R'(x',t') = \begin{cases} 0 \text{ for } t' \leq \frac{x}{V} \\ -1 \text{ for } t' \geq \frac{x}{V} \end{cases} \tag{27}$$

and $t' \leq t$. Hence, the total forcing function for the case of finite band width is

$$R+R' = \begin{cases} 0 \text{ for } t \leq \frac{x}{V} \\ 1 \text{ for } t \geq \frac{x}{V} \geq t' \\ 0 \text{ for } t' \geq \frac{x}{V} \end{cases} \tag{28}$$

Since our equations are linear, the solutions due to forcing from R' can be obtained from the solutions due to forcing from R by changing the sign of η and replacing t with $t' = t - T$.

Rao showed that the maximum possible elevation at the head is given by:

$$\bar{\eta}_{max} = \frac{2}{\bar{g}} \frac{\bar{L}}{\bar{h}} \bar{R}_o \tag{29}$$

He used the following representation for $\bar{R}_o$

$$\bar{R}_o = 4 \times 10^{-6} \, |W|W \tag{30}$$

where W is the wind speed (ft sec^{-1}) along the axis of the bay. If we take W=75 ft sec^{-1}, corresponding to 51 mph we get

$$\bar{\eta}_{max} = \frac{2 \times 230 \times 6080}{32 \times 32 \times 3.281} \times 4 \times 10^{-6} \times 75^2$$

$$= 18.8 \text{ feet}$$

For higher winds speeds the amplitude of the surge is even higher. The reason for the possibility of such large storm surges in James Bay is the long fetch and the relatively shallow water in the bay.

Thus the storm surges appear to attain very large amplitudes at the southern end of James Bay when intense storms pass over it. This result coupled with the fact that the shores are very flat could give rise to very serious storm surge inundation.

4. Circulation in James Bay

The circulation in James Bay cannot be studied completely independent of the circulation in Hudson Bay to which it is connected at its widest portion. As is expected for any large water body in the northern hemisphere, the circulation in Hudson Bay is dominated by a large counterclockwise cell (Murty and Yuen, 1970). Figure 3 shows the circulation in September

(left side) and in May (right side) calculated by these authors using wind stress computed through the balance equation. Figure 4 shows the corresponding water level deviations. These calculations were made using a steady state topographic model and pressure data averaged over several years has been used in the wind stress computation. It can be seen from Figures 3 and 4 that while the circulation itself is extremely weak and not too well defined in the southern portion of Hudson Bay and James Bay, the water level gradients in the equilibrium state are strong in James Bay (relative to Hudson Bay).

The Ice Central Office at Halifax which is responsible for predicting the movement of ice has faced some difficulty in their prediction because of anomalous drift patterns,

> "...The summer of 1968 provided another such situation. The general water circulation pattern is believed to be a cyclonic drift around the Bay. This would normally be expected to carry ice from the Cape Churchill area southeastward toward James Bay. A west to northwest wind accompanying such drift should move the ice southeastward rather quickly...the combination of wind and water current drifts is almost negligible.
>
> It must be assumed, therefore, that during the melting period thermohaline processes occur which tend to disrupt the normally weak circulation in the southern bay. The addition of fresh water through runoff from the rivers along the west and south coasts also tends to disrupt the salinity balance of the southern section of the Bay. However, the failure of ice to drift southeastward under the influence of persistently favourable winds leads again to the idea of a counter-current in the southern part of the Bay, especially during the period when melting ice is present.

Figure 3 Stream function (for the volume transport) in units of 10^{12} cm^3 sec^{-1} for September (left) and May (right) using data averaged over 30 years.

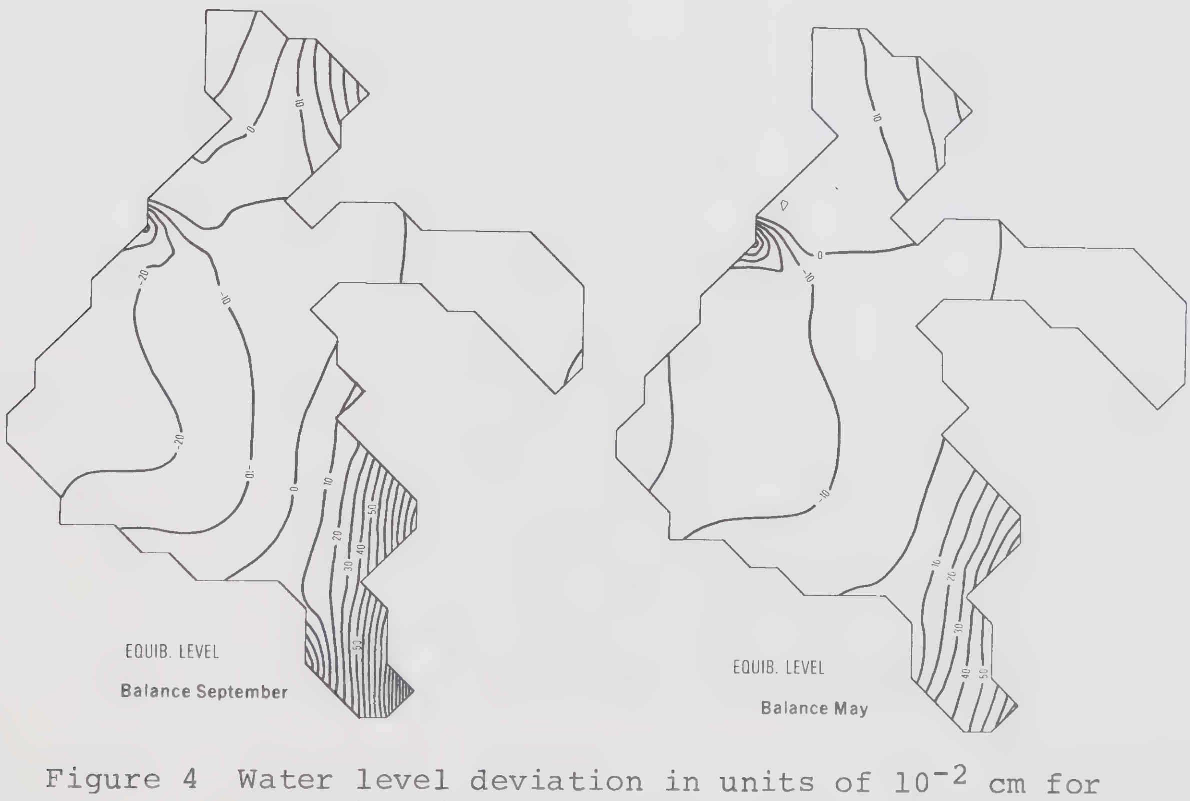


Figure 4 Water level deviation in units of 10^{-2} cm for September (left) and May (right) corresponding to Figure 3.

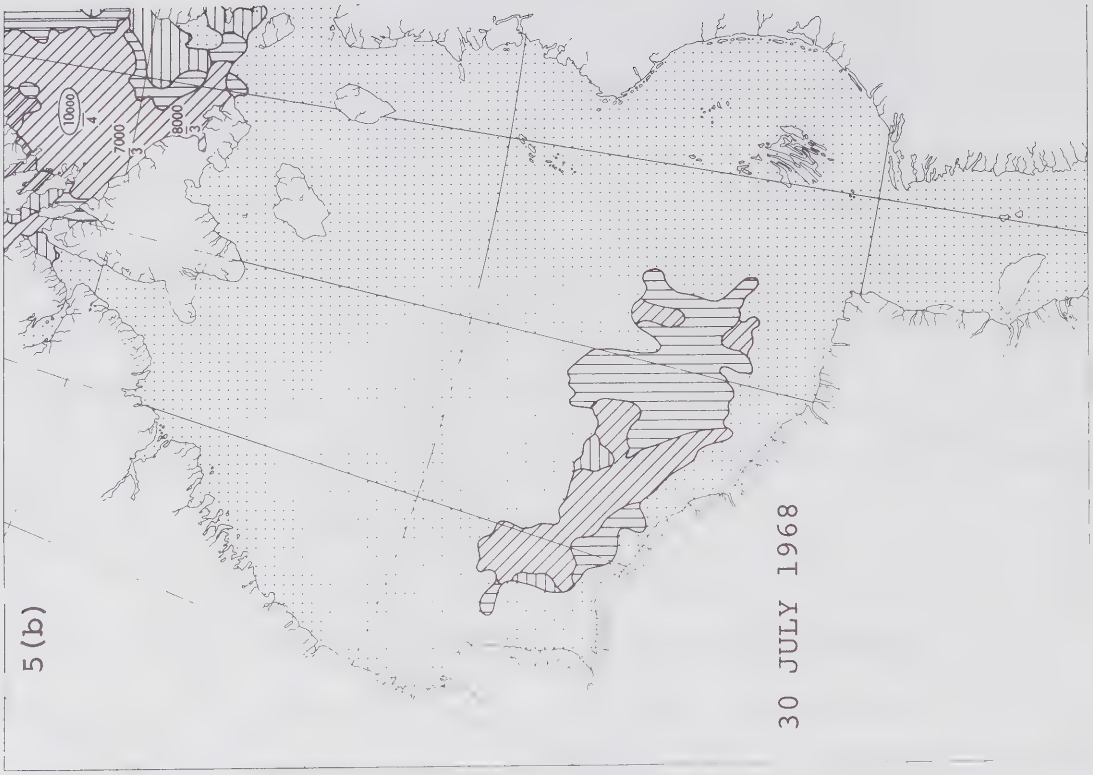
> It is obvious that a serious study of thermohaline relationships to water circulation in this area is necessary before useful forecasts can be provided for southern Hudson Bay". (Anon, 1970).

Figures 5(a) to 5(f) show a sequence of diagrams in which the ice conditions in Hudson Bay are shown on different days. These figures are respectively the figures 12, 14, 16, 18, 20 and 22 in the Ice Central Report for 1968. It can be seen from these that the ice simply melted away and did not move with the water current pattern.

It is clear that an estimation of the intensity of the thermohaline circulation is essential, especially to answer the question whether this could be strong enough to offset the wind-generated circulation. Another consideration is that because James Bay really is part of the complex consisting of Hudson Bay, Hudson Strait and Foxe Basin, this system should be studied together and any estimates made should involve the length and time scales representative of this system as a whole. The north-south extent of this system is large enough for β, the variation of the coriolis parameter with latitude to play a significant role. Because of the time limitations on this study no numerical model has been developed. Instead, estimates have been made using an analytical study made by Gates (1968). For convenience I will summarize Gates' theory.

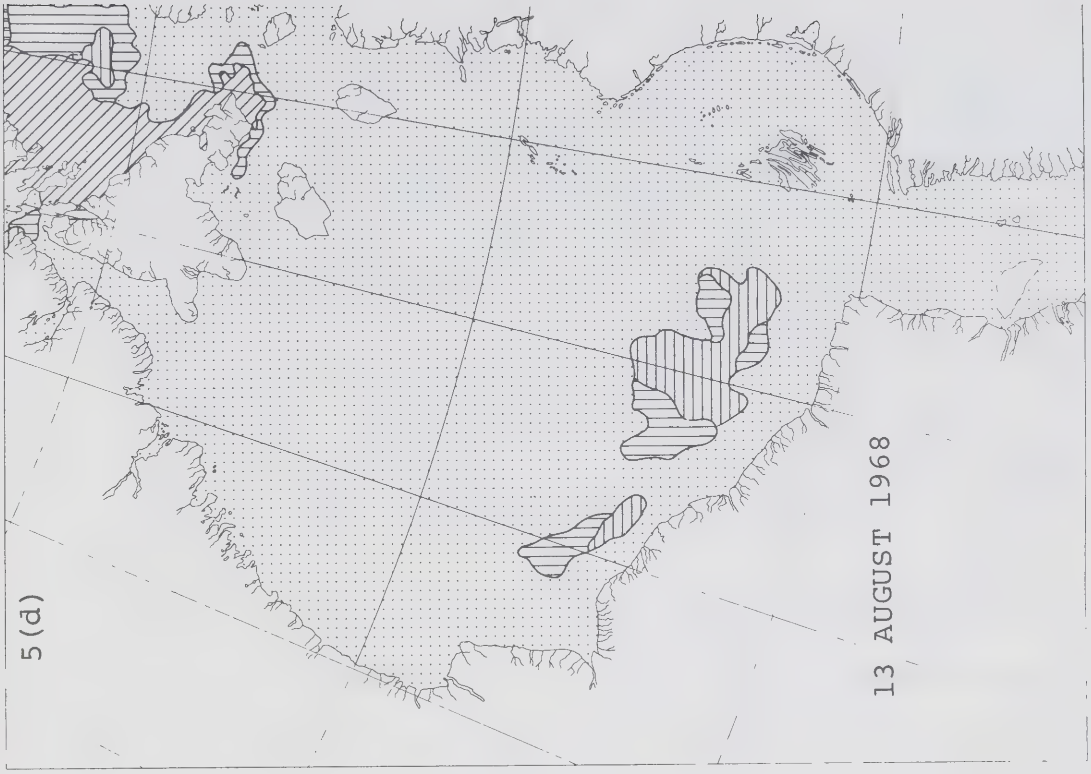
> "The action of surface wind stress on an underlying ocean has been extensively studied since the pioneering work of Ekman (1905). The characteristic depth

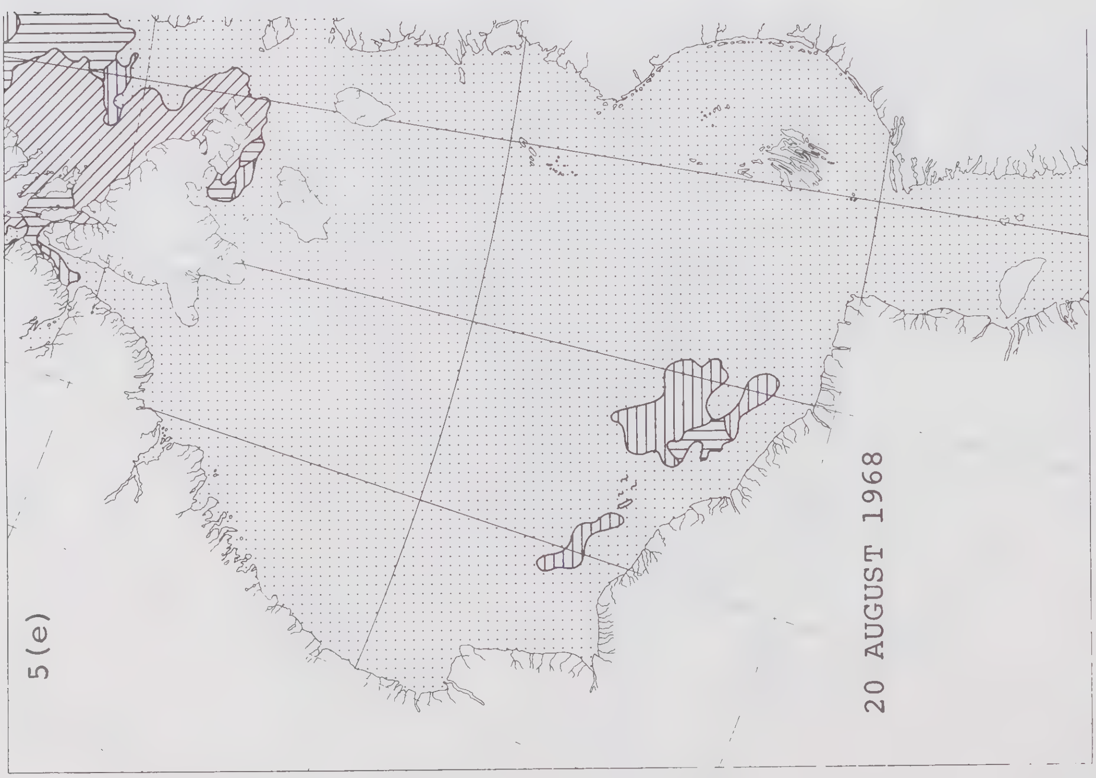
Figure 5 Ice conditions in Hudson Bay for the winter period of 1968-69 on six different dates.


5(b)
30 JULY 1968
10000
4
7000
3
8000
3


5(a)
23 JULY 1968


5(c)
6 AUGUST 1968


5(d)
13 AUGUST 1968


5(e)
20 AUGUST 1968


5(f)
27 AUGUST 1968

of penetration of wind-induced motion (Ekman depth) depends upon both the vertical eddy diffusion and the coriolis parameter, and the net transport in deep waters is normal to the surface wind stress with the horizontal velocity displaying the well known spiral structure. The modifications of these features in shallow water and near coast lines were also explored by Ekman by the introduction of a bottom boundary layer whose transport complements that of the surface layer. In addition to describing the horizontal transport in terms of the wind stress, the Ekman theory provides an estimate of the net vertical flux required beneath the surface boundary layer by mass continuity. This is the so-called Ekman vertical velocity."

Gates (1968) studied at first, a homogeneous ocean of uniform depth on a β-plane and then considered stratification. His assumptions include steady-state, neglect of inertial terms and horizontal friction. The homogeneous ocean is divided into an interior region in between two boundary layers, one at the top and one at the bottom. It should be cautioned that because of the shallow nature of the Hudson Bay System it is debatable whether there really is an interior region where the vertical eddy viscosity is negligible.

The characteristic thickness E of the Ekman boundary layer is:

$$E = \sqrt{\frac{2\nu_z}{f}} \tag{31}$$

where f is the coriolis parameter and ν_z is the vertical eddy viscosity. Gates defined E slightly differently from the conventional definition of Ekman depth, the difference being,

here E is thickness at which the speed is reduced by the factor e^{-1} and this is $1/\pi$ times the conventional definition. Since f increases with increasing latitude, E decreases with increasing latitude provided ν_z does not change. Another assumption involved in Gates' study is that the thicknesses of the boundary layers are small compared to the total depth H (assumed uniform).

The equilibrium pressure field P is determined from the following relation:

$$E\nabla^2 P + \frac{\beta}{f}(2H-E)\frac{\partial P}{\partial x} - \frac{\beta E}{f}\frac{\partial P}{\partial y}$$

$$= 2\,\mathrm{curl}_z\,\tau + \frac{\beta}{f}\tau_{sx} \tag{32}$$

where τ is the vector wind stress with x and y components τ_{sx} and τ_{sy}. It can be seen that, when $\beta=0$ this reduces to

$$\nabla^2 P = \frac{2}{E}\,\mathrm{curl}_z\,\tau \tag{33}$$

which was originally derived by Ekman (1923). The vertical velocity at the bottom of the surface boundary layer is

$$W_1 = \frac{1}{\rho_o f_o}\,\mathrm{curl}_z\,\tau \tag{34}$$

which agrees with the Ekman vertical velocity. Here, ρ_o is the density of the homogeneous fluid and f_o is the coriolis parameter at the standard latitude such that

$$f = f_o + \beta y \tag{35}$$

The vertical velocity in the interior region is given by

$$W_I = W_K + \left(\frac{\beta}{f^2 \rho_o} \frac{\partial P}{\partial x}\right) z \qquad (36)$$

where $W_K = W_K(x,y)$ is an arbitrary function to be determined knowing the wind stress. In the absence of β, this is uniform in depth.

Next, Gates introduced a surface temperature distribution and considered its effects on the Ekman vertical velocity. This temperature distribution is assumed to be maintained by surface heat exchanges and Gates assumed that the horizontal temperature gradient decreases linearly with depth, from a maximum at the surface, z=H, to zero at the bottom, z=0. Thus if T is the temperature field, then

$$\frac{\partial T}{\partial x} = \frac{z}{H}\left(\frac{\partial T}{\partial x}\right)_o \qquad (37)$$

$$\frac{\partial T}{\partial y} = \frac{z}{H}\left(\frac{\partial T}{\partial y}\right)_o$$

where the subscript o denotes the value at the surface. Let the equation of state be

$$\rho = \rho_o(1 - \alpha T) \qquad (38)$$

where α is the coefficient of thermal expansion of water. That

is, here we ignored the effect of salinity variations on the density. However, qualitatively speaking it will add one more term in equations (39) and (40). In (38), T is the temperature relative to a reference value corresponding to $\rho=\rho_o$.

Assuming that $\eta << H$ (where η is the perturbation of the free surface) and $\rho \approx \rho_o$ when η and ρ are undifferentiated we can write the following expressions for the interior geostrophic velocity components

$$U_I(z) = -\frac{g}{f}\frac{\partial\eta}{\partial y} + \frac{\alpha g(H^2 - z^2)}{2fH}\left(\frac{\partial T}{\partial y}\right)_o \tag{39}$$

$$V_I(z) = \frac{g}{f}\frac{\partial\eta}{\partial x} - \frac{\alpha g(H^2 - z^2)}{2fH}\left(\frac{\partial T}{\partial x}\right)_o \tag{40}$$

Thus the presence of a temperature gradient permits the reversal of the current with depth.

The surface boundary conditions are now

$$\frac{\partial U}{\partial z} = \frac{\tau_{sx}}{\rho_o\nu_z} + \frac{\alpha g}{g}\left(\frac{\partial T}{\partial y}\right)_o \tag{41}$$

$$\frac{\partial V}{\partial z} = \frac{\tau_{sy}}{\rho_o\nu_z} - \frac{\alpha g}{f}\left(\frac{\partial T}{\partial x}\right)_o \tag{42}$$

both at z=H. The vertical velocity in the interior is given by

$$W_I(z) = \frac{\beta g}{f^2}\frac{\partial\eta}{\partial x} z - \frac{\beta\alpha g}{2f^2H}\left(\frac{\partial T}{\partial x}\right)_o\left(H^2z - \frac{z^3}{3}\right) + W_K \tag{43}$$

Thus, in the non-homogeneous case the vertical velocity varies as the cube of the depth in the interior while in the homogeneous case it varies linearly.

The equation to determine η for the non-homogeneous case is given by

$$E\nabla^2\eta + \frac{\beta}{f}(2H-E)\frac{\partial\eta}{\partial x} - \frac{\beta E}{f}\frac{\partial\eta}{\partial y} = \frac{2}{g\rho_o}\,\mathrm{curl}_z\,\tau + \frac{\beta\tau_{sx}}{gf\rho_o} + \alpha E\left(\frac{H}{2} - E\right)\nabla^2 T_o$$

$$+ \frac{\alpha\beta H}{f}\left(\frac{2H}{3} - \frac{E}{2}\right)\left(\frac{\partial T}{\partial x}\right)_o + \frac{\alpha\beta E}{f}\left(\frac{3E}{2} - \frac{H}{2}\right)\left(\frac{\partial T}{\partial y}\right)_o \qquad (44)$$

NOTE: equation (32) gives a corresponding equation for the homogeneous case.

It can be seen that when $\beta=0$ this reduces to

$$E\nabla^2\eta = \frac{2}{g\rho_o}\,\mathrm{curl}_z\,\tau + \alpha E\left(\frac{H}{2} - E\right)\nabla^2 T_o \qquad (45)$$

Thus the effect of a local excess of temperature (for example, $\nabla^2 T_o < 0$) is analogous to that of the curl of the anticyclonic wind stress ($\mathrm{curl}_z\,\tau < 0$).

Without giving any more details we will simply state that starting with (45) Gates showed that the thermally forced contribution could be comparable to the wind-induced portion.

5. Possibility of coastal jets in James Bay

The dimensions of James Bay (length 230 nautical miles and average width of 100 nautical miles) are comparable to the Great Lakes. James Bay is very shallow (with an average depth of 32 meters) like Lake Erie. The main difference between James Bay and the Great Lakes, besides the saline water in the former and fresh in the latter, is that James Bay has a very wide mouth and

is connected to Hudson Bay while the Great Lakes are essentially closed systems with only some interconnecting rivers. Csanady (1967, 1968a, 1968b) studied the coastal jets in the Great Lakes and his concepts (Csanady, 1971) could be applied here to study the possibility of coastal jets in James Bay:

> "Theoretical studies of some simple model Great Lakes (Csanady, 1967, 1968a,b) have suggested that shoreward wind drift may set up concentrated boundary currents of essentially zero potential vorticity in the Great Lakes by the mechanism of vortex stretching. Along an infinite vertical shore, as shown by Charney (1955) in an extension of Rossby's (1938) work on geostrophic adjustment, some coastal jets may be generated in this manner, although these would possess practical significance only in the *baroclinic* modes. In a closed basin of constant depth, the implusive application of wind stress leads, by the same mechanism, to the generation of slow baroclinic Kelvin waves (Csanady, 1968b), the profile of which, perpendicular to shore, is very similar to the steady-state coastal jets.
>
> The application of these previous results to the real Great Lakes or other similar bodies of water is not immediate for at least two reasons: 1) depth variations in the shore zone are certain to modify the vortex stretching mechanism; and 2) a linear, frictionless theory (from which the above results were obtained) can hardly be expected to describe water movements accurately, particularly in the shallow coastal zones."

Csanady (1971) at first considered barotropic boundary currents near a sloping shore. He postulates that a boundary current is generated by the arrival of water in the shore zone, either driven by wind or due to the passage of a long slow wave. Csanady shows that no narrow boundary currents are possible in the barotropic case. Then Csanady considered a two-layer system

and showed that the analysis for the barotropic case with a modified scale factor can be used for the baroclinic case. Let ρ and ρ' be the densities of the top and bottom layers and h and h' be the depths. Then $\varepsilon = (\rho' - \rho)/\rho'$ is the fractional density defect. The top layer depth h is assumed to be uniform while the depth h' of the bottom layer is variable and is assumed to be

$$h' = \begin{cases} sy \text{ for } y \leq y_0 \\ h'_0 \text{ for } y \geq y_0 \end{cases} \tag{46}$$

where y denotes a coordinate normal to the shore and it is assumed that conditions are uniform along the shore (for example, $\partial/\partial x = 0$). Here, s is the beach slope.

Csanady showed that for this problem there are two important parameters, K and L where

$$L \equiv \left(\frac{g\varepsilon h}{f^2}\right)^{1/2} \tag{47}$$

and

$$K \equiv \frac{f}{2s}\left(\frac{h}{g\varepsilon}\right)^{1/2} \tag{48}$$

Next he defines

$$S \equiv \frac{sg\varepsilon}{f} \tag{49}$$

Here S is the slope length scale for effective gravity $g\varepsilon$ in the baroclinic case. Thus for the baroclinic case, there are two length scales S and L. A key parameter of this problem is κ defined in (48) which is also the same as

$$\kappa = \frac{L}{2S} \tag{50}$$

For James Bay, we can take

$$\begin{aligned} s &= 7 \times 10^{-4} \\ \varepsilon &= 10^{-2} \\ h &= 5 \text{ meters} \\ f &\sim 10^{-4} \text{ sec}^{-1} \end{aligned} \tag{51}$$

From (48) we get

$$\begin{aligned} \kappa &= 1/2 \\ L &= S = 7 \text{ km} \end{aligned} \tag{52}$$

Thus, the width scale of the boundary current L for James Bay is 7 km. Also the peak current occurs at a distance of order L from the point where the thermocline intersects the bottom.

6. Some miscellaneous topics

Here I will discuss the estuarine circulation in the rivers that drain into James Bay. Because of strong tidal action the saline water intrudes into the rivers and this might give rise to the type of estuarine circulation discussed by Hansen

and Rattray (1965). They derived a set of coupled partial differential equations to study the circulation and salt flux processes for estuaries in which turbulent mixing results primarily from tidal currents and they separated the circulation into modes analogous to the barotropic, baroclinic and Ekman modes of oceanic circulation. These authors state that their solutions (although derived for the Delaware river estuary) hold well for sea straits and narrows having strong tidal currents and well-defined water and density budgets.

The present analysis may be of some use in understanding the dispersion processes and circulation in those regions of the rivers that are affected by strong tidal action from James Bay.

Hansen and Rattray divided the estuary into the three regimes, namely the outer, central and inner regimes, the outer being the closest to the connecting sea. For the circulation in the central regime they derived the following relation using similarity theory.

$$\phi(\eta) = \frac{1}{2} \underbrace{(2 - 3\eta + \eta^3)}_{\mathrm{I}} - \frac{T}{4} \underbrace{(\eta - 2\eta^2 + \eta^3)}_{\mathrm{II}}$$

$$- \frac{\nu R_a}{48} \underbrace{(\eta - 3\eta^3 + 2\eta^4)}_{\mathrm{III}} \qquad (53)$$

where ϕ is a stream function, η is a non-dimensional vertical coordinate, T is a non-dimensional wind-stress, ν is a constant and R_a is an estuarine Rayleigh number. Thus, equation (53) expresses the circulation in the central regime as the sum of

three modes: the river discharge mode (term I), the wind-stress mode (term II) and the gravitational-convection mode (term III). The important result is that only the river-discharge mode allows a net transport of water. In deriving equation (53) it has been assumed that, in the central regime the width and depth of the river as well as the river discharge are constant. This assumption would be more questionable both in the outer and inner regimes. In the next section (Section 7) we will speculate on the relative influences of these three modes in some of the rivers and how the situation may change due to the proposed hydroelectric power project.

Next we consider the question of atmospheric water balance. A study incorporating this along with the hydrology of the various river basins before and after the man-made changes will help to evaluate the effects of the man-made changes. At present, although in principle, the study could be carried out for the present situation, because of the necessity to handle a large amount of data and because of the time limitations on this study, this part of the study is not carried out. However, I will outline a procedure described by Rasmussen (1970) and speculate on this in the next section. Rasmussen has developed the following model to study the atmospheric water balance and hydrology of the upper Colorado River Basin. Since any climatic effects are intimately connected with the atmospheric water balance, the following procedure developed by Rasmussen to study the hydrometeorology of a given river basin using data on precipitation and evaporation will be especially useful. I quote from Rasmussen:

> "Traditionally, studies of the hydrologic balance of river basins have been approached from the viewpoint of the terrestrial part of the hydrologic cycle. The factors determining the runoff from an area are precipitation, evaporation, change in ground-water storage, and underground seepage from the basin. Such an approach to the study of hydrologic problems is often plagued by measurement deficiencies... If a large mountainous region is studied, the measurement problem is maximized because not only is the density of observations low, but the observations are biased toward the lower elevations where precipitation is lower.
>
> Alternately, the atmospheric part of the hydrologic cycle may be studied to evaluate the net deposition of water over an area. A budget parallel to that of the terrestrial part of the hydrologic cycle must be observed. The atmospheric water balance may be expressed as the evaporation minus precipitation occurring at the ground over an area balanced by the net transfer of water mass through the atmospheric volume over the area and by the change in storage of water mass within the atmospheric volume. In theory then, given a continuous distribution in time and space of the atmospheric water mass, an accounting can be done to determine as a residual the quantity evaporation minus precipitation. In practice, however, the distribution of water in the atmosphere is not continuously known; rather only water in the vapour state is sampled and only at time intervals of 12 hours and over distances of hundreds of kilometers. The problem is to approximate the water balance from this imperfect sampling procedure, realizing that the computation is only meaningful over sufficiently large areas and for sufficiently large weather systems."

In a coordinate system with pressure as the vertical coordinate, the time rate of change of water and water vapour can be written as:

$$\frac{d}{dt}(q+r) = \frac{\partial}{\partial t}(q+r) + \mathbf{V}_2\nabla_2(q+r) + \omega\frac{\partial}{\partial p}(q+r) \tag{54}$$

where q is the specific humidity, r is the ratio of the mass of water (either in liquid or ice form) to the mass of air, $\mathbf{V}_2$ is the velocity vector on a pressure surface and $\omega = dp/dt$ is the vertical velocity in the isobaric coordinate system. Equation (54) can be integrated from the earth's surface to some pressure level at which the amount of water in any form is negligible. Then, making use of the continuity equation and the divergence theorem due to Gauss one obtains the so-called atmospheric water balance equation:

$$E - P = \frac{1}{g}\int_{dp}\int_{dA}\frac{\partial}{\partial t}(q+r)\,dA\,dp$$

$$+ \frac{1}{g}\int_{dp}\oint_{dt}(c_n q + c_n r)\,dl\,dp \tag{55}$$

where P and E are the rates of precipitation and evaporation at the earth's surface, g is gravity, dA is an area increment on a pressure surface, dl is a line increment on the vertical boundary and c_n is the component of the wind vector $\mathbf{V}_2$ normal to the walls of the volume such that it is positive upward. If the integrals in (55) can be properly evaluated through observational data, the exchange of water and water vapour at the earth's surface, given by E - P can be determined as a residual.

One should be able to obtain the same exchange of water at the earth's surface from the hydrological balance provided we

confine ourselves to surface waters. The hydrologic balance equation for a river basin is

$$P - E = R_0 + \Delta W + L \tag{56}$$

where R_0 is the runoff from the entire basin, ΔW is the total change in the surface and subsurface water storage and L is the depletion from the basin due to use within the basin itself or due to man-made diversion from the basin.

Thus the results from (55) and (56) should agree and this provides a check on the validity of the computation. Equation (56) is straightforward enough for computation but (55) has to be replaced by the following finite-difference form that is convenient for computation

$$P - E = - \frac{1}{g} \frac{\Delta}{\Delta t} \sum_{j=1}^{N} \sum_{i=1}^{M} q_{ij} \, \Delta A_{ij} \, \Delta P_{ij}$$

$$- \frac{1}{g} \sum_{j=1}^{N} \sum_{i=1}^{M} c_{nij} \, q_{ij} \, \Delta \ell_{ij} \, \Delta P_{y} \tag{57}$$

Here i denotes the grid point number (with maximum M) and j denotes the pressure level with N being the top most one.

7. Speculation on the possible effects due to man-made changes

It has been shown in Section 4 that the thermohaline processes could give rise to circulation as strong as that due to wind-generation in James Bay and the southern part of Hudson Bay. The discharge from the rivers flowing into James Bay will be

effected when the project proposed by the Quebec government is carried out and it appears that towards the end of the winter season, the thermohaline processes could become stronger and thus further impede the movement of ice. Though this is highly speculative it appears as though the movement of ice in the southern part of Hudson Bay may be even more restricted towards the end of the winter season.

Another speculation could be made on the ice pressure. It appears from Russian work (Kagan, 1967a, 1967b) that because of tidal movement, ice could concentrate in the regions near amphidromic points. To illustrate this we reproduced Figures 2 from Kagan's reports of 1967a and 1967b to produce our Figure 6. The left side shows the cotidal lines for the M_2 tide in the Okhotsk Sea while the right side shows the lines of convergence and divergence in the ice pattern. Figure 7 shows the semi-diurnal tide in the James Bay-Hudson Bay-Hudson Strait complex (Dohler, 1964). This figure shows that there are no amphidromic points in James Bay while there are two in Hudson Bay. Hence it is possible that while strong and well defined zones of convergence and divergence may not occur in James Bay, these could occur in Hudson Bay and thus create ice pressure situation. However, the proposed hydroelectric power project may not effect the tidal effects on ice.

Although the total water discharged from the rivers into James Bay annually will be essentially unaffected by the hydroelectric power project, it is very likely that the amount of spring discharge into James Bay will be reduced. Because of the

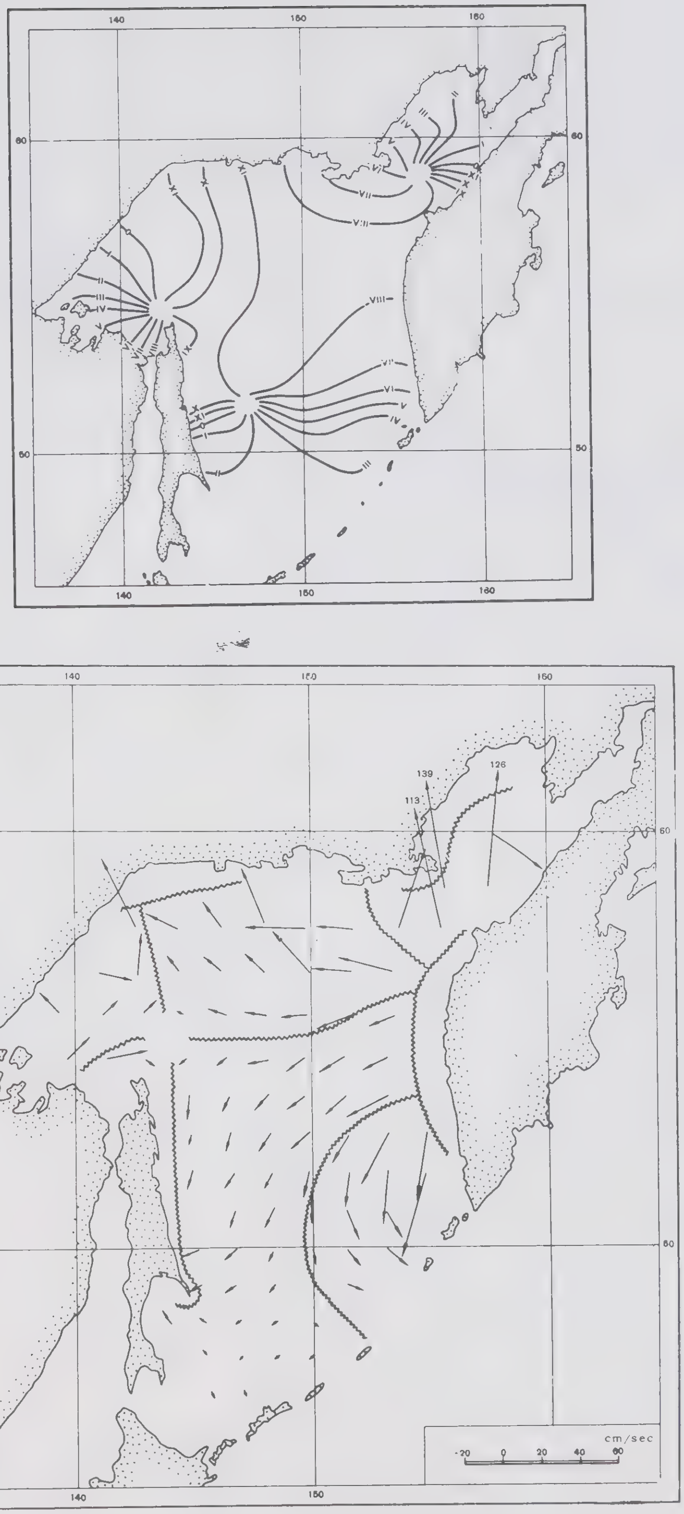


Figure 6 Cotidal lines for the M_2 tide (left side) and convergence-divergence pattern in the ice (right side) in the Okhotsk Sea.

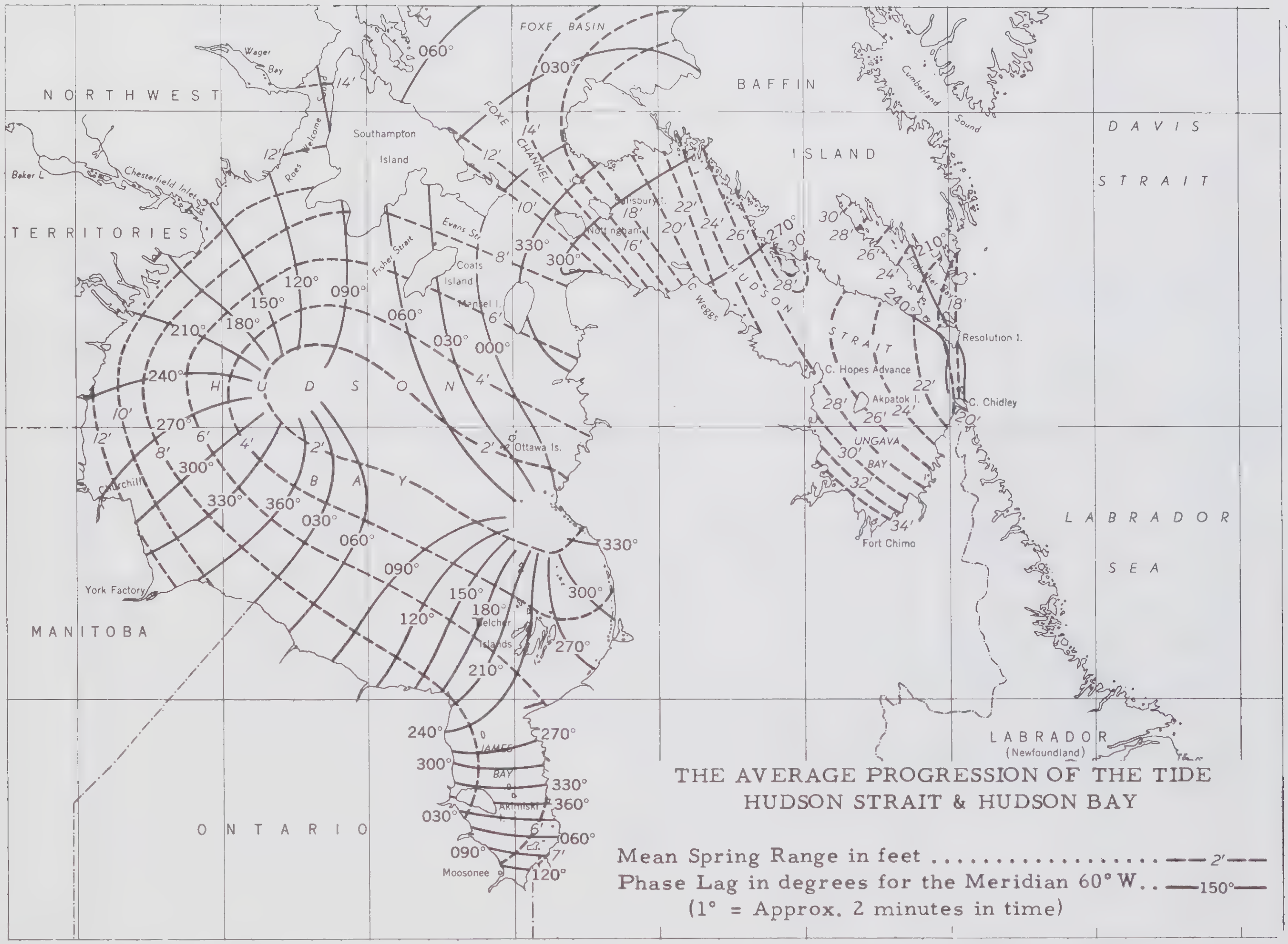


Figure 7 Cotidal lines for the semidiurnal tide in Hudson Bay-Hudson Strait-James Bay complex.

reduction in the freshwater content at the surface, the stability will be reduced and this may give rise to increased upwelling.

The storm surge activity is mainly due to wind stress effects and the effects of baroclinicity usually are negligible. Thus we do not expect any significant changes in the storm surge amplitudes after the construction of the hydroelectric power project.

It appears that the construction of the project may have noticeable effects on the baroclinic coastal jet in James Bay. Because of the reduced freshwater content, the density difference between the bottom and top layers may be reduced. Suppose ε is of the order of 5×10^{-3}, then the coastal jet will be somewhat weakened in the sense that it may have only a width of about 3.5 km.

Next we consider the effect of the project on the estuarine circulation in the central regime of the rivers. As was explained in Section 6 through equation (53), the circulation could be visualized as the combination of a river discharge mode, wind stress mode, and a gravitational convection mode. It is unlikely that the wind stress mode will be affected by the project. However, it is very likely that the dominant term will be the river discharge mode. Naturally the relative influence of this term varies not only with season but also from one river to the other. The gravitational convection mode may play an important role in determining the salinity pattern, noting that the strong tidal action will make some saline water intrude into the rivers.

One of the intractable problems a priori is the effect of the project on the local climate. Our approach to this is to

calculate the atmospheric water balance and compare this with the hydrologic water balance. In the equation for the hydrologic water balance given by (56) allowance is made for the change in water content due to man-made projects (L). However, in the atmospheric water balance equation given by (55) any consequence of human interference has to be felt only indirectly. Thus the only meaningful way to attempt to answer this is to calculate the atmospheric water balance over the river basin system using data before and after the power project construction.

8. Acknowledgements

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10. List of Figures